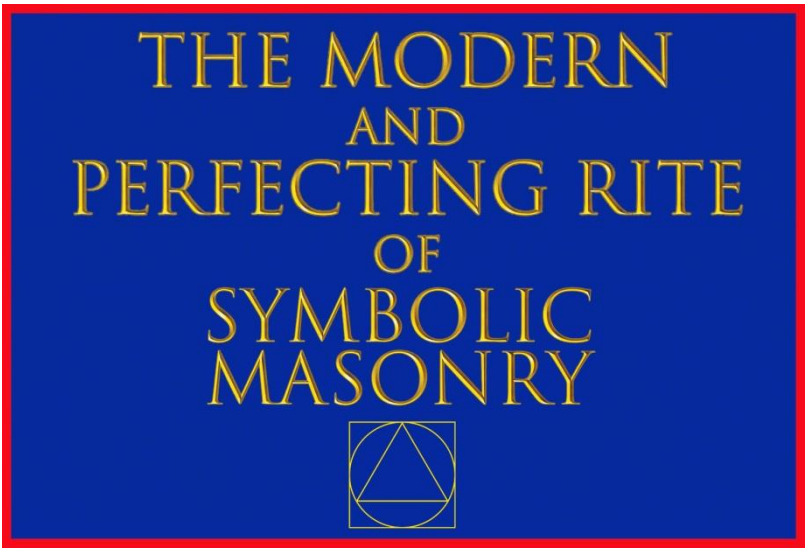

A METHOD OF PRACTICING THE DEGREES OF SYMBOLIC MASONRY AND OF THE ANCIENT AND ACCEPTED SCOTTISH RITE

By
Nicolas Laos

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PREFACE

"The world of ideas is not revealed to us in one stroke; we must both permanently and unceasingly recreate it in our consciousness."
René Thom, French mathematician (Fields Medal 1958).

By the term "Symbolic Masonry," I mean an initiatory organization that uses symbols, allegories, and dramas related to and inspired by ancient and medieval Operative Masonry in order to dramatize philosophical ideas, cultivate intellect, mold consciousness, teach a moral system, and forge fraternal relations between its members (the spirituality of Symbolic Masonry is enshrined in building terminology). Beyond the aforementioned definition of Symbolic Masonry, or Freemasonry, each particular Body of Symbolic Masonry (Freemasonic Obedience) has its own "landmarks" and ethos—in other words, there are different Freemasonries. For instance, the United Grand Lodge of England, the Grand Orient of France, the International Order of Co-Freemasonry Le Droit Humain, and the Swedish Order of Freemasonry comply with the aforementioned definition of Freemasonry, but each of them maintains different landmarks and initiatory customs and regulations from the rest. Just as the market of construction companies is pluralistic and competitive, so the market of symbolic construction companies—namely, organizations of Symbolic Masonry—is also pluralistic and competitive. When I study the history of Freemasonry, including the current state of affairs in Freemasonry, I do not see a battle between a unified "regular" Freemasonry versus splinter cells of "irregular" Freemasonries, but I see different varieties (and qualities) of Freemasonry all attempting to define their own social group. More specifically, defining the "in-group" and the "out-group." Furthermore, there are numerous "appendant" Masonic Orders, known as Orders "beyond the Craft" or Orders of the "higher degrees" (or, more accurately, the "side degrees"): these are initiatory organizations that seek to expand the degree structure and the teachings of Symbolic Masonry by superimposing other degrees on the basis of the three Craft degrees, and, understandably, they require that every candidate for admission must be in possession of the third and supreme degree of Symbolic Masonry, namely, a Master Mason. In its narrow sense, the term "Freemasonry" refers only to Symbolic Masonry, the "Craft," but, in its broader sense, it refers to both Symbolic Masonry and the group of

appendant Masonic Orders (ultra-Craft Orders). In my opinion, the following books are necessary readings in order for one to have a systematic and creative understanding of Freemasonry:

- Di Bernardo, G., *Filosofia della Massoneria e della tradizione iniziatica*, Venezia, Marsilio, 2016.
- Di Bernardo, G., *La conoscenza Umana: Dalla fisica alla sociologia alla religione*, Venezia, Marsilio, 2010.
- Di Bernardo, G., *Freemasonry: A Philosophical Investigation*, Pittsburgh, Pennsylvania, Dorrance, 2020.
- Di Bernardo, G., *La mia vita in Massoneria*, Amazon, 2021.

In general, Freemasonry is one of the most important constituent components of Western esotericism. The term "esotericism" derives from the Greek root "eso-," which means inner. Plato, in his dialogue *Alcibiades*, uses the expression "ta ēso," meaning "the inner things," and, in his dialogue *Theaetetus*, he uses the expression "ta ēxo," meaning "the outer things." The Greek adjective "esoterikōs" (esoteric) was coined by the rhetorician and satirist Lucian of Samosata (second century C.E.) in his book *The Auction of Lives* (paragraph 26). The term "esoteric" first appeared in English in Thomas Stanley's *History of Philosophy*, which was published in 1701. Thomas Stanley used the term "esoteric" in order to describe the mystery-school of Pythagoras, since the Pythagoreans were divided into the exoteric circle (under training) and the esoteric circle (admitted into the "inner" circle). The corresponding noun "esotericism" was coined by the French philosopher and historian Jacques Matter in his book *Histoire Critique du Gnosticisme* (1828), and it was popularized by the nineteenth-century French occult author and ceremonial magician Eliphas Lévi (born Alphonse Louis Constant).

From my perspective, esotericism, or rather the essence of esotericism (irrespective of the differences between particular esoteric institutions), consists of bearing witness to the reality of the human being and, particularly, to the autonomy of humanity, by focusing on the power of the intentionality of human consciousness and on the submissiveness of external reality to the intentionality of human consciousness. Hence, *I interpret esotericism as a program of personality creation and spiritual emancipation and as a manifestation of inner rectitude.* My approach to esotericism emanates from the argument that every object of consciousness exists not only *in itself* but also *for someone*, in the sense that it is united with the meaning that is assigned to it by consciousness. Every manipulation of the meanings that are assigned to beings and things, making the world meaningful, is equivalent to a manipulation of the reality of the world by consciousness. This is, according to my viewpoint, the

main underlying "secret" of esotericism. Furthermore, inherent in my previous argument is a powerful political message, since the intellectual and moral autonomy that is achieved through and underpinned by my interpretation of esotericism signifies a high level of personal and social autonomy, highlighting humanity as the creator and manager of its destiny. In fact, I am fascinated by Freemasonry in this context because of my aforementioned approach to esotericism and my conception of humanity as a creative and rational being. Thus, my approach to Freemasonry is deeply philosophical, and *I interpret Freemasonry as an esoteric system of dramatized philosophy and philosophical dramaturgy.* This is also my major point of differentiation from several contemporary Freemasonic organizations.

My Masonic Project

This book contains the Official Final Edition of the "Modern and Perfecting Rite of Symbolic Masonry" (M∴P∴R∴S∴M∴), which I have personally authored with a clear vision and a clear set of goals, which are fully explained in the present book. The Rites of Symbolic Masonry and the Ancient and Accepted Scottish Rite have all been revised and absorbed by the Modern and Perfecting Rite of Symbolic Masonry in a philosophically rigorous and consistent way.

In few words, what is the "Modern and Perfecting Rite of Symbolic Masonry" (M∴P∴R∴S∴M∴)? It is a method of practicing the degrees of Symbolic Masonry and of the Ancient Accepted Scottish Rite.

In particular, what I have done in this volume consists of the following threefold work:

- Firstly, I diligently consulted and applied several Freemasonic ritual books: rituals of the three degrees of Symbolic Masonry as it is practiced by the United Grand Lodge of England, the Grand Orient of France, the Grand Orient of Italy, and several Grand Lodges of the U.S.A.; rituals of the thirty-three degrees of the Ancient and Accepted Scottish Rite as it is practiced by the Supreme Councils of the Ancient and Accepted Scottish Rite for the U.S.A. (Southern and Northern Jurisdictions) and the Supreme Council for France that is affiliated with the Grand Orient of France; the rituals of the ninety-seven degrees of the Ancient and Primitive Rite of Memphis–Misraim that have been composed by John Yarker (1880s) and Allen H. Greenfield

(2014); the Aldersgate Royal Arch Ritual (as taught in the Aldersgate Royal Arch Chapter of Improvement under the auspices of the Supreme Grand Chapter of England); the rituals of the thirteen degrees of the York Rite as it is practiced in the U.S.A.; the rituals of the Grand Lodge of Mark Master Masons of England and Wales and Its Districts and Lodges Overseas; the rituals of the Grand Council of Royal and Select Masters of England and Wales and Its Districts and Lodges Overseas; and the rituals of the United Religious, Military and Masonic Orders of the Temple and of St. John of Jerusalem, Palestine, Rhodes and Malta of England and Wales and Its Provinces Oversees.

- Secondly, I fused the degrees of Symbolic Masonry, the degrees of the Ancient and Accepted Scotish Rite, and the English Order of the Holy Royal Arch of Jerusalem into one, unified Masonic system, which I called the "Modern and Perfecting Rite of Symbolic Masonry" (M∴P∴R∴S∴M∴), and it treats the degrees of the Ancient and Accepted Scotish Rite and the English Order of the Holy Royal Arch of Jerusalem as educational supplements to the third degree of Symbolic Masonry and as degrees with which we honor members of our Order who have worked in an excellent manner. The exact way in which I achieved this result is fully explained in this volume.
- Thirdly, I revised the rituals of the aforementioned degrees, and I authored the ritual, the lectures, and the catechisms of the Modern and Perfecting Rite of Symbolic Masonry—thus, creating a new Freemasonic method—in order to achieve the following goals: to rid Symbolic Masonry of any direct or indirect religious element and characteristic; to put Freemasonry in the service of a rigorously formulated cosmopolitan and humanistic philosophical vision; to make Freemasonry a mechanism for the creation and support of polymaths; to change the spiritual focus and strategy of Western esotericism and, particularly, of Freemasonry by bringing about a structural transition from old ritualism, mythology, and daydreaming to philosophy, psychoanalysis, sociology, politology, and positive science; to overcome the confusion that is caused by the existence of a multitude of different Masonic Rites, including different Masonic cultures and hundreds of different Masonic degrees; to eliminate the blind spots of contemporary Masonic institutions (caused by intellectual flaws, petty-bourgeois mentalities, vainglory, instituted corruption, psychological operations conducted by

particular power oligopolies, etc.); and to fuse the notions of an esoteric fraternity, a think-tank, a research institute, and a scientific club into a peculiar institution of "enhanced" Symbolic Masonry in the context of which thorough philosophical, theological, scientific, and political discussions and projects take place. In the present volume, you can read the official final edition of the rituals, the catechisms, the lectures, as well as the rules and the regulations of the Modern and Perfecting Rite of Symbolic Masonry, which, as I have already mentioned, consists of a substantial revision and a grand synthesis of the degrees of Symbolic Masonry, the degrees of the Ancient and Accepted Scotish Rite, and the English Order of the Holy Royal Arch of Jerusalem. The Modern and Perfecting Rite of Symbolic Masonry expresses a keen longing for unifying, all-embracing knowledge and for instituting a Freemasonic system that creates, unites, and supports polymaths for the sake of knowledge and a better world order. Thus, it includes an interdisciplinary system of education that spans philosophy, psychology, psychoanalysis, political theory, political economy, sociology, mathematics, and the natural sciences.[1]

[1] Given the philosophical, psychological, psychoanalytical, and political aspects of the Modern and Perfecting Rite of Symbolic Masonry, in order to compose the rituals of its three degrees, I consulted the following books: Bateman and Holmes, *Introduction to Psychoanalysis*; Borghini and Casetta, *Brill's Companion to the Philosophy of Biology*; Boyd, Gasper, and Trout, eds, *The Philosophy of Science*; *Cambridge Companions to Philosophy*; Coon, Mitterer, and Martini, *Introduction to Psychology*; Copleston, *A History of Philosophy*; W. Durant and A. Durant, *The Story of Civilization*; Edwards, ed., *The Encyclopedia of Philosophy*; Freud, *A General Introduction to Psychoanalysis*; Gabbard, Litowitz, and Williams, *Textbook of Psychoanalysis*; Gaus and D'Agostino, eds, *The Routledge Companion to Social and Political Philosophy*; Gelder, Gath, and Mayou, *Oxford Textbook of Psychiatry*; Hastings, ed., *Encyclopedia of Religion and Ethics*; Marcus and Mukherjee, eds, *A Concise Companion to Psychoanalysis*; *McGraw-Hill Encyclopedia of Science and Technology*; Sabine and Thorson, *A History of Political Theory*; B. J. Sadock and V. A. Sadock, *Kaplan and Sadock's Synopsis of Psychiatry*; Samuels, Biddle, and Davis, eds, *A Companion to the History of Economic Thought*; Shapiro, ed., *The Oxford Handbook of Philosophy of Mathematics and Logic*. Given the mathematical and other scientific aspects of the Modern and Perfecting Rite of Symbolic Masonry, in order to compose the rituals of its three degrees and, especially, in order to write Chapter 5, I consulted the following books: Altland and von Delft, *Mathematics for Physics*; Anton, *Elementary Linear Algebra*; Blei and Odian, *An Introduction to General Chemistry*; Fitzpatrick, *Advanced Calculus*; Fraleigh, *Calculus with Analytic*

I invented the Modern and Perfecting Rite of Symbolic Masonry, and I gave it this name in order to fuse Freemasonry, the spiritual core of modernity, and a vision of humanity's perfection into a consistent, secular, scholarly rigorous, historically aware, politically responsible, humanistic, and rational Masonic Order. The Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry advocates stern scholarly integrity and aims to historicize Freemasonry in order to create Freemasons capable of improving themselves and the world within the context of history, by becoming polymaths and morally responsible persons.

A peculiar esoteric steganography resides in the Modern and Perfecting Rite of Symbolic Masonry. In general, "steganography" means concealing messages and other information within non-secret texts or data, and ancient Greeks used to call steganography a page within a page, or writing within writing. By publishing this Rite, we disseminate a *corpus* of knowledge and a strategic vision as well as an invitation to an esoteric path, thus making an intervention in the public sphere, while maintaining an Initiatory Secret that is understood only by the persons who are initiated into this Rite. In the case of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, steganography means that there is a Ritual within the Ritual, an Initiatory Secret that can be accessed only from the inside, and the path to this Initiatory Secret starts with the study and the endorsement of the public and published aspects of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, which are contained in the presennt book.

The structure of this book

The present book tells a coherent story, and, therefore, one should start at the beginning and work through to the end.

- In **Chapter 1** (entitled "The Philosophy of the Modern and Perfecting Rite of Symbolic Masonry: The Complete Official Edition"), I delineate the intellectual underpinnings and the methodology of my work, my Masonic and scholarly credentials

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Geometry; Gowers, ed., *The Princeton Companion to Mathematics*; Haaser and Sullivan, *Real Analysis*; Lang, *Basic Mathematics*; Maurer and Ralston, *Discrete Algorithmic Mathematics*; Pedoe, *Geometry*; Sears, Zemansky, and Young, *College Physics*; Simmons, *Introduction to Topology and Modern Analysis*; Spiegelhalter, *The Art of Statistics*; Struik, *A Concise History of Mathematics*; Swokowski and Cole, *Algebra and Trigonometry with Analytic Geometry*; Toeplitz, *The Calculus*.

(which "legitimize" my bold and innovative Project), and my mindset regarding Freemasonry. **–pp.: 1–90.**

- In **Chapter 2** (entitled "Entered Apprentice of the M∴P∴R∴S∴M∴"), I expose and explain the ritual and the lectures of the First Degree of the M∴P∴R∴S∴M∴ (of which I am the author). Moreover, in this chapter, I expose and explain the structure as well as the fundamental rules and regulations of the Autonomous Order of the M∴P∴R∴S∴M∴. **–pp.: 91–226.**
- In **Chapter 3** (entitled "Fellow Craft of the M∴P∴R∴S∴M∴"), I expose and explain the ritual and the lectures of the Second Degree of the M∴P∴R∴S∴M∴ (of which I am the author). **–pp.: 227–289.**
- In **Chapter 4** (entitled "Master Mason of the M∴P∴R∴S∴M∴"), I expose and explain the ritual of the Third Degree of the M∴P∴R∴S∴M∴ (of which I am the author) as well as the rituals of the appendant degrees that our Order offers for the further training of Master Masons (namely, the rituals of the degrees 4º–33º of the Ancient and Accepted Scottish Rite as I have revised them in order for these degrees to fit the reasoning, the structure, and the overall ethos of the Modern and Perfecting Rite of Symbolic Masonry). **–pp.: 290–361.**
- In **Chapter 5** (entitled "A Course of Mathematics and Natural Sciences for the Members of the Modern and Perfecting Rite of Symbolic Masonry"), I expose and explain the basic scientific training that the Autonomous Order of the M∴P∴R∴S∴M∴ offers to all its members. **–pp.: 362–486.**
- **Bibliography –pp. 487–494.**

A∴  **P∴**

© Nicolas Laos, M∴P∴R∴S∴M∴ 3º-33º
Author and Grand Master of the
Modern and Perfecting Rite of Symbolic Masonry
Civis Mundi, April 2023.

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FREQUENT ABBREVIATIONS

In this book, I often use the following abbreviations for the sake of convenience:

A.A.S.R.: Ancient and Accepted Scottish Rite
A.D.C.: Assistant Director of Ceremonies
D.C.: Director of Ceremonies
E.A.: Entered Apprentice
F.C.: Fellow Craft
I.G.: Inner Guard
J.O.: Junior Overseer
L.G.C.: Lieutenant Grand Commander
M.M.: Master Mason
O.G.: Outer Guard
S.G.C.: Sovereign Grand Commander
S.O.: Senior Overseer
V.M.: Venerable Master
M∴P∴R∴S∴M∴: Modern and Perfecting Rite of Symbolic Masonry

Notice: Throughout this book, I follow the spelling "Kabbalah," but there also exist the following spellings: Qabal(l)ah (often preferred in the context of Western occultism) and Cabala(h) (often preferred by Christian Kabbalists—that is, in the context of Christian mysticism).

CHAPTER 1

THE PHILOSOPHY OF THE MODERN AND PERFECTING RITE OF SYMBOLIC MASONRY: THE COMPLETE OFFICIAL EDITION

In this Chapter, I shall refer to a few fundamental breakthroughs of my Masonic and scholarly journeys, which help one to understand the intellectual underpinnings and the ethos of my Masonic Project, namely, the creation of the Modern and Perfecting Rite of Symbolic Masonry, as well as the relation between this Masonic Project and the Enlightenment Project.

By the term "Enlightenment," we refer to a movement of intellectual change that penetrated Europe (and America) during the eighteenth century. It aimed, essentially, to emancipate human reason from the thraldom of prejudice and superstition (and especially from the feudal ethos and institutions) and to apply it to the cause of social and political reform. A common characteristic of the Enlightened thinkers was a belief in "progress." According to the idea of progress, humanity has begun its history in ignorance, squalor, and fear. Thereafter, it has risen slowly and continuously to ever-higher levels in the arts and sciences, its command of environment, and knowledge generally. Thus, the French epistemologist Gaston Bachelard maintains that rationality is a continuous process of overcoming primary impulses, and, in particular, he argues as follows: "In point of fact I see no solid basis for a natural, direct, elemental rationality . . . Rationalist? That is what we are trying to *become*."[1]

In its broadest sense, science is a system of behavior by means of which humans become masters of their environment. For this reason, no human society can exist without science. In a narrower sense, science is not so much a system of behavior as a system of knowledge which, specifically, aims to conceptualize, describe, and interpret the phenomena of the macrocosm and the microcosm according to a clearly determined and

[1] Bachelard, *The New Scientific Spirit*, p. 171.

robust method, as well as to create the necessary intellectual tools for understanding the aforementioned phenomena, logic, and mathematics.[2]

The origins of science can be traced to the Greek civilization that blossomed in Ionia in the sixth century B.C.E. In his *Metaphysics* (983b), Aristotle argues that the rigorous investigation of the nature of things started with the work of Thales of Miletus. Thales of Miletus and other Ionian intellectuals, such as Anaximander and Anaximenes, utilized more ancient knowledge and theories, both Greek and Oriental, but their thought followed a new, totally different path, signaling a radical departure from the past. For this reason, according to the prevailing consensus among historians of science and philosophy, the origins of science and philosophy can be traced to the "Ionian school." The two most important characteristics that separate the Ionian intellectuals from their predecessors are the following: firstly, the discovery and rigorous formulation of natural laws and, secondly, the exercise of rational criticism and dialogue.[3]

Ancient Greek intellectuals overcame the immobility of myth and the traditional collective mode of perception, and they set human thought in motion according to the rules and methods of science and philosophy. Moreover, ancient Greek intellectuals emerged as distinct, *eponymous* persons, as self-conscious "othernesses," breaking the web of anonymity that covered the collective thinking of the most archaic societies. In fact, in ancient Greek history, we can find thousands of names of Greek intellectuals. Ancient Greek philosophy, in particular, transformed ancient *empirical* theories of nature into *conceptual* ones. The Pythagorean fraternity was one of the major pioneers of the aforementioned intellectual trend, which, in ancient Greece, culminated with the philosophy of Aristotle and the scientific treatises of Euclid, Archimedes, and Apollonius of Perga.

Aside from my argument that we should hold the content of Freemasonry to rigid scientific and philosophical standards, I hold the view that we should maintain a similar attitude towards occultism in general, pursuing a creative reconciliation between reason and myth. Let me, for example, bring to memory that the French occultist and alchemist François Jollivet-Castelot (1874–1937) has explained the meaning of magic as follows:

> Magic is by no means, as most outsiders imagine, the negation of Science. Quite on the contrary Magic is *Science*, but Science with syntheses, almost integral Science, its horizons being the Absolute, the Infinite in Unity . . . In truth *Magic is the knowledge of the action and the combination of the*

[2] Clagett, *Greek Science in Antiquity*.
[3] Lloyd, *Early Greek Science*.

> *forces of the Universe . . . the study of their conduct, their involution, their evolution*.[4]

Hence, the quaternary of ancient magic: "Know, dare, will, keep silent." From the aforementioned "magical" perspective, the Egyptian Sphinx (a mythical animal with a human head, a woman's breast, the loins of a bull, the claws of a lion, and the wings of an eagle) can be interpreted as follows: its human head symbolizes intelligence and knowledge; its claws symbolize daring and action; its loins symbolize will-power, perseverance, and labor; its folded wings symbolize silence.

In addition, I should stress that Plato utilized myth in his philosophical works because he realized that, in the context of a myth, knowledge is not the result of a static representation. Instead, due to a myth's plot, it is an itinerary towards the reason of beings, or the "logos" (in ancient Greek, the term "logos" means both thought and language). Myth does not serve ideas in a passive way, but, due to its plot, it endows ideas with inner life. Thus, myth is not an allegory, which is something intrinsically static; myth is actually a symbol. Instead of simply referring to something external, a symbol shows, within itself, what the symbolized object is. In other words, a symbol is not an exoteric, formalist reproduction of the symbolized object, but it participates in the spiritual reality (significance) of the symbolized object without, however, encompassing the entire reality of the symbolized object. The knowledge that derives from symbols is always characterized by a combination of faith, intuition, and reason. Therefore, it can be called "holistic." Plato's encounter with this holistic type of knowledge took place in the context of his initiation into the Orphic Mysteries; and, in fact, his philosophy is an attempt to transform the mythological language of the Orphic Mysteries into a philosophical one and, thus, to formulate the wisdom of the Orphic Mysteries in terms of philosophical concepts and syllogisms, and to develop it further in a rational and rigorous way, while maintaining a holistic approach to knowledge. My conception of Freemasonry as "an esoteric system of dramatized philosophy and philosophical dramaturgy" is underpinned by the aforementioned Platonic understanding of holistic knowledge, in the context of which myth is united with reason in a rational manner, in a fusion without admixture or confusion.

[4] Quoted in: *The Encyclopedia of Occult Sciences*, p. 305.

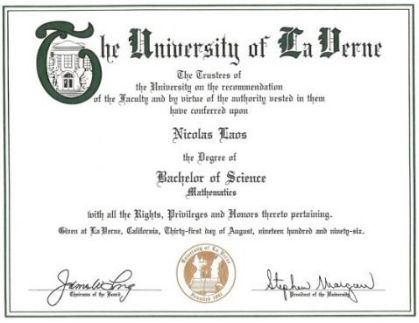
My journey in Mathematics and Philosophy

The two major pillars of my education are Mathematics and Philosophy. Regarding my competence in mathematics, mathematical modelling, and the epistemology of the natural sciences, I would like to acknowledge the contribution of the following persons to my scientific education: Dr. Themistocles M. Rassias (Fellow of the Royal Astronomical Society of London and Accademico Ordinario of the Accademia Tiberina in Rome), who taught me Calculus I, II & III, Advanced Calculus, Linear Algebra, and Differential Equations, and supervised my research work in the foundations of mathematical analysis and differential dynamics at the University of La Verne, where I completed my studies in mathematics (a part of the research work and the dissertation that I completed at the University of La Verne under the supervision of Dr. Th. M. Rassias was published in 1998 as the volume no. 24 of the scientifically advanced Series in Pure Mathematics of the World Scientific Publishing Company); Dr. Christos Koutsogeorgis, who taught me Discrete Mathematics, Abstract Algebra, and Probability Theory (University of La Verne, 1994–96); and Dr. Chamberlain Foes, who taught me PASCAL (programming language) and introduced me to mathematical informatics and management information systems (University of La Verne, 1995).

Furthermore, my approach to esotericism is underpinned and determined by my education in mathematics and by my work as a consultant and trainer in mathematical modelling. My love for mathematics derives from the fact that mathematics is inextricably linked to inner life and, specifically, to deep and systematic thinking. Additionally, inherent in mathematics is a sense of mystery, in the sense that, in mathematics, we play with concepts and syllogisms and try to assemble them according to concrete rules of correctness in order to create models of reality. In the context of a mathematical proof, we may use several techniques, such as direct proof (involving arguing step by step, starting from what we know until we have demonstrated the truth of some conclusion), mathematical induction, counterexamples (since a single counterexample suffices to prove that a statement claiming necessity and universality is wrong), *reductio ad absurdum* (i.e., the form of argument that attempts to prove a statement by proving that the negation of the given statement leads to absurdity or contradiction), proof by contraposition (i.e., inferring a conditional statement from its contrapositive; the contrapositive of the statement "if A, then B" being the statement "if not B, then not A"), etcetera. In addition, the concept of a mathematical proof is inextricably

linked to the concept of a definition—that is, to a deep and rigorous understanding of the essence of the object under consideration.
In fact, the beauty, the peculiar aesthetics of mathematics, which emanates from the art of proof and reasoning, has led me to philosophy, which, in turn, has led me to the greatest heights of intellectual abstraction and to the greatest depths of the psychic apparatus.

Figure 1-1: Dr. Nicolas Laos at the University of La Verne. On the left: Dr. Nicolas Laos's degree in Mathematics from the University of La Verne, California. On the right: Dr. Laos (on the left) with Professor Themistocles M. Rassias (on the right), who supervised Dr. Laos's dissertation on mathematical analysis and differential geometry, at the graduation ceremony, on 31 August 1996, at the University of La Verne.


The University of La Verne

The Trustees of
the University on the recommendation
of the Faculty and by virtue of the authority vested in them
have conferred upon

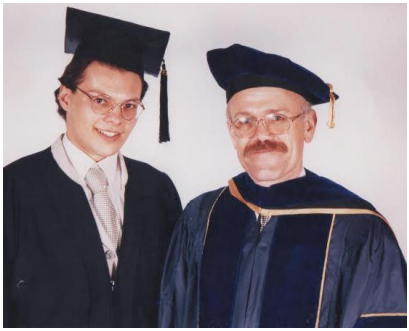
Nicolas Laos

the Degree of

Bachelor of Science
Mathematics

with all the Rights, Privileges and Honors thereto pertaining.

Given at La Verne, California, Thirty-first day of August, nineteen hundred and ninety-six.



No material object or system of objects—nor any connection or interaction that exists between them in material reality—is the direct object of mathematical study. In order for mathematical tools to be used to study the processes, the phenomena, and the individual objects that exist in reality, it is necessary to construct the corresponding mathematical models.

By the term "mathematical model," we mean a system of mathematical relations that symbolically describes the processes or the phenomena under study. For the construction of mathematical models, a variety of mathematical tools are used—such as: equations (algebraic, differential, and integral ones), graphs, matrices and determinants, relations of mathematical logic, geometric constructions, etc. In fact, the basic type of mathematical activity, the fundamental problem of mathematics, is the construction, the study, and the application of mathematical models.

No model can represent all the properties and all the relations of the original object. In other words, a model is a simplification, an approximate representation of the original object, but, simultaneously, a model highlights and describes an important pattern of the properties and the

relations of the original object. The dialectical process of knowledge of reality consists of two processes: firstly, the replacement of existing models by others in which a more complete representation of the properties of the original object is achieved; and, secondly, the combined application of various models.

As I have already mentioned, mathematics is concerned with the construction of such models of objects (namely, of things, processes, and phenomena) that reflect the corresponding objects' quantitative and/or qualitative attributes as well as their spatial and structural peculiarities. For instance, geometry is the scientific study of the quantitative and the qualitative properties of spatial forms and relations (the criteria for equality of triangles provide instances of qualitative geometric knowledge, and the computation of lengths, areas, and volumes exemplifies quantitative geometric knowledge).

The constituent elements of a model are symbols and signs. Symbols are forms that express commonly accepted intentions and actions, and they can be organized into particular systems that are called codes, and the elements of such a code are called signs. In the context of mathematical modelling, the character of these signs can vary, since these signs can be schematic images (namely, shapes, drawings, and graphs), collections of numerical symbols, and elements of artificial or natural languages. Furthermore, symbols are subject to transformations according to specific symbol transformation rules. The symbols and their transformations are definitely interpreted in terms of the original objects. The combinations of symbols used and their transformations are dictated and determined by the properties of the original objects and by the associations selected and included in the corresponding model.

Mathematical models—which, with the help of the human senses, are directly extracted from material objects—usually express the primary simplest abstractions of a quantitative and spatial character, such as, for instance, enumeration, dimensions, form, position in space, etc. If a human being relies only on the sense organs, then he/she cannot achieve deep knowledge of the world around him/her nor of his/her inner world. Nature, acting on the sense organs, can only produce in humans a limited set of sensations, impressions—namely, that type of knowledge which we call "empirical."

The accumulation of empirical data constitutes the basis of generalizations and abstractions. The formulation of generalizations and abstractions provides the intellectual setting in which the application of mathematical tools becomes possible and meaningful. In the course of the historical development of mathematics, the construction of models of

increasingly complex systems has been achieved, including systems that consist of multiple abstractions. With regard to its theoretical essence, mathematics can be construed as a science of modelling; and, therefore, both the reality of the world and the reality of consciousness are fundamental to mathematics.

The demand for computational precision goes hand in hand with the demand for conceptual precision and logical rigor. As the renowned French mathematician and philosopher René Thom has pointed out—in mathematics and, generally, in science—in addition to descriptive accuracy, explanatory accuracy is also required. Furthermore, Cybernetics and Systems Science have given rise to a transdisciplinary approach to scientific modelling, since they are characterized by an attempt to build general, domain-independent theories.

The scientifically rigorous conception of mathematical modelling is based on the concepts of homomorphism and isomorphism. In mathematics, the term "homomorphism" describes the transformation of one data set (or "system") into another while preserving relationships between elements in both sets. In other words, a homomorphism is a structure-preserving mapping. The mathematical term "isomorphism" is more specialized, since it refers to a structure-preserving mapping between two systems of the same type that can be reversed by an inverse mapping. In other words, an isomorphism is a special type of homomorphism, a bijective homomorphism. In fact, homomorphisms can lose some information about the object, but isomorphisms always preserve all the information. In view of the foregoing, a general definition of a mathematical model can be formulated as follows: Given two data sets, or systems A and B, each is a model of the other if there exist a homomorphism from data set A to a data subset X of B and a homomorphism from data set B to a data subset Y of A, where systems X and Y are isomorphic to each other.

According to such renowned mathematicians and logicians as Jacques Hadamard, Nicolas Bourbaki (the collective pseudonym of a group of mathematicians), René Thom, Hermann Weyl, Ljubomir Iliev, Andrey Kolmogorov, and Leonid Kantorovich, the order of operations involved in the construction of mathematical models can be summarized as follows:

1. Determining and formulating the problem as clearly as possible.
2. Identification of the variable quantities that determine the process under study or are chosen for the study of the given problem.
3. Definition of the relations between these variables and the parameters on which the state of the process under study depends.

4. Formulation of a hypothesis (or hypotheses) about the nature of the conditions under study.
5. Construction of the model so that its properties coincide with the initially defined ones.
6. Conducting experimental tests.
7. Checking the hypothesis accepted for the construction of the model, and evaluating it according to the outcome of experimental tests.
8. Acceptance, rejection, or modification of the hypothesis on the basis of repeated experimental tests and conclusions.

How could one describe and explain the relationship between mathematics and philosophy? Moreover, how could one explain the role that inquisitiveness and the pursuit of scientific rigor play in the development of mathematical thought?

First of all, I should mention that the symbolic language of mathematics is equipped with rules for handling concepts. In addition, the logical construction of mathematical models is rigorously determined in the context of, and my means of, a hypothetico-deductive system. In a "hypothetico-deductive" (or "axiomatic") system, there are two requirements that must be met in order that we agree that a proof is correct: (i) acceptance of certain statements, called "axioms," without proof, on the basis of their intrinsic merit, or because they are regarded as self-evident; and (ii) agreement on how and when one statement "follows logically" from another, that is, agreement on certain rules of reasoning. Inextricably linked to the aforementioned two requirements is the requirement that every person who applies hypothetico-deductive reasoning to a particular discourse understands the meaning of the words and the symbols that are used in that discourse. The more consistent and the more complete a hypothetico-deductive system is, the more its imposition is safeguarded. By the term "consistency," we mean that the axioms of a hypothetico-deductive system neither contain nor produce contradictions. By the term "completeness," we mean that the truth value of any proposition that belongs to a hypothetico-deductive system can be determined within the given hypothetico-deductive system (that is, according to the terms and the rules of the given hypothetico-deductive system). All these are philosophical questions.

In general, there is a close affinity between mathematics and philosophy. Mathematics, like philosophy, is done by consciousness. Mathematics provides a model of knowledge of a particular kind, and, in fact, philosophers have highlighted the particular nature of mathematical

knowledge and have argued that all knowledge could possibly aspire to the particular nature of mathematical knowledge. According to the German mathematician and philosopher Friedrich Ludwig Gottlob Frege, unlike other kinds of knowledge, mathematical knowledge is characterized by rigor and objectivity, because mathematics is constituted as a logical system.

The model of knowledge that is provided by mathematics has the following characteristics: (i) certainty (in the sense that, if something is true and known in mathematics, then it is undoubted), (ii) incorrigibility (in the sense that the development of mathematical knowledge is internally consistent), (iii) eternity (in the sense that mathematical knowledge is not subject to time), and (iv) necessity (in the sense that mathematical truths are not contingently true but necessarily true). Being aware of these attributes of mathematical knowledge, Plato had the phrase "Let no one ignorant of geometry enter" engraved at the door of his Academy. In the context of Plato's philosophy, geometry is concerned with the understanding of the reason ("logos") of the world. Thus, *Plato*, in his *Republic*, 527c, argues that "geometry is the knowledge of the eternally existent," and that, therefore, geometry "would tend to draw the soul to truth, and create the spirit of philosophy, and would be productive of a philosophical attitude of mind."

One of the reasons why ancient Greeks regarded geometry, rather than arithmetic, as the more foundational and superior branch of mathematics is the crisis that erupted in the foundations of Pythagoras's mathematical theory. Pythagoras is famous for finding out that, for any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. However, this geometric theorem, which is known as the Pythagorean Theorem, presented a problem to Pythagoras and his disciples. Pythagoras was a philosopher, a mathematician, and a sort of cult leader in ancient Greece. Pythagoras and his school (the so-called "Pythagoreans") were dedicated to the mysticism of numbers, and they maintained that everything in the world could be expressed as a number. But they had a rather inadequate understanding of the concept of a number.

From the Pythagorean perspective of mathematics, the relations between the objects of the world (e.g., magnitudes) correspond to the relations between natural (and, generally, integral) numbers. However, it was soon realized that things are not so simple, since it was realized that there exist magnitudes that do not have a common measure. According to the Pythagoreans, two objects (magnitudes) are "commensurable," that is, they have a common measure, if and only if there is a magnitude of the

same kind contained an integral number of times in both of them. In other words, two magnitudes are "commensurable" if and only if their ratio is a rational number. However, the Pythagoreans encountered "incommensurable" magnitudes, namely, magnitudes whose ratio is an irrational number. For instance, the length of a diagonal of a unit square, namely, of a square whose sides have length 1, is, according to the Pythagorean Theorem, equal to $\sqrt{2}$, which is an irrational number; similarly, a circle's circumference and its diameter are incommensurable. Pythagoras swore his disciples to secrecy about the existence of irrational numbers. Nevertheless, the awareness that there exist incommensurable magnitudes compelled ancient Greek mathematicians to inquire into the relations between incommensurable magnitudes. This event marked a major crisis in ancient mathematics.

According to ancient Greek mathematicians, quantities (magnitudes) are continuous and uniform objects, which are best represented by straight line segments, whereas their division into parts, namely, their measurement in terms of a "unit of measurement" (that is, a definite magnitude of a quantity), represents the notion of discreteness. Ancient Greek mathematicians used the term "ratio of magnitudes" in order to refer to the relation between two magnitudes that can be measured in terms of a common unit of measurement, and, thus, the ancient Greek concept of a ratio is most similar to the more abstract modern concept of a number. In the context of ancient Greek mathematics, the objects of mathematics were quantities (represented by straight line segments), and the ratio between two quantities was a meta-object, namely, something that was used in order to study mathematical objects without being treated as a mathematical object itself. In other words, in the context of ancient Greek mathematics, a ratio (namely, a number) was construed as a measuring relationship between two quantities, and such a measuring relationship could be built up (and, hence, proved) in finitely many steps, using a common unit of measurement. Nevertheless, the discovery of incommensurable ratios demonstrated that a ratio could not be interpreted as a measuring relationship in the aforementioned way. In fact, as a result of the discovery of incommensurable ratios, the concept of a ratio (or a number) acquired its conceptual autonomy, and, instead of being treated as a meta-object, it started being treated as an object of mathematics. Therefore, ancient Greek mathematicians had to transcend the system of mathematics that was based on commensurable ratios (note that a commensurable ratio could easily become an object of mathematical theory, since it is a rational number, and, therefore, it can be constructed in finitely many steps, whereas the decimal representation of an irrational

number neither terminates nor infinitely repeats but extends forever without regular repetition).

The discovery of irrational numbers undermined the faith in numbers as the foundational conceptual system of mathematics, and led to the belief in the superiority of geometry. The belief in the superiority of geometry was reinforced by the realization that, in geometry, an irrational number can be constructed—as is the case, for example, when we draw a diagonal of a unit square, namely, $\sqrt{2}$—whereas, in arithmetic, an irrational number, such as $\sqrt{2}$, cannot be expressed as a ratio of two whole numbers. The aforementioned crisis in the foundations of mathematics was ultimately overcome by Eudoxus's theory of proportions and by the method of exhaustion, which derives from Eudoxus's theory of proportions, and it was used by Archimedes.

Indeed, the overcoming of crises in the foundations of mathematics is intimately related to set-theoretical concepts and axioms and to the concept of infinity in particular. New philosophical problems arise.

The concept of modern mathematics that is semantically most similar to Aristotle's concept of a "potential infinity" is the convergence of a sequence of natural numbers. Thus, from the perspective of ancient Greek mathematics, infinity is not a being (specifically, it is not an actual state)—namely, it cannot be simultaneously considered in its whole extension—but it can only be considered as a becoming (specifically, as a process). In this way, the concept of an infinite approach helps us to overcome the contradiction between incommensurable ratios and commensurable ratios, since we can think of an incommensurable ratio infinitely approaching a commensurable ratio and vice versa. Similarly, the concept of an infinite approach helps us to overcome the contradiction between broken lines and curves as well as the contradiction between continuity and discreteness. This reasoning is endorsed by Euclid, and, therefore, in his *Elements*, he does not consider infinitely extended straight lines, but he always works with straight line segments, which, as he says, can be extended as much as one needs. Moreover, in view of the foregoing, Archimedes was very careful in the use of infinite processes, and, therefore, he approximated the irrational number π (i.e., the ratio of a circle's circumference to its diameter) by using the fact that the circumference of a circle is bounded by the perimeter of an *inscribed* polygon and the perimeter of a *circumscribed* polygon. According to Eudoxus's theory of proportions and Archimedes's method of exhaustion—which incorporate the Aristotelian concept of a "potential infinity" and the modern mathematical concept of a convergent sequence—there is always a ratio between any two

magnitudes, and we can always make any magnitude smaller or greater than a given magnitude.

Whereas, from the perspective of ancient mathematicians, numbers are things by means of which we count, Cartesianism (as the intellectual "school" of Descartes is known) is based on the algebraization of geometry, thus giving rise to the idea that numbers can be thought of as positions on the number line. Fusing geometry and arithmetic is an arduous task. In order to understand the difficulties that originate from fusing geometry and arithmetic, let us consider, for instance, the famous irrational number $\sqrt{2}$, which was discovered by Pythagoreans when they attempted to compute the length of a diagonal of a unit square. The Pythagoreans realized that the diagonal of a unit square is not commensurable with the side of the given square, but, by keeping geometry and arithmetic separate from each other (that is, by refusing to identify numbers with lengths of straight line segments), ancient Greek mathematicians could argue as follows: given a straight line segment whose length is one, we can construct a straight line segment whose length is $\sqrt{2}$, and, in general, irrational numbers are geometrically constructible (and, hence, geometrically explicable and manageable), even though, from the perspective of arithmetic, irrational numbers are ideal quantities, in the sense that the calculation of irrational numbers (such as $\sqrt{2}$) is an infinite process (since irrational numbers have infinitely many decimal digits). On the other hand, in the nineteenth century, having endorsed the Cartesian approach to mathematics, mathematicians realized that they had to clarify some still ambiguous fundamental concepts (such as that of a real number), to formulate new methods of doing mathematics in a logically rigorous way, and to create a rigorous theory of the arithmetic continuum, specifically, a rigorous theory of real numbers and their arithmetic.

Another important crisis in the foundations of mathematics broke out in the seventeenth century. Whereas ancient Greek mathematics (as it is expounded and systematized by Euclid in his *Elements*) is based on a geometric way of thinking, modern European mathematics is more inclined to an algebraic way of thinking (and, hence, it tends to give primacy to arithmetic over geometry). This shift was typified by the reduction of geometry to arithmetic in the context of analytic geometry, which is characterized by the use of coordinates and by the correspondence between curves and equations. The use of coordinate systems implies that space itself is encoded by n-tuples (namely, by sequences, or ordered lists, of n numbers), and, specifically, that the 2-dimensional space, the "plane," is encoded by pairs of numbers, so that the conception of space becomes subordinate to the conception of arithmetic.

In the seventeenth century, mathematicians (primarily the initiators and developers of calculus) were preoccupied with such geometric problems as the computation of areas and volumes of arbitrary geometric figures and the construction of tangents to curves as well as with such physical problems as the formulation of the law that determines the rate of change of velocity and of acceleration (with respect to time) when one knows the law that determines the rate of change of displacement of an object (namely, its velocity) and vice versa. The tendency towards the study of the aforementioned types of geometric and physical problems was reinforced by Galileo's physical theory, which constrained Aristotle's theory of motion (according to which the term "motion" referred to any kind of change, development, and growth) to the study of change in the relative position of physical objects.

The most prominent seventeenth-century mathematicians realized that, when we treat geometric figures and the motions of physical bodies as "wholes," we cannot demonstrate significant apparent similarities between them, but, when we analyze them into (sufficiently) "small" pieces, they display great similarities to each other. Hence, the major problem of seventeenth-century mathematics consisted of determining the proper processes for dividing the "whole" into "small" parts, which would be more easily and more rigorously studied than the "whole," as well as of determining the proper processes for resynthesizing the behavior of the "whole" from the behavior of its "small" parts. However, the "small" parts into which an object of scientific research is divided are similar to the "small" parts into which another object of scientific research is divided, and, thus, they give rise to generalizations (such as natural laws), only when the dimensions of such "small" parts tend to zero, and, thus, only when the number (namely, the population size) of such "small" parts tends to infinity. Therefore, the need for the use of infinite processes, specifically, infinitesimals, became prominent again.

Even though "infinitesimal methods" could lead to correct results and useful applications, they lacked the logical rigor that characterized ancient Greek mathematics, particularly, Euclid's *Elements*, and they were susceptible to contradictions. Some mathematicians argued that lengths consisted of (infinitely many) infinitesimal lengths, areas consisted of (infinitely many) infinitesimal areas, and volumes consisted of (infinitely many) infinitesimal volumes, while other mathematicians argued that lines consisted of an infinite number of points, surfaces consisted of an infinite number of lines, and solid bodies consisted of an infinite number of surfaces. In that era, namely, in the seventeenth century, the mathematical concept of a limit was not yet clarified. It is worth pointing out that the

famous French Enlightenment scholar Voltaire described infinitesimal calculus as "the art of measuring exactly a thing whose existence cannot be conceived," thus expressing his bewilderment at the fact that the seventeenth-century infinitesimal calculus was a useful and powerful scientific instrument, but the actual things that it was talking about were almost beyond conception (quoted in: Andrew Simoson, *Voltaire's Riddle*, U.S.A.: The Mathematical Association of America, 2010, p. 51).

In infinitesimal calculus, the usual derivative, denoted by $\frac{dy}{dx}$, is an operator (actually, a function) that describes how a function $y = f(x)$ changes relative to its argument x. Newton defined the derivative of a function as the "ultimate ratio" of "vanishing quantities," and Leibniz argued that the quantities dy and dx, which appear in the definition of the derivative of a function, are infinitely small yet non-zero quantities. These ambiguities ignited heated debates regarding the foundations of infinitesimal calculus. In fact, the major problem pertaining to the development of infinitesimal calculus in the seventeenth and the eighteenth centuries was the reduction of a continuous entity, namely, a "whole," to discrete entities, namely, infinitesimals (meaning infinitely small parts of the corresponding "whole"), by means of a non-well-defined concept, namely, the concept of infinity. However, the effectiveness of the application of infinitesimal methods to physics and astronomy played a significant role in the acceptance and the further development of infinitesimal calculus. In general, many eighteenth-century mathematicians drew their subject matter from many branches of physics, astronomy, navigation, cartography, commerce, and finance. Infinitesimal calculus was put in a rigorous conceptual setting by the French mathematician Augustin-Louis Cauchy (1789–1857), who explained the concept of a limit of a function in a clear, formal, and arithmetic, rather than geometric, way by arguing as follows: "when the successive values attributed to a variable approach indefinitely a fixed value so as to end by differing from it by as little as one wishes, this last is called the limit of all the others" (quoted in: Carl B. Boyer, *The History of Calculus and Its Conceptual Development*, New York : Dover, 1959, p. 272).

In view of the peculiar model of knowledge that is provided by mathematics, as I have delineated it, the following question emerges: which are the underpinnings of mathematical knowledge, and what exactly is it that endows mathematical knowledge with the characteristics that I have mentioned—namely, certainty, incorrigibility, eternity, and necessity?

Firstly, we have to consider mathematical Platonism. According to mathematical Platonism, numbers are forms, specifically, abstract,

objectively existing objects. This thesis seems to be corroborated by the fact that numbers are not intrinsic characteristics of objects, but they are applicable to objects, and they seem to be the contents of objective truths, irrespective of any contingency and any particular object of the sensible world. From this perspective, numbers are objects themselves. In particular, according to mathematical Platonism, numbers are a peculiar kind of objects, since they exist objectively, but they cannot be grasped by the senses, they are not part of the material space-time, and they are not subject to the laws of material space-time. Far from negating the thesis that numbers are objects, the fact that numbers are not subject to the spatio-temporal structure of our sensible world corroborates the Platonic thesis that the world of forms is the reality *par excellence*, which underpins the logical constitution of our sensible world, which, in Platonic parlance, can be regarded as a "shadow" of the world of forms. This reasoning underpins the Platonic argument that, whereas the knowledge that is provided by the senses is subject to revision, the knowledge that is provided by forms, such as numbers, is incorrigible; and, therefore, reason ("logos"), which consists of thought and language, is superior to the senses. This is how mathematical Platonism explains the peculiar characteristics of the mathematical truth—namely, the certainty, the incorrigibility, the eternity, and the necessity of the mathematical truth.

Mathematical Platonism is a variety of dualistic realism. In philosophy, the term "realism" refers to a philosophical model that is based on objectively existing objects, thus giving primacy to a consciousness-independent world, as opposed to "idealism," which gives primacy to the reality of consciousness. According to philosophical realism, the fact that experience furnishes consciousness with images—even unrelated to each other—of a reality that seems to lie outside the dominion of consciousness implies that the reality of the world is the cause of the particular images of the world that are present within consciousness. From the realist perspective, the principle of causality points us in the direction of the claim that the autonomous existence of reality is naturally and logically necessary. Even though the aforementioned reasoning is sound, dualistic realism, with its doubling of the world, leads to contradictions and logical gaps, especially regarding the existence of, and the relationship between, the world of forms and the world of "shadows," namely, the forms' sensible images.

Aristotle attempted to overcome the contradictions and the logical gaps of Plato's dualistic realism by reformulating dualistic realism in a way that does not depend on a Platonic doubling of the world and bridges the gap

between the world of forms and the human mind. In particular, Aristotelianism highlights the structural mode of being.

The cohesive bond between substance and form is the structure of a being. The deepest reality of a being is its substance, the external aspect and the existential otherness of that reality are the form of the given being—namely, an element that animates the given being—and these two elements (modes of being) concur with each other in the context of the structural mode of being. From the perspective of structuralism, Platonic realism corresponds to the *ante rem* structuralism ("before the thing"), in the sense that, according to Platonism, the ideational structure of mental life is a real but transcendent principle vis-à-vis the mind itself and the sensible world, and philosophical consciousness tries to partake of and progress in the world of forms, while Aristotelian realism corresponds to the *in re* structuralism ("in the thing"), in the sense that, according to Aristotelianism, structures are held to exist inasmuch as they are exemplified by some concrete system, and the mind itself, not the world of forms, is a real and transcendent principle vis-à-vis the sensible world, and it conceives forms as abstractions. According to Plato's dualistic realism, forms are objectively existing objects, of which the objects of the sensible world are images, or "shadows." According to Aristotle's dualistic realism, forms are mental abstractions, the objects of the sensible world are material exemplifications, materializations of forms, forms are conceived by the mind, and the mind, rather than the world of forms itself, is transcendent to the sensible world. For this reason, Aristotle argued that the mind is the "entelechy"—that is, the program of actualization—of the body, generally, of the human organism.

According to mathematical Aristotelianism, mathematics refers to truths of the sensible world, in the sense that, even though numbers are not sensible things, they are properties of sensible things—specifically, abstract entities which can be predicated of sensible things. In other words, numbers are not objects themselves, they do not exist independently of objects, but they are features of objects, and they exist within objects. For instance, when we see ten people, the number ten is a property of the given collection of people that we see.

In the context of mathematical Aristotelianism, numbers are not self-subsistent forms, objects, but still numbers are properties of other things in an objective way. In general, according to Aristotle and according to Thomas Aquinas's variety of Aristotelianism (in the context of medieval scholasticism), consciousness is a passive mirror of reality, and truth refers to an objective correspondence between thinking consciousness and its object. But Descartes reversed the aforementioned relation between the

intellect and its object, arguing that understanding (or intellection) is the basic reality, and that understanding is activated by conceiving itself; hence, Descartes's famous *dictum*: "cogito ergo sum," meaning "I think therefore I am." By assigning this active role to consciousness, Descartes emerged as the rigorous initiator and founder of modern philosophy.

Gradually, modern philosophy gave rise to a new general model, which is known as idealism. According to modern philosophical terminology, there are two general models whereby philosophers interpret the world: one gives primacy to the reality of the world, and it is known as philosophical realism, whereas the other gives primacy to the reality of consciousness, and it is known as philosophical idealism. According to idealism, the nature of consciousness is not totally different from or opposite to the nature of extra-conscious reality. The idealists' way of thinking can be summarized as follows: if the nature of reality were totally different from the nature of consciousness, then the human being would be unable to know reality. Thus, ultimately, idealism construes and studies the world not as something reflected in consciousness, but as an extension and a projection of consciousness outside itself and as part of consciousness.

In the nineteenth century, the German mathematician and philosopher Friedrich Ludwig Gottlob Frege departed from the traditional realist philosophy of mathematics, and, in contrast to mathematical Aristotelianism, he argued that, even though mathematical knowledge is objective, numbers are not objective, consciousness-independent properties of other things. According to Frege, any number n can be used in order to count any n-membered set, but the formulation of a claim concerning which number belongs to a set is determined by the way in which mathematical consciousness conceptualizes that set. For instance, consider the Tarot. The Tarot consists of 78 cards. Moreover, it has two distinct parts: the Major Arcana, consisting of 22 cards without suits, and the Minor Arcana, consisting of 56 cards divided into 4 suits of 14 cards each. Depending of whether we are thinking in terms of Tarot cards in general, or in terms of the Major Arcana Tarot cards, or in terms of the Minor Arcana Tarot cards, or in terms of the suits of the Minor Arcana Tarot cards, different numbers will belong to that particular set of cards. Hence, we have to decide if that particular set has the property 78, or the property 22, or the property 56, or the property 4. Similarly, a pair of shoes is one pair of shoes, but it consists of two shoes, and, therefore, we have to decide which number belongs to this physical object: the number one or the number two. Thus, according to Frege, numbers are not objective properties of objects, but objects acquire numbers as properties according

to the ways in which consciousness thinks of the corresponding objects. Frege's argument about the active role of consciousness in mathematical creation—especially in the light of Kant's philosophy—may lead one to the conclusion that we have to do away with mathematical objectivity completely. Before explaining the way in which Frege prevented mathematical philosophy from sinking into arbitrary idealism, it is important to summarize Kant's theses.

Immanuel Kant—who wrote the seminal book *Critique of Pure Reason* (1781/1787) and is one of the paradigmatic representatives of the European Enlightenment—formulated a theory of mathematical philosophy that is focused on the following question: given that mathematical knowledge is necessarily, intrinsically true, and, simultaneously, it is applicable to the sensible world—since the sensible world seems to conform to the laws of arithmetic, which transcend the sensible world—how is it possible to know something about the world that is necessarily true, or, in other words, how can we have knowledge of the world independent of recourse to experience? In order to tackle this question, Kant distinguished between two kinds of sentences: analytically true sentences and synthetically true sentences.

An analytically true sentence is necessarily true on purely logical grounds—that is, solely in virtue of its meaning—and, in reality, it elucidates meanings already implicit in the subject. For instance, the sentence "Pediatricians are medical doctors who specialize in the medical care of infants, children, adolescents, and young adults" is an analytic statement, because it is true by definition. By contrast, the sentence "Pediatricians are rich" is not necessarily true; since it is not part of the definition of a pediatrician that a pediatrician is rich, but it is part of the definition of a pediatrician that a pediatrician is a medical doctor who specializes in the medical care of infants, children, adolescents, and young adults. The sentence "Pediatricians are rich" is a synthetic statement.

The distinction between analytic and synthetic statements is based on whether we are dealing with one concept or two concepts. If you say that "Pediatricians are rich," you are making a synthesis of two unrelated concepts—namely, the concept of being a medical doctor specialized in pediatrics and the concept of being rich. By contrast, if you say that "Pediatricians are medical doctors who specialize in the medical care of infants, children, adolescents, and young adults," you are not synthesizing two unrelated concepts, but you are analyzing a feature of one concept—namely, the concept of being a pediatrician.

Furthermore, Kant made another important epistemological distinction in order to clarify the manner in which we know things to be true—

specifically, he distinguished between *a priori* philosophical methods and *a posteriori* philosophical methods. The major attribute of the *a priori* methods is that they are based on primitive hypotheses usually intuitively conceived and axiomatically accepted, which deductively give rise to series of syllogisms, which, in turn, lead to ultimate conclusions, which are related to the preceding propositions in a logically rigorous way. For instance, we know that "pediatricians are medical doctors who specialize in the medical care of infants, children, adolescents, and young adults" *a priori*, that is, prior to any testing and any surveying. On the other hand, *a posteriori* philosophical methods are based on empirical research. For instance, the truth value of the statement that "pediatricians are rich" can only be determined *a posteriori*, that is, on the basis of doing some empirical research.

In view of the aforementioned Kantian epistemological distinctions, analytic statements are *a priori*, and synthetic statements are *a posteriori*. But mathematical knowledge exhibits the following peculiar feature: it is necessarily true, and, therefore, *a priori*, but, simultaneously, it is true of the world, and, therefore, *synthetic*. In fact, Kant observed that mathematical knowledge is a peculiar hybrid, in the sense that it is synthetic *a priori*. In other words, according to Kant, mathematical propositions, such as "$1 + 2 = 3$," are synthetic statements, abstractions from the sensing of objects, and, yet, they are *a priori*, in the sense that we do not need to do any experiments in order to verify them. Thus, Kant came up with the following question: how can we know things that are synthetic *a priori*? In order to answer this question, he developed a whole system of metaphysics that he called transcendental idealism and expounded in his *Critique of Pure Reason*.

Kant's metaphysical system is founded on the thesis that we do not know, and cannot know, the essence of things, the things-in-themselves, which he called "noumena"—meaning objects or events that exist independently of human sense and/or perception—but we can only know things as they appear to consciousness, which are called "phenomena." In Kant's philosophy, a phenomenon is a faded, dissolved declaration of the corresponding noumenon, the manner in which the corresponding noumenon (thing-in-itself) appears to an observer. According to Kant, phenomena have been put through a kind of mental filter, which is the way in which consciousness perceives the world, and mathematics is that kind of mental filter. In particular, Kant maintains that geometry is the spatial form through which consciousness perceives the world, and arithmetic—specifically, the one-dimensional sequence of numbers—is the temporal form through which consciousness perceives the world. Hence, according

to Kant, we do not receive mathematics from the system of space-time itself, but we use mathematics, our spatio-temporal intuitions and intellectual glasses, in order to understand and organize the world, and this is the reason why mathematics is *a priori*. Geometry is the way in which we organize space, and arithmetic is the way in which we organize time, and, when we combine geometry with arithmetic, we obtain the intellectual framework of the spatio-temporal world that we experience.

But how did Gottlob Frege respond to Kant's transcendental idealism, especially regarding the nature of mathematics?

First of all, I have to point out that Kant has correctly highlighted and elucidated the active role of consciousness in cognition, and the distinction between cognition and the object of cognition. The distinction between cognition and the object of cognition plays a central role in the so-called analytic philosophy. However, analytic philosophy may lead to an impasse, because it urges one to repeat the distinction between cognition and the object of cognition *ad infinitum* (forever). Inherent in analytic philosophy is the risk of using Kantian philosophy in an abortive way, in the sense that the attempt to define the presuppositions of the presuppositions of philosophy can continue *ad infinitum*, annihilating epistemology. To mitigate this risk, Kant resorted to a formalist view of idealism: Kant's *Critique* is characterized by formal idealism, in the sense that it maintains that the *form* of objects is due to consciousness, but not their *matter*. Furthermore, to avoid the excesses of analytic philosophy, I would say that, at some point, a mature philosophical-scientific mind must make a final, epistemologically responsible decision, instead of transforming philosophy into a meaningless game of words. After all, the very fact that the object of cognition, the world, exhibits a sort of resistance to cognition, and consciousness has to try hard in order to know the world and impose the intentionality of consciousness on the word, implies that—even though, under certain conditions, the world is submissive to the intentionality of consciousness—the world is not merely a projection and extension of consciousness. Endorsing the arguments that Ludwig Wittgenstein put forward in his book *On Certainty*, I should add that the distinction between cognition and the object of cognition is meaningful only if it arrives at a specific conclusion-end, instead of becoming a communication game or a rhetoric obsession.

The way in which Frege attempted to do justice to the objectivity of mathematics and to the reality of the world was logicism, which, as I mentioned earlier, brings together logic and arithmetic. Logicism resorts to Plato's philosophical realism regarding the objectivity of mathematics, but logicism differs from classical Platonism in two ways. Firstly, in contrast

to classical Greeks, Frege and logicism in general regard arithmetic, rather than geometry, as the foundational branch of mathematics, because of the following two reasons: in the seventeenth century, Descartes's analytic geometry, adapting Viète's algebra to the study of geometric loci, showed that algebra can be used in order to model geometric objects in a systematic and rigorous way, thus establishing a correspondence between geometric curves and algebraic equations; and, in the nineteenth century, Nikolai Lobachevski, János Bolyai, and Bernhard Riemann invented rigorous and consistent alternatives to Euclidean geometry. Hence, for Frege and the logicists in general, the central problem in mathematical philosophy is to understand the meaning of a number. In particular, logicists endow arithmetic with the objectivity that characterizes Platonic forms, but they do so in an indirect way—through logic—trying, in a sense, to achieve a creative synthesis between Kant's transcendental idealism and Plato's philosophical realism. The role that logic plays in the "school" of logicism is the second issue with regard to which logicism differs from classical Platonism. In particular, Frege thought that we can do justice to mathematical Platonism, according to which arithmetic is about things that are forms, if we show that mathematics—particularly, arithmetic—is reducible to logic, and if we take a Platonic view of logic; hence, the name of this "school" of mathematical philosophy is logicism.

Frege fused logic and arithmetic by formulating a theory of numbers that is based on the concept of a class of objects and on structural linguistics. Hence, Frege synthesized Aristotle's work on logic and language with Plato's theory of forms. In particular, Frege thought as follows: Let us consider a variable x, meaning that x is either a symbol representing an unspecified term of a theory, or a basic object of a theory that is manipulated without referring to its possible intuitive interpretation. Thus, given a class of sentences that have the same form, we can capture their common form by replacing their specific subjects with a variable x. For instance, given sentences such as "Plato is a philosopher," "Aristotle is a philosopher," "Kant is a philosopher," "Frege is a philosopher," etc., which have the same form, we can replace the name of the subject with a variable x, thus formulating the sentence "x is a philosopher," which captures the common form of the aforementioned sentences. In this way, we obtain a class: all the things that can satisfy the sentence "x is a philosopher," whenever we replace x with a name, belong to the class of philosophers. Hence, Plato, Aristotle, Kant, Frege, and any other person whom we could substitute for x are members of the class of philosophers.

According to Frege's terminology, whereas propositions are declarative statements that are either true or false, such as the statement "Plato is a

philosopher," a statement that contains a variable x and expresses a proposition as soon as a value is assigned to x is a propositional function, such as the statement "x is a philosopher." In other words, propositions and propositional functions differ from each other by the fact that propositional functions are ambiguous, in the sense that a propositional function contains a variable of which the value is unassigned. A class is the extension of a propositional function; for instance, the collection of all philosophers constitutes the extension of the propositional function "x is a philosopher," and is a class. Frege used the so defined concept of a class in order to refer to numbers and study the foundations of arithmetic.

According to Frege, numbers are classes. In his seminal book *Basic Laws of Arithmetic* (1893, 1903), Frege explained that any number n can be used in order to count any n-membered class. For instance, the number two can be thought of as the class of all 2-membered things, namely, as the class of all pairs, independently of the nature of the objects that constitute each pair. Similarly, the number three can be thought of as the class of all triples, namely, as the class of all those things that have three members; the number four can be thought of as the class of all quadruples, namely, as the class of all those things that have four members, etc. Collect all those things that have n members, and that, according to Frege, is the number n. Note that this way of defining numbers is substantively different from the thesis that a number is a property of a collection of objects, because, according to Frege's conception of numbers, a number is a particular kind of object, it is a class. Frege built a whole system of logic on the aforementioned concept of a class.

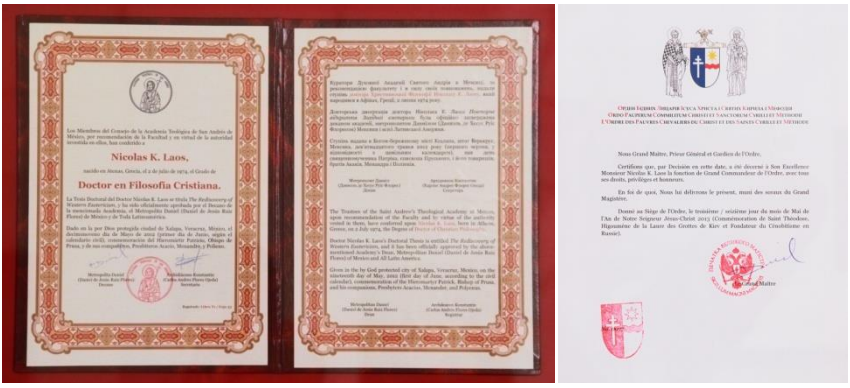
In order to define the concept of a natural number, in particular, Frege defined, for every 2-place relation R, the concept "x is an ancestor of y in the R-series," and this new relation is known as the "ancestor relation on R." The underlying idea can be easily grasped if we interpret Frege's 2-place relation R as "x is the father of y in the R series." For instance, if a is the father of b, b is the father of c, and c is the father of d, then Frege's definition of "x is an ancestor of y in the fatherhood-series" ensures that a is an ancestor of b, c, and d, that b is an ancestor of c and d, and that c is an ancestor of d. More generally, given a series of facts of the form aRb, bRc, and cRd, Frege showed that we can define a relation R^* as "y follows x in the R-series." Thus, Frege formulated a rigorous definition of "precedes," and he concluded that a "natural number" is any number of the predecessor-series beginning with 0.

Figure 1-2: Dr. Nicolas Laos at the "Angel Kanchev" University of Ruse. From left to right: Dr. Nicolas Laos with Professor Svetoslav Jordanov Bilchev (former Head of the Department of Algebra and Geometry at the "Angel Kanchev" University of Ruse) and Professor Stepan Tersian (faculty member of the Department of Mathematics at the "Angel Kanchev" University of Ruse and of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences) at the Fifth International Conference on Differential Equations and Applications, which took place during 24–29 August 1995 at the "Angel Kanchev" University of Ruse (Ruse, Bulgaria), and which Dr. Nicolas Laos (then 21 years old) addressed, presenting his research paper entitled "A Comparative Study of Linear and Nonlinear Differential Equations with Applications" (published in the volume of the proceedings of that conference). Moreover, Dr. Nicolas Laos and Professor Svetoslav Jordanov Bilchev jointly conducted various studies and discussions in the fields of geometry and differential equations, and they co-authored a research paper on algebraic geometry, which was published in the Proceedings of the Twenty Fifth Spring Conference of the Union of Bulgarian Mathematicians, Kazanlak, April 6–9, 1996.

Regarding my perseverance in inquiring into several challenging philosophical and theological questions, I would like, first of all, to express my gratitude to my colleagues at the Faculty of Philosophy at the Theological Academy of Saint Andrew (Academia Teológica de San Andrés), Veracruz, Mexico, where I completed the following Ph.D. courses: Methodology of Philosophical Investigation I & II, Theology and Philosophy I–IV, Selected Topics in Christian Philosophy I–IV, Seminar on Investigation in Christian Philosophy I–IV, and Interpretation of Philosophical Texts, I & II. In particular, the Dean of that Theological Academy, Metropolitan Dr. Daniel de Jesús Ruiz Flores of Mexico and All Latin America of the Ukrainian Orthodox Church (Iglesia Ortodoxa Ucraniana en México), has helped me to explore and appreciate the

interdisciplinary nature of Patristics and its interplay with Philosophy, and he has bestowed upon me the ecclesiastical titles of a Knight Grand Cross and a Grand Commander of his Ecclesiastical Metropolis's Order of the Poor Fellow-Soldiers of Christ and Saints Cyril and Methodius (Ordo Pauperum Commilitum Christi et Sanctorum Cyrilli et Methodii). My gratitude extends to the following persons: the historian Dr. Vassilios Christides (affiliated with the Institute for Advanced Study, Princeton, U.S.A.), who taught me a comprehensive set of courses on the history of world civilization during my studies at the University of La Verne; the historian Dr. Paul Angelides, who taught me the courses "U.S. Intellectual History" and "Development of American Democracy" during my studies at the University of La Verne; and the theologian and philosopher Dr. Conor Cunningham (Faculty of Arts of the University of Nottingham, U.K.), who has helped me to publish in the fields of philosophy and theology, and whose research work is a great source of inspiration for me.

Figure 1-3: Dr. Nicolas Laos at the Academia Teológica de San Andrij (Andrés). On the left: Dr. Nicolas Laos's Doctorate in Christian Philosophy from the Academia Teológica de San Andrij (Andrés), Mexico, of the Ukrainian Orthodox Church (Iglesia Ortodoxa Ucraniana en México). On the right: Dr. Nicolas Laos's brevet certifying his appointment as a Grand Commander of the Order of the Poor Fellow-Soldiers of Christ and Saints Cyril and Methodius (Ordo Pauperum Commilitum Christi et Sanctorum Cyrilli et Methodii) by Metropolitan Dr. Daniel de Jesús Ruiz Flores of Mexico and All Latin America of the Ukrainian Orthodox Church (Iglesia Ortodoxa Ucraniana en México).



My views have been refined and reinforced by my experience as a consultant and educator in the fields of mathematical modelling and noopolitics. In the context of my collaborations with research and

educational institutions, private intelligence and investment services companies, as well as in the context of my work as an independent consultant in mathematical modelling and epistemology, I have worked in the fields of biomathematics (mathematical applications in population biology, ecology, microbiology, and genetics), mathematical psychology (mathematical modelling of perceptual, thought, cognitive, and motor processes), mathematical modelling of social conflicts and war problems (i.e., mathematical modelling of conflicts between social groups, arms races between nation-states, and military conflicts), and mathematical modelling of economic problems (in particular, mathematical models of economic planning and financial engineering).

As regards noopolitics, I should mention that I define and practice noopolitics as the systematic study and management of personal and social life in the context of the information-cognitive field that is created by the communication between conscious beings, spanning philosophy of anthropology, of psychology, of sociology, of politics, and of economics. As a matter of fact, the term "noopolitics" (or "Noopolitik") was coined by the American defense experts John Arquilla and David Ronfeldt in a seminal RAND Corporation study of 1999. It derives from the Greek words "nous" (i.e., mind) and "politikī" (i.e., politics).

The levels at which noopolitics can be conducted and studied are the following:

- *Cyberspace:* this is the global system of the Internet-connected computers, communications infrastructures, online conferencing entities, databases, and information utilities. The most important characteristic of the cyberspace is the communication between conscious beings and the social interactions involved rather than the computational medium.
- *Infosphere:* it encompasses the cyberspace and information systems that may not be part of the Internet, such as the "mediasphere" (broadcast, print, and other media), libraries, military information infrastructures (Command, Control, Computer, Communications, Intelligence, Surveillance, and Reconnaissance Systems), etc.
- *Noosphere:* the term "noosphere," which is the intellectually highest level at which noopolitics can be conducted, was coined in the 1920s by the Jesuit philosopher Pierre Teilhard de Chardin, and, according to Teilhard de Chardin, it describes a globe-circling realm of the mind, or a "thinking circuit."

In view of the foregoing, I maintain that Freemasonry, being essentially about "building" minds and societies, can, under certain conditions, exist

and operate as an institution that promotes humanity's intellectual development and existential nobility. True initiatory institutions help and guide people to become conscious of the contents that press upward from the unconscious, upward from the level of instinct, or blind nature, and to ascend to the level of reason, thus becoming what Plato has called a "Philosopher King" over themselves and the world. The essential core of civilization is nothing but the transformation of nature into the dominion of humanity's reason and will. Thus, there is an ongoing contradiction between the "initiatory secret" and the "profane": the former leads to the expansion of consciousness, freedom, and illumination, whereas the latter leads to inner shrinking, serfdom, and obscurantism.

According to ancient Greek mythology, Orpheus founded the Orphic Mysteries, a system of mystical religious anthropology. The rites of those mysteries were based on the myth of Dionysus Zagreus, the son of Zeus and Persephone. When Zeus proposed to make Zagreus the ruler of the universe, the Titans disagreed, and they dismembered the boy and devoured him. Athena saved Zagreus's heart and gave it to Zeus, who swallowed the heart, from which was born the second Dionysus Zagreus. Moreover, Zeus destroyed the Titans with lightning. From the ashes of the Titans sprang the human race, who were part divine (Dionysus) and part evil (Titan). This double aspect of human nature, the Dionysian and the Titanic, plays a key role in Orphism. The Orphics affirmed the divine origin of the soul, but they believed that the soul could be liberated from its Titanic inheritance and could achieve eternal bliss through initiation into the Orphic Mysteries.[5] Thus began the history of Western esotericism.

At this point, we must explain somewhat more fully the philosophical difference between the ancient public polytheistic religion and Orphism, from which the ancient Greek mysteries and Platonic philosophy originated. The deities of ancient Greek public religion had no personal dimension and therefore no autonomous will, because their status was identified with an individual cosmic energy which they embodied. Hence, an ancient god (resp. goddess) caused a situation, not because he (resp. she) was acting according to one's free will, but because he himself (resp. she herself) was that situation. For example, the goddess Aphrodite is a personification of the sexual instinct and represents it in its good and painful consequences, but she does not seek to, nor can she, transmute it into a civilized form of love (it is a physical fact and not a state of consciousness). Because of this inherent limitation of ancient public religion, polytheism was accepted, and, thus, the ancient Greeks were

[5]Guthrie, *Orpheus and Greek Religion*.

ready to accept any new deity to the extent that it expressed a natural force which up to that time had not been included (represented) in the established polytheistic belief system. This spiritual weakness of ancient public religion tended to be cured by the ancient mysteries, which highlighted a personal, "inner" way of worship. The ancient mysteries in general and Orphism in particular offered an outlet for the inner demand for existential fulfilment and bliss of the subject, a demand which public religion overlooked, since public religion covered certain basic needs of existence, but precluded the inner life of both gods and men. On the other hand, the God of the Mysteries is a pure spirit who addresses the human spirit and, by establishing a spirit-to-spirit relationship, offers a vision of humanity's personal salvation (existential integration and completion).

Figure 1-4: Dr. Nicolas Laos addressing Rotary Clubs. Dr. Nicolas Laos (fifth from the left) presents his scholarly work in noopolitics as the key-note speaker at a meeting of the District No. 2470 of Rotary International, which took place in Glyfada, Greece, on the 24th of January 2019, and it was a joint initiative of the Rotary Clubs of Vouliagmeni, Athinai-Zappeion, Athens-Ilisia, Athens-South, Pagrati, and Salamina "King Aias."

My Freemasonic Lineages and the Foundation of the Esoteric Initiatory Grand Lodge of Greece

I authored and instituted the Modern and Perfecting Rite of Symbolic Masonry, as it is exposed and explained in the present book, on the basis of the knowledge, the inspiration, and the vision that I received from the Illuminati Grand Master the Most Eminent Brother Professor Giuliano Di Bernardo and on the basis of the Masonic Authority and Power vested in me by the Gran Loggia Iniziatica under the leadership of its Grand Master and Sovereign Grand Commander the Most Venerable and Most Puissant Sister Ginevra Di Nicola, 33º. Before I address these issues, which mark the culmination of my career in esoteric-initiatory societies, I would like to briefly expose and explain my Freemasonic background.

In 1997, at the age of 23, I was initiated into Freemasonry in London, specifically in the "Honor Per Onus" Lodge No. 6586 of the United Grand Lodge of England (UGLE), having been "proposed" by the late W. Bro. Theodore Frangos, who was a Grand Officer of the United Grand Lodge of England, a Grand Elected Knight Kadosh (under the auspices of the Supreme Council 33º for England and Wales), a prominent member of the Cypriot-British community of London, and an Archon (honoree) of the Ecumenical Patriarchate of Constantinople (my "seconder" being the late W. Bro. Constantine Drakos, PM, Grand Elected Knight Kadosh). Within six months of my initiation, I was raised to the sublime degree of a Master Mason. From 1997 to 1999, at Lodge meetings (which were taking place at the Freemasons' Hall, the headquarters of the United Grand Lodge of England, which has been at 60 Great Queen St, London since 1775), I had several opportunities of meeting interesting and well-intended people and of reading many authoritative Masonic books and historical documents at the Grand Library, where I had the opportunity to personally interact with two brilliant senior members of the UGLE's staff: V.W. Bro. John Hamill (who was the then Grand Librarian of the UGLE) and W. Bro. Julian Perry (who was the then media relations coordinator of the UGLE).

Moreover, from 1999 to 2000, I had the opportunity to practice the English form of Symbolic Masonry in the National Grand Lodge of Greece (specifically, in the "Greek-Speaking Hotel" Lodge No. 67). During that period, I also explored appendant Masonic Orders, specifically the Order of the Holy Royal Arch of Jerusalem, the Order of Mark Masonry, the Order of the Royal and Select Masters, and the Orders of the Knights Templar and the Knights of Malta, affiliated with the National Grand Lodge of Greece on the basis of Charters issued to them by the

corresponding British Masonic Bodies. From 2001 to 2003, I accumulated a few more Freemasonic experiences by participating in the labors of the "Etairia ton Filikon" Lodge No. 116 under the auspices of the Grand Lodge of Greece. Moreover, from 1999 to 2003, I investigated the history of the Greek Masonic schism of 1986, which led to the formation of the National Grand Lodge of Greece by Lodges that seceded from the Grand Lodge of Greece in an effort to consolidate Greek Freemasonry, which had been extensively debased and corrupted by the policy of the Grand Lodge of Greece. Efstathios Liakopoulos, the founding Grand Master of the National Grand Lodge of Greece (which was founded in 1986 by Master Masons who renounced the Grand Lodge of Greece and decided to follow strictly the British practice in their rituals and operation) has written a book entitled *The Masonic Schism* (in Greek: *To Tektoniko Schisma*, published in 2009, in Greek, by the Ionian Philosophical Society), in which he explains the factionalism and profane types of behavior that prevailed in Greek Freemasonry during the 1980s. Additionally, in the 2010s and the 2020s, major schisms occurred in the Body of the Scottish Rite that is affiliated with the Grand Lodge of Greece, which, with its actions and business culture, caused negative publicity for Freemasonry (in the 2010s and the 2020s, these Greek Masonic factions resorted to the civil courts of Greece, entrusting an institution of the profane society to settle their Masonic disputes). Another important insight I gained from my interaction with Greek Freemasons, Greek mainstream journalists, and Greek "alternative researchers" is that I analyzed and understood the ways in which scammers infiltrate Freemasonry and the ways in which agents of the Greek state intelligence agencies produce and disseminate nationalist neo-mythologies and co-opt Greek Freemasonic groups for the conduct of psychological operations (I have identified and studied similar phenomena in the Anglo-American world, too, regarding the attempts of state intelligence agencies to manipulate esotericism and fictional narratives).

By the middle of the 2000s, I had already forged important friendships in the United Grand Lodge of England, the National Grand Lodge of Greece, and the Grand Lodge of Greece, and, indeed, in these and other Masonic institutions, I met and talked with several interesting and remarkable persons (unfortunately sitting by several Masons ill-suited to Masonry). However, I had already decided that, under the circumstances, the established Greek Masonic institutions (such as the Grand Lodge of Greece, the National Grand Lodge of Greece, and a few other Greek Masonic organizations for men and women) did not suit me, both for philosophical and aesthetic reasons, and I had come to the conclusion that,

in general, a peculiar, yet elusive, kind of nihilism lurks in the so-called "regular Freemasonry."

Those Freemasonries which describe themselves as "regular" while considering any other type of Freemasonry as "irregular" are based on certain English Masonic traditions established by the United Grand Lodge of England as well as on certain American Masonic traditions established by the Scottish Rite Supreme Council of the 33rd degree for the Southern Jurisdiction (U.S.A.). These so-called "regular" Freemasonries perceive "regularity" as adherence to a fatal constitutive principle and, therefore, deny the creative integration of Freemasonry into historical becoming. Hence, the aforementioned conception of "regularity" underpins strict ritualism and rigid traditionalism, which veil historical nihilism.

The word nihilism comes from the Latin terms "ne" (= "not") and "hilum" (= "a hilum"). It signifies "uprooted" or spiritually hovering and disconnected people. In the context of contemporary, regular Anglo-American Freemasonry, what I call "Masonic nihilism" manifests itself in the following three ways: (i) hyper-conservatism: the Freemasons who belong to this group have created such a strong dependence on certain formal procedures that, in order to be able to wear their regalia and work under the auspices of self-proclaimed oligopolies of "regular Freemasonry," they are willing to sell out the essence of Freemasonry and profane the Temple; (ii) "millennial" mindset: the Freemasons who belong to this group merely want to follow exact orders, comply with specific regulations, and be fairly assessed according to their performance in executing the given orders, avoiding any meaningful discussion about essence, teleology, and genuine creativity; (iii) "snowflake" mindset: the Freemasons who belong to this group subscribe to an absolute and indiscriminate principle of inclusivity and have as a priority not to offend or provoke or intellectually complicate each other, to such an extent that they avoid having any philosophically, politically, socially, or culturally serious discussion with each other and mainly confine themselves to insignificant forms of interaction.

It is worth adding that many Freemasonries have lost their significance, many Freemasons actually sail in the chaos of diverse experiences that they accumulate without being able to synthesize them into a consistent existential strategy and world-conception, and several Freemasonic factions operate as networks of white-collar crime or even as agents of state bureaucracies that are known as the "deep state." For instance, a famous case of a "deep state" institution was the Italian Lodge "Propaganda Due," better known as "P2," under the leadership of the

Italian financier and CIA asset Licio Gelli.[6] It is important to mention that, as the French Professor of Legal History and literary critic Jacques de Saint Victor has argued, the mafia (specifically, the European and American transnational system of organized crime) was born in the "décombres" (ruins) of the feudal regime, and it was developed further as a consequence of the advent of bourgeois democracy and capitalism from the nineteenth century onwards.[7] In fact, the essence of the modern mafia is the result of the merger between a rotten nobility and a criminal bourgeoisie. Consequently, various secret/"esoteric" societies (such as the notorious Italian Masonic Lodge "P2") and private exclusive membership clubs operate as front organizations for the mafia, often in collaboration with state bureaucracies.

In the 2000s and the 2010s, one of my most important friends in the English Masonic Obedience was the late Fra John Baron Dudley von Sydow von Hoff (1937–2016), who was also a professional member of the staff of the Grand Secretary's Office at the Mark Masons' Hall, in London. The Mark Masons' Hall is the administrative headquarters of ten appendant Freemasonic Orders affiliated with the United Grand Lodge of England. Moreover, Fra John Baron Dudley von Sydow von Hoff was the Master-General of the United Grand Priories of the Hospitaller Order of Saint Lazarus of Jerusalem, a British, non-Masonic, chivalric, charitable organization based on the legacy of the medieval Hospitaller Order of Saint Lazarus of Jerusalem. Fra John Baron Dudley von Sydow von Hoff has shared with me some of his important experiences and insights regarding the administrative aspects of Freemasonic institutions, and he bestowed upon me the title of a Knight of Saint Lazarus of Jerusalem (KLJ) at the 26th Investiture Ceremony of the Grand Priory of England, Wales, Isle of Man and Chanel Islands, held on the 27th of September 2008 in the Priory Church of Saint Bartholomew the Great in the City of London.

In the Middle Ages, the Hospitaller Order of Saint Lazarus of Jerusalem was one of the five major Crusading Orders (the other four being the Knights of the Holy Sepulchre, the Knights Templar, the Knights of Saint John, and the Teutonic Knights). The history of the Lazarite Hospitallers in the twentieth century goes back to the 1930s. In fact, in 1929, a "Council of Officers" of the "Association" of the Lazarite Hospitallers approached the Duke of Seville, and offered him the "Grand Mastership of the Order." It was not, however, until the 12th of December 1935 that he

[6] For a systematic study, see: Ganser, *NATO's Secret Armies*.
[7] Saint Victor, *Un Pouvoir Invisible*.

was proclaimed the "44th Grand Master." The Grand Priory of England, Wales, Isle of Man and Channel Islands of the Hospitaller Order of Saint Lazarus of Jerusalem was formed, consecrated, and established on the 1st of January 1995 under the leadership of a highly experienced Lazarite Knight, Fra John Baron Dudley von Sydow von Hoff. From there, it was gradually developed into the United Grand Priories of the Hospitaller Order of Saint Lazarus of Jerusalem, which was formed on the 13th of May 1999 as an independent British branch of the Lazarite Hospitallers.

Figure 1-5: Dr. Nicolas Laos, KLJ, attending an investiture ceremony of the United Grand Priories of the Hospitaller Order of Saint Lazarus of Jerusalem. Dr. Nicolas Laos (on the left) and Fra John Baron Dudley von Sydow von Hoff (on the right), wearing their distinctive insignia of the Hospitaller Order of Saint Lazarus of Jerusalem, at an investiture ceremony of the United Grand Priories of the Hospitaller Order of Saint Lazarus of Jerusalem that took place in the Anglican Church of St. Paul in Athens, Greece, in 2008, with the blessings of Canon Malcolm Bradshaw, MBE (first photograph, on the far right), Chaplain of St. Paul's Anglican Church, Greater Athens.

The English Freemasonic tradition is founded on Operative Masonry, and, indeed, this approach to Freemasonry has put an indelible imprint on the way I think about Freemasonry. However, wishing to expand my Freemasonic horizons and to enrich my Freemasonic journey with more philosophical elements, I decided to join the Ancient and Accepted Scottish Rite. I did so in the context of my participation in an Italian Masonic Obedience known as the Ordine Massonico Tradizionale Italiano (Italian Traditional Masonic Order). In 2017, the Grand Master of the Ordine Massonico Tradizionale Italiano, Luigi Pruneti, acting according to the authorities with which he was vested, put me forward to be elevated to the 33rd and *ne plus ultra* degree of the Ancient and Accepted Scottish Rite. On the 27th of July 2020, the Supreme Council of the Ancient and Accepted Scottish Rite that is affiliated with the Ordine Massonico

Tradizionale Italiano, officially known as the Supremo Consiglio del 33∴ ed Ultimo Grado Rito Scozzese Antico e Accettato Tradizionale d'Italia (under its Sovereign Grand Commander Br. Giovanni Constantini 33∴ and its Grand Secretary General Br. Nestore Bartoletti 33∴), officially appointed me as an active member of the 33rd and *ne plus ultra* degree of the Ancient and Accepted Scottish Rite under its auspices (Fig. 1-6). On the 11th of January 2017, Grand Master Br. Luigi Pruneti, acting in his official capacity as the Grand Master of the Ordine Massonico Tradizionale Italiano, had already issued two Charters (Patent Letters) to me: with the first Charter, he authorized me to found an independent Body of Lodges of Symbolic Masonry in Greece (working in the degrees 1º–3º), thus investing me with the authorities and powers of a Grand Master of Symbolic Masonry (Fig. 1-7); and with the second Charter, he authorized me to found an independent Body of the Ancient and Accepted Scottish Rite in Greece (working in the degrees 4º–33º), thus investing me with the authorities and powers of a Sovereign Grand Commander of the Ancient and Accepted Scottish Rite (Fig. 1-8).

Regarding Br. Luigi Pruneti's Masonic background and expertise, it is worth mentioning that, before becoming the Grand Master of the Ordine Massonico Tradizionale Italiano, he served as the Grand Master of the Grand Loggia d'Italia degli ALAM (Antichi Liberi Accettati Muratori) from 2007 to 2013, and was elevated to the 33rd degree of the Ancient and Accepted Scottish Rite in 1992 in that Obedience. The Grand Loggia d'Italia degli ALAM was founded in Italy in 1910 as a mixed Masonic Obedience by a group of Scottish Rite Masons who split from the Grande Oriente d'Italia in 1908 in order to reform Italian Freemasonry. In 2017, he was regularly installed as the Founding Grand Master of the Ordine Massonico Tradizionale Italiano, which is an independent Masonic Obedience.

Figure 1-6: Dr. Nicolas Laos's certificate of appointment as an active member of the 33rd and ne plus ultra degree of the Ancient and Accepted Scottish Rite under the auspices of the Supremo Consiglio del 33∴ ed Ultimo Grado Rito Scozzese Antico e Accettato Tradizionale d'Italia, which is affiliated with the Ordine Massonico Tradizionale Italiano (on the 27th of July 2020).


A∴U∴T∴O∴S∴A∴G∴

Supremo Consiglio del 33∴ ed Ultimo Grado
Rito Scozzese Antico e Accettato Tradizionale d'Italia

Noi, Giovanni Costantini 33∴

Sovrano Gran Commendatore del Supremo Consiglio
del Rito Scozzese Antico e Accettato Tradizionale d'Italia,

Visto lo Statuto del Supremo Consiglio,

In virtù dei poteri a Noi conferiti abbiamo decretato e decretiamo:

Il pot.mo Fr. Nikolas Laos

Viene di nostro Moto Proprio elevato al 33∴ e Ultimo Grado,

Sovrano Grande Ispettore Generale

Il presente Decreto, registrato al n° 2760 dell'Albo dei Decreti Sovrani e munito di Bolli,
entra in vigore nel giorno della sua emissione.
Dato dalla Sede del Supremo Consiglio allo Zenit di Roma, nella Valle del Tevere,
sotto la volta Celeste al 41° 54' 39" di latitudine Nord e a 12° 28' 54" di longitudine Est
Dallo Zenit, oggi 27 Luglio 2020

Gran Segretario Generale
Fr.∴ Nestore Bartoletti 33∴

Sovrano Gran Commendatore
Fr.∴ Giovanni Costantini 33∴

Figure 1-7: Dr. Nicolas Laos's Grand Lodge Charter. The Charter (Lettera Patente) with which, on the 11th of January 2017, Grand Master Br. Luigi Pruneti, acting in his official capacity as the Grand Master of the Ordine Massonico Tradizionale Italiano, authorized Dr. Nicolas Laos to found an independent Body of Lodges of Symbolic Masonry in Greece (working in the degrees 1º–3º), thus investing Dr. Laos with the authorities and powers of a Grand Master of Symbolic Masonry.

Lettera Patente

a

Nikolas Laos 33°

Repubblica di Grecia

A.·.G.·.D.·.G.·.A.·.D.·.U.·.
L.·.U.·.F.·.

Zenit di Roma, Valle del Tevere 6 novembre 2020 E.·.V.·.

noi
Luigi Pruneti
33°.66°.90°.95°.98°

Imperatore Gran Conservatore del Rito Egizio di Misraim, Gran Cofto del Rito Egizio di Cagliostro,
Dux Regis et Magister Templi dell'Ordine Fratellanza Regulus Compagnia Orientale Cavalieri Mediterranei,
Sovrano Gran Maestro Sovrano Principe, Sovrano Gran Maestro dell'Ordine dell'Aquila d'Oro Gran Priorato Scozzese,

Avendo ricevuto dal Ven.mo e Pot.mo Fratello Gianfranco Pecoraro 33°
Sovrano Gran Commendatore del Supremo Consiglio della Massoneria di Rito Scozzese Antico ed Accettato
e Gran Maestro dell'Ordine della Serenissima Gran Loggia Nazionale Italiana per l'Italia e sue dipendenze

con Decreto n. 001/2017 del giorno 11 gennaio 2017 E.·.V.·.

Confluenza nella mia persona quale Sovrano Gran Commendatore *ad vitam*

Del Suo Supremo Consiglio,

In virtù di tali poteri che ci furono conferiti,

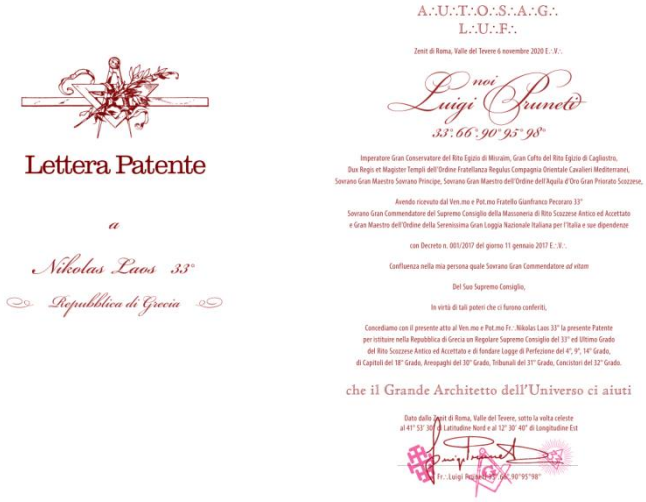
Concediamo con il presente atto al Ven.mo e Pot.mo Fr.·.Nikolas Laos 33° la presente Patente
per fondare nella Repubblica di Grecia Logge Azzurre che lavorino in 1°, 2° e 3° Grado Simbolico.

che il Grande Architetto dell'Universo ci aiuti

Dato dallo Zenit di Roma, Valle del Tevere, sotto la volta celeste
al 41° 53' 38" di Latitudine Nord e al 12° 30' 40" di Longitudine Est

Fr.·.Luigi Pruneti 33°.66°.90°.95°.98°

Figure 1-8: Dr. Nicolas Laos's Supreme Council Charter. The Charter (Lettera Patente) with which, on the 11th of January 2017, Grand Master Br. Luigi Pruneti, acting in his official capacity as the Grand Master of the Ordine Massonico Tradizionale Italiano, authorized Dr. Nicolas Laos to found an independent Body of the Ancient and Accepted Scottish Rite in Greece (working in the degrees 4º–33º), thus investing Dr. Laos with the authorities and powers of a Sovereign Grand Commander of the Ancient and Accepted Scottish Rite.


Lettera Patente

a

Nikolas Laos 33°

Repubblica di Grecia

A.·.U.·.T.·.O.·.S.·.A.·.G.·.

L.·.U.·.F.·.

Zenit di Roma, Valle del Tevere 6 novembre 2020 E.·.V.·.

noi Luigi Pruneti

33° 66° 90° 95° 98°

Imperatore Gran Conservatore del Rito Egizio di Misraim, Gran Cofto del Rito Egizio di Cagliostro,
Dux Regis et Magister Templi dell'Ordine Fratellanza Regulus Compagnia Orientale Cavalieri Mediterranei,
Sovrano Gran Maestro Sovrano Principe, Sovrano Gran Maestro dell'Ordine dell'Aquila d'Oro Gran Priorato Scozzese,

Avendo ricevuto dal Ven.mo e Pot.mo Fratello Gianfranco Pecoraro 33°
Sovrano Gran Commendatore del Supremo Consiglio della Massoneria di Rito Scozzese Antico ed Accettato
e Gran Maestro dell'Ordine della Serenissima Gran Loggia Nazionale Italiana per l'Italia e sue dipendenze

con Decreto n. 001/2017 del giorno 11 gennaio 2017 E.·.V.·.

Confluenza nella mia persona quale Sovrano Gran Commendatore *ad vitam*

Del Suo Supremo Consiglio,

In virtù di tali poteri che ci furono conferiti,

Concediamo con il presente atto al Ven.mo e Pot.mo Fr.·.Nikolas Laos 33° la presente Patente
per istituire nella Repubblica di Grecia un Regolare Supremo Consiglio del 33° ed Ultimo Grado
del Rito Scozzese Antico ed Accettato e di fondare Logge di Perfezione del 4°, 9°, 14° Grado,
di Capitoli del 18° Grado, Areopaghi del 30° Grado, Tribunali del 31° Grado, Concistori del 32° Grado.

che il Grande Architetto dell'Universo ci aiuti

Dato dalla Zenit di Roma, Valle del Tevere, sotto la volta celeste
ai 41° 53' 30" di Latitudine Nord e ai 12° 30' 40" di Longitudine Est

Fr.·.Luigi Pruneti 33° 66° 90° 95° 98°

In the late 2000s, my interests in comparative mythology, cultural psychology, and psychology of religion ignited my interest in the Ancient and Primitive Rite of Memphis–Misraim, since this Masonic Rite comprises degrees that are concerned with several ancient European, Egyptian, and Asian mythologies and systems of mysticism (such as Norse mythology, ancient Greek and ancient Roman mythologies, Zoroastrianism, ancient Egyptian mythology, Jewish mysticism and mythology, Hindu mysticism and mythology, etc.). In general, the term "civilization" refers to a mode of being, a way of life. Civilization is a structure that consists of institutions and technologies, and, in broad terms, it includes culture. Culture is the result of humanity's reflection on human life, and it is historically objectified through artistic creation, philosophy, religion, and science. Culture is embodied in civilization and underpins civilization, and civilization underpins the integration of culture into history. The French cultural anthropologist and ethnologist Claude Lévi-Strauss, in his book *The Raw and the Cooked: Mythologiques*, notes that

culture (corresponding to the notion of spiritual creation) and civilization (corresponding to the notion of technological construction) are inextricably related to each other, regarding both their essences and their manifestations. In particular, culture is a reflective attitude towards institutions and an attempt to transcend institutions through myth. Myth's complex structure reflects the structure of institutions, and it represents the core of culture. Additionally, the Swiss psychoanalyst Carl Gustav Jung, in his book *Psychology of the Unconscious*, points out that myth translates experienced reality into a symbolic language, and, in this way, it is conductive to the participation of a society as a whole in the same experience of reality, since myth integrates all areas of conscious and unconscious life into a common experience of reality. In view of the foregoing, the management of social affairs and any meaningful attempt to bring about changes in an established civilization are inextricably linked to the adequate management of the mythological contents of people's collective mind.

I was initiated into the Ancient and Primitive Rite of Memphis–Misraim by Tau Dr. Allen Bar Kohenim Greenfield, 33º-95º-97º. Tau Dr. Allen Bar Kohenim Greenfield is the Founder and Facilitator of the Mother Lodge of the "Sons and Daughters of Aaron," in Atlanta, in the State of Georgia, U.S.A. He is also a regular Initiate of the Ancient and Primitive Rite of Memphis–Misraimby partaking of the initiatory lines of succession of Michael Paul Bertiaux and the Hermetic Brotherhood of Light, which derives from the Order of the Golden and Rosy Cross (which, in turn, was founded in the 1750s by the German Freemason and alchemist Hermann Fictuld). Regarding Michael Paul Bertiaux (born in 1935), I should mention that he studied philosophy and history at the Jesuit University in Seattle, received instruction in philosophy at Tulane University in New Orleans, studied theology at the Anglican Theological Center in Vancouver, and was ordained in the Old Catholic Church by Forest E. Gregory Barber (whose ecclesiastical lineage goes back to Archbishop Gerardus Gul). He was initiated into the Ancient and Primitive Rite of Memphis–Misraim by the Grand Hierophant of one of its branches, Hector-François Jean-Maine, the son of Lucien-François Jean-Maine, in 1964, in Haiti. Lucien-François Jean-Maine was a student of Gérard Anaclet Vincent Encausse, whose esoteric pseudonym was Papus. Papus was a renowned occultist, and he succeeded John Yarker (the Founding International Grand Master of the British branch of the Ancient and Primitive Rite of Memphis–Misraim) in the leadership of this Masonic institution after the latter's death in 1913 (moreover, John Yarker was a Master Mason of the Lodge of Integrity No. 189, under the auspices of the

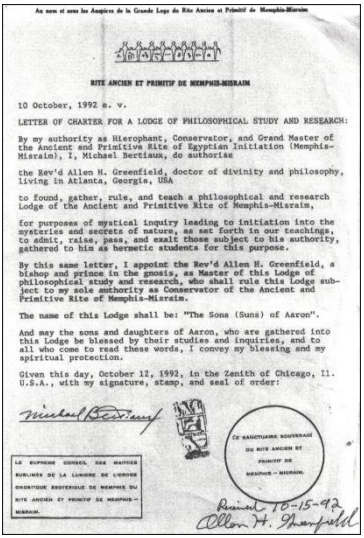
United Grand Lodge of England). Tau Dr. Allen Bar Kohenim Greenfield, 33º-95º-97º, has played an important role in the Ancient and Primitive Rite of Memphis–Misraim, and he has written a relevant ritual book entitled *The Compleat Rite of Memphis* (CreateSpace Independent Publishing Platform, second edition, 2014). On the 10th of October 1992, acting as a regular Grand Hierophant of the Ancient and Primitive Rite of Memphis–Misraim, Michael Paul Bertiaux authorized Tau Allen Bar Kohenim Greenfield "to found, gather, rule, and teach a philosophical and research Lodge of the Ancient and Primitive Rite of Memphis–Misraim" —specifically, the aforementioned Mother Lodge of the "Sons and Daughters of Aaron" (in Atlanta, Georgia, U.S.A.), by which I was awarded the 95th degree of the Ancient and Primitive Rite of Memphis–Misraim on the 8th of September 2012 (Fig. 1-9). Finally, on the 14th of June 2015, I was awarded the 97th and *ne plus ultra* degree of the Ancient and Primitive Rite of Memphis–Misraim (namely, the degree of a Grand Hierophant) by the "Misir" Lodge, in Vršak, Serbia, presided by its Grand Hierophant Tau Neven Daničić, 33º-95º-97º (Fig. 1-10). The aforementioned Serbian "Misir" Lodge has been consecrated by and is regularly affiliated with the aforementioned American Mother Lodge of the "Sons and Daughters of Aaron." In the "Misir" Lodge, I developed a creative Masonic communication with Slobodan Škrbić, an architect, artist, author, and a Masonologist, working and residing in Novi Sad, Serbia; he is also a Masonic Patriarch Grand Hierophant 97º of the Ancient and Primitive Rite of Memphis–Misraim. In response to my request, Brother Slobodan Škrbić wrote and send me the following insightful remarks regarding the history of Freemasonry:

> Irrespective of the causes of the events that took place in 1717, the new stage was created, signaling the transition from "operative" Masonry to "speculative" Masonry and a shift from a highly secretive way of transmitting knowledge to broader initiatory networks and later to global ones. These developments originated from the decision of particular Masonic groups to organize themselves and, generally, Freemasonry in a different way. I mean the Grand Lodge of London and Westminster. The fundamental reasons for this historical change may be analyzed from several perspectives, but, eventually, this historical change brought about a tremendous increase in the influence of the British Empire over all other existing countries and their newly disclosed and manifested Freemasonries. Even though Freemasonry existed in many other regions of Europe, the Grand Lodge of London and Westminster and, subsequently, the United Grand Lodge of England grabbed the banner of Freemasonry and proclaimed that the United Grand Lodge of England is the "Mother Grand Lodge of the world." This is a purely political, economic, and

> administrative decision, overruling the authorities and the decisions of smaller or alternatively structured Masonic groups. The history of this development is well documented, and we can learn a lot from it

Furthermore, Tau Dr. Allen Bar Kohenim Greenfield has succinctly explained the significance of "ritual initiations," as practiced, for instance, by Freemasons as follows: firstly, an initiatory ritual is the product of its composer, who seeks to get the candidate from a point A to a point B according to a specific process; and, secondly, an initiatory ritual is a product of action. More specifically, it is a product of the act of initiation by a trained initiate, charged with the duty of conducting the candidate through a ceremony in such a way that a receptive candidate will experience the intended inner alteration and personal growth.

Figure 1-9: Dr. Nicolas Laos's initiatory lineage in the context of the Memphis–Misraim Rite (Greenfield–Bertiauxlineage). On the left: The Charter with which, on the 10th of October 1992, Michael Paul Bertiaux authorized Tau Allen Bar Kohenim Greenfield "to found, gather, rule, and teach a philosophical and research Lodge of the Ancient and Primitive Rite of Memphis–Misraim," specifically, the Mother Lodge of the "Sons and Daughters of Aaron" (in Atlanta, Georgia, U.S.A.). On the right: The Masonic Certificate and Charter with which Tau Allen Bar Kohenim Greenfield, on the 8th of September 2012, bestowed upon Dr. Nicolas Laos the 95th degree of the Ancient and Primitive Rite of Memphis–Misraim and authorized Dr. Laos to found and rule a research Lodge of this Masonic Rite (in addition to Tau Allen Bar Kohenim Greenfield's signature, this certificate bears the signatures of other senior members of the Mother Lodge of the "Sons and Daughters of Aaron").



Au nom et sous les Auspices de la Grande Loge du Rite Ancien et Primitif de Memphis-Misraim

RITE ANCIEN ET PRIMITIF DE MEMPHIS-MISRAIM

10 October, 1992 e. v.

LETTER OF CHARTER FOR A LODGE OF PHILOSOPHICAL STUDY AND RESEARCH:

By my authority as Hierophant, Conservator, and Grand Master of the Ancient and Primitive Rite of Egyptian Initiation (Memphis-Misraim), I, Michael Bertiaux, do authorize

the Rev'd Allen H. Greenfield, doctor of divinity and philosophy, living in Atlanta, Georgia, USA

to found, gather, rule, and teach a philosophical and research Lodge of the Ancient and Primitive Rite of Memphis-Misraim,

for purposes of mystical inquiry leading to initiation into the mysteries and secrets of nature, as set forth in our teachings, to admit, raise, pass, and exalt those subject to his authority, gathered to him as hermetic students for this purpose.

By this same letter, I appoint the Rev'd Allen H. Greenfield, a bishop and prince in the gnosis, as Master of this Lodge of philosophical study and research, who shall rule this Lodge subject to my sole authority as Conservator of the Ancient and Primitive Rite of Memphis-Misraim.

The name of this Lodge shall be: "The Sons (Suns) of Aaron".

And may the sons and daughters of Aaron, who are gathered into this Lodge be blessed by their studies and inquiries, and to all who come to read these words, I convey my blessing and my spiritual protection.

Given this day, October 12, 1992, in the Zenith of Chicago, Il. U.S.A., with my signature, stamp, and seal of order:

Michael Bertiaux

CE SANCTUAIRE SOUVERAIN DU RITE ANCIEN ET PRIMITIF DE MEMPHIS – MISRAIM.

LE SUPREME CONSEIL DES HAUTES SUBLIMES DE LA LUMIERE DE L'ORDRE DIGNITAIRE ESOTERIQUE DE MEMPHIS DU RITE ANCIEN ET PRIMITIF DE MEMPHIS – MISRAIM.

Received 10-15-92

Allen H. Greenfield

ANCIENT AND PRIMITIVE RITE OF MEMPHIS-MISRAIM

Grand Areopagus of the Celestial Empire of the Sublime Magi 90°

CHARTER

the Most Puissant Brother Dr Nicolas Laos,

Sovereign Grand Inspector General - 33°,

Grand Architect of the Mysterious City, Grand Consecrator - 60°,

Sublime Master of the Great Work - 90° and

Grand Conservator - 95°.

the Most Puissant Brother Dr Nicolas Laos,

Given this day, September 8, 2012, in the Zenith of Atlanta, USA

with my Signature, Stamp and Seal of Order

T ALLEN Bar Kohenim GREENFIELD, DO, 33°, 90°, 95° A&P.R.SM

Figure 1-10: Dr. Nicolas Laos's certificate of elevation to the 97th degree of the Ancient and Primitive Rite of Memphis–Misraim. The Masonic Certificate and Charter with which, on the 14th of June 2015, the "Misir" Lodge of Serbia of the Ancient and Primitive Rite of Memphis–Misraim (Greenfield–Bertiaux lineage) bestowed upon Dr. Nicolas Laos the 97th and ne plus ultra degree of the Ancient and Primitive Rite of Memphis–Misraim (namely, the degree of a Grand Hierophant) and authorized Dr. Laos to found and rule a sovereign Grand Sanctuary of the Ancient and Primitive Rite of Memphis–Misraim.

In addition, my education in the Egyptian Rite was enriched in Italy. On the 10th of October 2020, the Grand Master of the Ordine Massonico Tradizionale Italiano (OMTI) Br. Luigi Pruneti, holding also the degree of an Emperor Grand Conservator (Imperatore Gran Conservatore) of the Egyptian Rite of Misraim and the Office of the National Grand Hierophant of the Sovereign Italian Sanctuary of the Egyptian Rite of Misraim (Sovrano Santuario Italiano del Rito Egizio di Misraim), personally appointed me as an honorary 97th-degree member of the executive committee of the aforementioned institution of the Egyptian Rite, in the Zenith of Rome, Italy (Fig. 1-11). Br. Luigi Pruneti has written the book entitled *Rituali della Massoneria Egizia di Cagliostro*, studying the history and the ethos of the Egyptian Rite of Freemasonry.

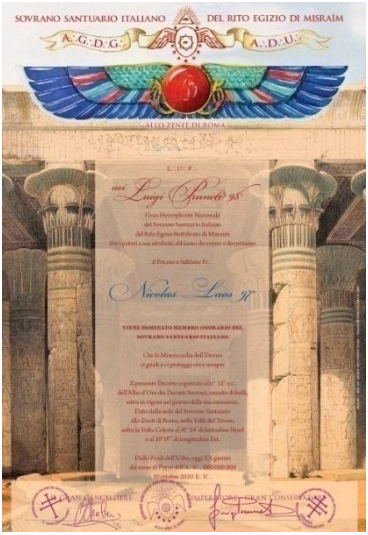
The origins of "Egyptian Masonry" can be traced back to the Egyptian Rite of Masonry that was developed by the Italian mystic Count Alessandro di Cagliostro at the end of the eighteenth century. Regarding Cagliostro, it should be mentioned that he was an agent of the Bavarian Illuminati and, through his "Egyptian Rite," he attempted to fuse Scottish Rite Masonry and revolutionary social ideas. Moreover, Cagliostro's Ritual was an attempt to fuse Freemasonry and Alchemy (the Hermetic Art) to provide a humanistic initiatory system.[8] Napoleon Bonaparte was initiated by Cagliostro into the Egyptian Rite of Freemasonry and the Masonic Rites of Memphis, Misraim, and Memphis–Misraim which came down from it.

A maxim given in Cagliostro's Rite was the following: "Qui agnoscit martem, cognoscit artem," meaning "whoever recognizes Mars, knows art." Mars was an ancient Greco-Roman god of war. In alchemy, Mars also symbolizes separation, which refers to the isolation of the components of dissolution by filtration and, subsequently, to the discarding of any ungenuine or unworthy material. The aforementioned maxim implies the struggle that an initiate must make in one's quest to understand the reality of oneself and the reality of the world. As we try to transmute our primal self into a higher and better self, and as we try to improve our existential conditions, we must face all our negative qualities and all negative historical forces, and win. This is the reason why we need the destructive force of "Mars." Additionally, the art of transforming a "Rough Ashlar" into a "Perfect Ashlar" requires struggle, discipline, and perseverance.

It is worth mentioning that, through his "Egyptian Rite," Cagliostro attempted to fuse the Ancient and Accepted Scottish Rite and the revolutionary visions of the Bavarian Illuminati. The three letters L.·.P.·.D.·. on Cagliostro's seal were the initials of the words "Lilia Pedibus Destrue," meaning "tread under foot the Lilies," and the Lily was the emblem of the French Crown (when Pope Leo III crowned Charlemagne as emperor in 800 C.E., he is reported to have presented him with a blue banner covered with golden lily flowers; in French, "fleurs-de-lis").

[8] Faulks and Cooper, *The Masonic Magician*; McCalman, *The Last Alchemist*.

Figure 1-11: Dr. Nicolas Laos's certificate of appointment to the Sovereign Italian Sanctuary of the Egyptian Rite of Misraim. The Certificate of Dr. Nicolas Laos's appointment as an honorary 97th-degree member of the Sovereign Italian Sanctuary of the Egyptian Rite of Misraim, issued in Rome, Italy, on the 10th of October 2020 (signed and sealed by the Imperatore Gran Conservatore and the Gran Cancelliere of the aforementioned Obedience of the Egyptian Rite of Misraim).



After the late 1780s and especially during the 1790s, modern Egyptology blossomed, because Napoleon the Great extended the French Empire into Egypt and brought Egyptian treasures and artifacts to Europe. Moreover, in 1799, the Rosetta Stone was discovered. The Rosetta Stone is a granodiorite stele inscribed with a decree issued at Memphis in Hellenistic Egypt in 196 B.C.E. on behalf of King Ptolemy V. The decree appears in three scripts: in ancient Egyptian hieroglyphs, in the demotic Egyptian language, and in ancient Greek. Thus, the Rosetta Stone provided the key to the modern understanding of Egyptian hieroglyphs. At the end of the eighteenth century and at the beginning of the nineteenth century, Europe experienced a phase of intense fascination with ancient Egypt. As I have already mentioned, the origins of "Egyptian Masonry" can be traced back to the Egyptian Rite of Masonry that was developed by the Italian mystic Count Alessandro di Cagliostro at the end of the eighteenth century. The first mention of the Misraim Rite was in Venice in 1788, and it spread rapidly to Milan, Genoa, and Naples.

The Rite of Misraim with its ninety degrees, was originally structured and established in a formal way in Italy in 1804–05. Its degrees were collected from several Masonic, Hermetic, Neoplatonic, Kabbalistic, Alchemical, and Rosicrucian sources. In 1813, the Milan "Mother" Lodge of Misraim granted Patents of the 90th Degree to a few French Masons. From 1814 onwards, under the leadership of brothers Joseph, Michel, and Marc Bédarride, the Rite of Misraim was quickly developed in France. Initially, the Grand Orient of France accepted the authority of the Misraim Rite, but, in December 1817, the Misraim Rite asserted its independence, and, thus, the Grand Orient of France started treating the Misraim Rite in a rather inimical way.

During the nineteenth century in France, the Rite of Misraim served as a meeting-place of opponents to the regime. Moreover, during the nineteenth century the Misraim Rite also transmitted the required Third Degree to the Carbonari, and the Carbonari recruited members in Misraim and Memphis Lodges. The Carbonari were an Italian revolutionary secret society which, in 1814, attempted to obtain a constitution for the Kingdom of the Two Sicilies by force and, in 1820–21, began resisting the French occupiers, notably Joachim Murat, the Bonapartist King of Naples. Thus, finally, the Carbonari became a liberal nationalist secret society expressing militant opposition to Austria's regime (precisely, Emperor Francis I of Austria and Austrian State Chancellor Prince Klemens von Metternich), and to the Holy Alliance in general. They were instrumental in organizing revolutions in Italy in 1820–21 and in 1831. In the nineteenth century, nationalism was used by liberal political forces as a means of fighting against imperialist and authoritarian regimes (such as those of Napoleon and the Powers of the Holy Alliance) and creating liberal republics.[9]

The Memphis Rite (also known as the Oriental Rite) is a modification of the Misraim Rite, and it was originally organized in Paris, in 1838–39, by Jacques-Étienne Marconis de Négre, who had previously been a member of the Rite of Misraim. The Memphis Rite was composed of ninety working and six honorary and/or administrative degrees. References to an Egyptian system of Masonry were made in a pamphlet entitled *Master of Masters*, which appeared in Paris in 1815. In fact, by that time, French Freemasonry had already been established in Egypt by the armies of Napoleon the Great, and, from thence, mingled with Hermetic and other local mystical traditions, it was transplanted to Montauban in France in 1816 by the French Freemasons Gabriel-Mathieu Marconis de Négre, Baron Dumas, J. Petite, H. Labrunie, Samuel Honis,

[9] Hobsbawm, *The Age of Revolution: 1789–1848*.

and more. Thus, the Rite of Memphis was formed. After an interval of inactivity, this system of Egyptian Masonry was revived in Brussels and Paris between 1838 and 1839 by Jacques-Étienne Marconis de Négre, son of Gabriel-Mathieu Marconis de Négre.

In 1872, John Yarker merged the Rite of Memphis and the Rite of Misraim, and he founded a Sovereign Grand Sanctuary of the Rite of Memphis and Misraim in his hometown of Manchester. Additionally, at the end of the nineteenth century John Yarker established the "Ancient Universal Pansophic Rite," in the context of which he attempted to synthesize all esoteric European lineages of the various "Illuminati" currents and organizations. Like Papus after him, Yarker wanted to create a universal initiatory movement by uniting all the rites, the orders, and the currents of the Western mystery tradition of the Illuminati into one global superstructure. The traditional English Freemasonry was one of the fiercest opponents of such initiatives and ideas, and this is one of the reasons why, during the nineteenth and the twentieth century Anglo-Saxon world, the so-called regular Freemasonries consolidated their policy of considering the Rite of Memphis–Misraim as an "irregular" type of Masonry.

Another initiatory line of the Ancient and Primitive Rite of Memphis–Misraim was founded by the Italian general and politician Giuseppe Garibaldi (1807–82), who is considered, with Camillo Cavour, Victor Emmanuel II, and Giuseppe Mazzini, as one of Italy's "fathers of the fatherland." He played a protagonist role in the Risorgimento (namely, the political and social movement for the unification of Italy), and he was appointed general by the provisional government of Milan in 1848, and General of the Roman Republic in 1849 by the Italian Minister of War. In 1881, Giuseppe Garibaldi had already initiated a process of merging the Rite of Memphis and the Rite of Misraim with the Scottish Rite; Garibaldi's aforementioned initiative was accomplished in 1889, and he was proclaimed International Grand Master of the Rite of Memphis–Misraim.

To some extent, the bridge between the ancient Egyptian Mysteries and the Egyptian Rite of Freemasonry has been clarified by Jacques-Étienne Marconis de Négre 97º through the myth of Ormus, also known as Ormesius. According to this myth, in the year 46 of the Common Era, St. Mark the Evangelist converted Ormus, a Priest of the already very ancient Egyptian Mysteries, to Christianity. After this, rather than abandoning the mysteries, Ormus Christianized the mystery rites. In doing so, he became the mythic and eponymous forefather of the Hermetic Brotherhood of

Light, the Egyptian Rite of Memphis, and even Rosicrucianism. In Jacques-Étienne Marconis de Négre's words:

> The followers of Ormus, up to 1118, were the only trustees of the ancient doctrines of Egypt, purified, of course, by Christianity and the teachings of Solomon. Those doctrines they communicated to the Templars. They were in consequence known under the name of "Knights of Palestine" or "Brethren of the Rosy Cross of the Orient"; these Brethren are claimed by the "Rite of Memphis" as its immediate founders.
> In 1150, eighty-one of them under Garimont went to Sweden, presented themselves to the archbishop of Upsala and explained their Masonic doctrines to him. It was these eighty-one Masons who established Masonry in Europe.

Thus, Ormus is the personification of a mystery tradition and, particularly, of the survival of ancient Egyptian mysteries in Christianity, after those ancient Egyptian mysteries had been properly adapted to Christianity. Such stories come down to us from the Hierophants of Memphis of the nineteenth and the early twentieth centuries, from J.-E. Marconis 97º to John Yarker 97º.

Figure 1-12: The transition from the Rough Ashlar (shown at the bottom of the photograph) to the Perfect Ashlar and the Pyramid (in Dr. Nicolas Laos's Mother Lodge "Misir" of the Ancient and Primitive Rite of Memphis–Misraim in the Zenith of Vršak, Serbia).

Figure 1-13: Emblems of high Masonic degrees. On the left: the Emblem of the Sovereign Grand Inspectors General (namely, the recipients of the 33rd degree) of the Ancient and Primitive Rite of Memphis–Misraim (this degree is almost identical with the 33rd degree of the Ancient and Accepted Scottish Rite). On the right: the Emblem of the Grand Conservators (namely, the recipients of the 95th degree) of the Ancient and Primitive Rite of Memphis–Misraim. These emblems have been designed and are used by the Mother Lodge "Misir" of Serbia of the Ancient and Primitive Rite of Memphis–Misraim. The double-headed eagle is the most characteristic symbol of the Scottish Rite, and it symbolizes spiritual and material powers working together. The winged egg (which, in ancient Egyptian religious art, was known as the "Kneph") is the most characteristic symbol of the Egyptian Rite, and it symbolizes universal spirit acting on primordial matter, and, generally, it represents the productive world.

I should add that my Freemasonic career has been indirectly yet significantly reinforced by my engagement in chivalric institutions, such as the Hospitaller Order of Saint Lazarus of Jerusalem (Fig. 1-14), and by my study of chivalry. The term "chivalry" derives from the French word "chevalier," which means horseman. Medieval Crusades had their glory and their sense of nobility, since they expressed particular communities' dedication to a higher goal and a clear strategic vision, contributed to the development of international relations, and broadened the intellectual horizons of medieval Western Europe. However, far from endorsing any

type of religious war, I support the conduct of a Humanistic Crusade for the advancement of science and ethics.

Figure 1-14: Dr. Nicolas Laos, KLJ, with Lord and Lady Lingfield. From left to right: The Rt Hon the Lord Lingfield (Robert Balchin, Baron Lingfield, Kt, DL), who is a distinguished British educator, a member of the UK House of Lords, and the Patron of the Saint Lazarus Hospice Association (namely, the charitable apparatus of the United Grand Priories of the Hospitaller Order of Saint Lazarus of Jerusalem), Dr. Nicolas Laos, wearing the chain collar of a Knight of Saint Lazarus of Jerusalem, and Lady Lingfield, in the Church of Saint Bartholomew the Great in the City of London, in 2008, at an investiture ceremony of the United Grand Priories of the Hospitaller Order of Saint Lazarus of Jerusalem.

The aforementioned experiences and milestones of my jounry in Freemasonic and chivalric institutions constitute the Masonic "arch" on which my Project is based; and the keystone of the Masonic "arch" of my Project is the foundation of the Esoteric Initiatory Grand Lodge of Greece ("Gran Loggia Iniziatica Esoterica di Grecia" in Italian) and my installation as the Grand Master of the Esoteric Initiatory Grand Lodge of Greece and as the Sovereign Grand Commander of the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the aforementioned Grand Lodge. My scholarly background, which I delineated in the previous section of this chapter, constitutes the scholarly "arch" on which my Project is based.

In April 2023, the Grand Master and Sovereign Grand Commander 33º of the Italy-based Gran Loggia Iniziatica Esoterica the Most Venerable and Most Puissant Sister Ginevra Di Nicola (Fig. 1-15) officially authorized me to found the Gran Loggia Iniziatica Esoterica di Grecia (Esoteric Initiatory Grand Lodge of Greece), and she installed me as the Grand Master and Sovereign Grand Commander 33º of the Gran Loggia Iniziatica Esoterica di Grecia (Fig. 1-16). Thus, on the basis of a Masonic Charter and a Masonic Decree issued to me by the Gran Loggia Iniziatica Esoterica and by the Supreme Council 33º of the Scottish Rite Body that is attached to it, I became the Grand Master of the Gran Loggia Iniziatica Esoterica di Grecia and the Sovereign Grand Commander of the Supreme Council 33º of the Scottish Rite Body that is attached to the Gran Loggia Iniziatica Esoterica di Grecia. As regards the scholarly and the Masonic background of my distinguished Sister Ginevra Di Nicola, I should mention that she is a brilliant intellectual, she graduated in Religious Sciences with Magisterium at the Pontifical Gregorian University, and she was initiated into Freemasonry in the "Argillano" Lodge of Ascoli Piceno under the auspices of the Grand Lodge of Italy of the Ancient Free and Accepted Masons, piazza del Gesù, Palazzo Vitelleschi.

Figure 1-15: On the left: The Grand Master and Sovereign Grand Commander 33º of the Italy-based Gran Loggia Iniziatica Esoterica the Most Venerable and Most Puissant Sister Ginevra Di Nicola. On the right: The coat of arms of the Gran Loggia Iniziatica Esoterica (https://granloggiainiziaticaesoterica.org/).

Figure 1-16: The Decree of the Grand Master and Sovereign Grand Commander 33º of the Italy-based Gran Loggia Iniziatica Esoterica the Most Venerable and Most Puissant Sister Ginevra Di Nicola with which she installed Dr. Nicolas Laos as the Grand Master and Sovereign Grand Commander 33º of the Gran Loggia Iniziatica Esoterica di Grecia (this Decree dated 3 April 2023 bears the signatures and the seals of the Sovereign Grand Commander-Grand Master, the Grand Secretary, and the Grand Keeper of the Seal of the Gran Loggia Iniziatica Esoterica).


GRAN LOGGIA INIZIATICA ESOTERICA

A.'.G.'.D.'.G.'.A.'.D.'.U.'.

Massoneria Universale di Rito Scozzese Antico e Accettato

NOI
Ginevra Di Nicola 33°
Sovrano Gran Commendatore Gran Maestro

visti gli Statuti dell'Ordine e del Supremo Consiglio ed in ragione dei poteri che Ci furono conferiti

ABBIAMO DECRETATO E DECRETIAMO

La nomina del Ven.mo e Pot.mo Fr . ' . Nicolas Laos 33° a Sovrano Gran Commendatore Gran Maestro per la Grecia della GRAN LOGGIA INIZIATICA ESOTERICA DI GRECIA, con giurisdizione assoluta su tutto il territorio della Repubblica Ellenica. Il Ven.mo e Pot.mo Fr . ' . Nicolas Laos 33° è autorizzato, in Nostro nome e per Nostro conto, a concedere a tutti i Fratelli delle Logge da lui governate nulla osta per iniziazioni, passaggi di grado,elevazioni e aumenti di Luce.
Ha altresì facoltà di concedere, Motu Proprio, qualunque grado fino al 32° secondo quanto la Sua Saggezza riterrà opportuno. Attraverso questo decreto al Ven.mo e Pot.mo Fr . ' . Nicolas Laos 33° viene conferito mandato di rappresentarCi su tutto il territorio ellenico.
Decreto nr.011/2023 del 30 marzo 2023
Che il Grande Architetto dell'Universo ci assista.

Zenith di Chieti, terzo giorno del secondo mese dell' A . ' . L . ' . 6023 , 03 aprile 2023 Era Volgare

IL SOVRANO GRAN COMMENDATORE GRAN MAESTRO

IL GRAN SEGRETARIO **IL GRAN GUARDASIGILLI**

The Gran Loggia Iniziatica Esoterica di Grecia (Esoteric Initiatory Grand Lodge of Greece), under my Grand Mastership and in communion with the Italy-based Gran Loggia Iniziatica Esoterica under the Grand Mastership of the Most Venerable and Most Puissant Sister Ginevra Di Nicola, was the first Grand Lodge to endorse and practice the Modern and Perfecting Rite of Symbolic Masonry, which I authored, and, therefore, in the context of (and for) the Modern and Perfecting Rite of Symbolic Masonry, the Gran Loggia Iniziatica Esoterica di Grecia is elevated to the rank of "Grand Lodge of the Modern and Perfecting Rite of Symbolic Masonry," whose role and structure are delineated in this volume. In this case, I have followed the precedent of the Royal Order of Scotland: In 1750, the English Masonic authorities in London granted a Scotchman by the name of William Mitchell a Charter to create a Lodge of the Royal Order of Scotland in The Hague, where he was working as a teacher of the English language, and, about 1752 or 1753, by virtue of that Charter, Mitchell ultimately set up a Lodge of the Royal Order of Scotland in Edinburgh, and, in due course, that body was to elevate itself to the rank of Grand Lodge of the Royal Order of Scotland (in 1767), thus becoming the supreme international custodian of this particular Masonic system (i.e., the Royal Order of Scotland). Similarly: (i) I was granted a Charter and a Decree by the Italy-based Gran Loggia Iniziatica Esoterica, which officially installed me as the Grand Master and the Sovereign Grand Commander 33º of the Gran Loggia Iniziatica Esoterica di Grecia; (ii) the Gran Loggia Iniziatica Esoterica di Grecia endorsed and applied the Modern and Perfecting Rite of Symbolic Masonry as a method of practicing the degrees of Symbolic Masonry and of the Ancient and Accepted Scottish Rite; and (iii) as regards the Modern and Perfecting Rite of Symbolic Masonry, the Gran Loggia Iniziatica Esoterica di Grecia is elevated to the rank of "Grand Lodge of the Modern and Perfecting Rite of Symbolic Masonry," and, thus, it is the supreme international custodian of this Rite in particular (maintaining of course its allegiance to the Gran Loggia Iniziatica Esoterica, from which it has received the Masonic Light).

The creation of a new Masonic Rite should be judged on its own merits, and not according to the interests and the profane calculations of an established "cartel" of Masonic organizations. After all, the Rites, the Rituals, and the Orders that emerged in the previous centuries were developed, reformed, rewritten, and applied in different ways in the context of their acceptance by different Masonic groups. Let us also rember that, in the eighteenth-century England, the Moderns' Grand Lodge (officially known as the Grand Lodge of London and Westminster)

and the Ancients' Grand Lodge (officially called the "Grand Lodge of Free and Accepted Masons of England according to the Old Constitutions") used different Rites and Rituals, which were ultimately fused into the Emulation Rite as a result of the union between the two aforementioned Grand Lodges that took place in 1813 (giving birth to the United Grand Lodge of England). Additionally, we know that the prominent German Masonic leader Friedrich Ludwig Schröder rewrote the Ritual of Symbolic Masonry, and that the fact that several German Grand Lodges and individual Lodges adopted Schröder's revision of the Ritual of Symbolic Masonry set the stage for the establishment of a great German and international Masonic Alliance. In general, the Rites of Freemasonry develop through the development of human understanding regarding both Masonic knowledge and the dynamics of civilization. With this mindset, and with a deep awareness of the dynamic history of Freemasonry, I created the Modern and Perfecting Rite of Symbolic Masonry, revising the Rituals of the degrees of Symbolic Masonry and of the Ancient and Accepted Scottish Rite accordingly, as delineated in this book.

Ne Plus Ultra: My Initiation Lineage in the Illuminati and my mentor Grand Master Dr. Giuliano Di Bernardo

When we have to talk about the "Illuminati," we primarily think of the "Illuminati" of Bavaria.[10]That secret society was founded in 1776 by Adam Weishaupt, Professor of Law at the University of Ingolstadt in Bavaria. Some people connect the Bavarian Illuminati with the Spanish mystical Christian association of the "Alumbrados" ("Illuminated") of the sixteenth century, or the French "Société des Illuminés d'Avignon" ("Society of the Illuminati of Avignon"), which was founded around 1784 by Dom Antoine-Joseph Pernety and a Polish Count called Tadeusz Leszczyc-Grabianka. This argument is not correct; the name was similar, but nothing else. In fact, there are several varieties of "Illuminati" movements. In addition, it is hard to prove that all of the ideological assumptions and arguments of the Bavarian Illuminati were originally formulated and proclaimed by the Czech philosopher, educator and theologian Comenius (1592–1670), who addressed the first formal scientists as "Illuminati," and by other Renaissance scholars and mystics, as some researchers maintain. The truth is that the secret society that was created by Law Professor Adam Weishaupt was originally called the "Order of the

[10] For more details, see: Melanson, *Perfectibilists*; Wäges and Markner, eds, *The Secret School of Wisdom*.

Perfectibilists," and it was only later that it was called the "Order of the Illuminati (of Bavaria)." Initially, it consisted of students of the University of Ingolstadt, but later it became bigger and stronger, with a structure similar to that of Freemasonry and a vertical and ritualistic system. They wished to establish a new world order founded on the idea of a European Republic and a peculiar type of socialism/communism. However, as I have already mentioned, there are different varieties of "Illuminati" and different varieties of "illuminist" narratives.

Very soon after the foundation of the Order of the Illuminati of Bavaria, a fierce disagreement broke out between Adam Weishaupt, who was the Chairman of the Illuminati's "Areopagus" (executive apparatus), and Adolph Freiherr von Knigge, who was a rich and powerful member of the Illuminati's "Areopagus." In 1784, the clash between Weishaupt and Knigge led to Knigge's departure from the Order of the Illuminati, threatening as he left that he would reveal the Illuminati's secrets. This internal dissension was one of the main reasons for the demise of Weishaupt's Order of the Illuminati. Another important reason for the demise of Weishaupt's Order of the Illuminati was the fact that, in 1777, the relatively liberal Prince-Elector of Bavaria Maximilian I Joseph was replaced by Karl Theodor, whose outlook was more conservative. In 1785, Karl Theodor banned all secret societies, including the Illuminati. Thus, the Order of the Illuminati was shattered in Bavaria. Weishaupt fled Bavaria and settled in Gotha. With the cooperation of the Jesuits, who were strong opponents of Weishaupt's work, the authoritarian Bavarian political regime that was established by Karl Theodor conducted a vicious propaganda campaign against Adam Weishaupt and the Illuminati in Bavaria throughout Europe. Moreover, many Freemasons, mainly out of fear of being persecuted by political and religious authorities, denounced the Illuminati. I was made and installed an Illuminatus of the highest rank by the Illuminati Grand Master the Most Eminent Brother Professor Giuliano Di Bernardo.

Giuliano Di Bernardo occupied the Chair of Philosophy of Science and Logic at the Faculty of Sociology of the University of Trento (Università degli Studi di Trento), Italy, from 1979 to 2010. He is a member of the "International Academy for Philosophy of Science" ("Académie Internationale de Philosophie des Sciences"), which is based in Brussels, and one of its founders was Albert Einstein. He is also a member of several other international academies. At the University of Trento, he has served as Vice Rector and Director of the Department of Social Sciences. The fundamental object of the philosophical research that has been conducted by Professor and Academician Giuliano Di Bernardo is the

epistemological foundation of the social sciences. He has authored numerous studies in deontic logic. He has published numerous studies on epistemology and, particularly, on the philosophy of the social sciences, including the following books: *Introduzione alla logica dei sistemi normativi* (Il Mulino, 1972), *L'indagine del mondo sociale* (Angeli, 1979), *Le regole dell'azione sociale* (Il Saggiatore, 1983), and *Il futuro di Homo sapiens* (Marsilio, 2020).

In 1992, in Switzerland, Giuliano Di Bernardo, in collaboration with distinguished international personalities (such as Spencer Compton, 7th Marquess of Northampton and Member of the House of Lords of the United Kingdom, Hans Rudolf Meyer, lawyer and Mayor of Lucerne, Walter Hess, Professor of Surgery at the University of Basel, Giorgio Cavallo, Professor of Microbiology at the University of Turin, Vittorio Mathieu, Professor of Philosophy at the University of Turin, Mario Conde, president of the Spanish bank Banco Español de Crédito, Felix Ruben, commodity trader, and Joseph Guggenheim, Zurich lawyer), founded the Dignity Foundation on which the Dignity Order is based. The Dignity Order is an international esoteric and initiatory fraternity for the defense of human dignity with Giuliano Di Bernardo as its president (the first general assembly of the Dignity Foundation-Order took place at Ashby Castle, Lord Northampton's second residence, on 31 January 1993). Meanwhile, in 1990, Giuliano Di Bernardo was installed as the Grand Master of the Grande Oriente d'Italia (Grand Orient of Italy), the largest and oldest Italian Freemasonic institution. In 1993, he resigned from the Grand Orient of Italy and founded the Regular Grand Lodge of Italy (Gran Loggia Regolare d'Italia), which was immediately recognized by the English Freemasonry, and he remained the Grand Master of the Regular Grand Lodge of Italy until 2002. In 2002, having decided that he had completed his work in conventional regular Freemasonry, Giuliano Di Bernardo dedicated himself to the Illuminati.

As I have already mentioned, according to the terminology of the 17th century Czech philosopher, educator and theologian Comenius, "Illuminati" is a generic Latin term meaning conscientious scientists, but, in the 18th century, the term "Illuminati" became famous (and controversial) because of the turbulent history of the Bavarian secret society of the "Perfectibilists," which was founded by Adam Weishaupt and later became known as the Illuminati, charging this word with a more specific meaning, as well as because of the French mystical fraternity "Société des Illuminés d'Avignon."

In 2002, Giuliano Di Bernardo founded the Accademia degli Illuminati (Academy of the Illuminati), seated in Rome, at number 33 Piazza di

Spagna. It is noteworthy that, on the 11th of July 2002, when the "Accademia degli Illuminati" was officially founded as a legal entity at the notary's office of Giovanni Pocaterra in Rome, monsignor Giorgio Eldarov, a high-ranking and highly educated Vatican official, was one of the co-founders of the "Accademia degli Illuminati." Under the Grand Mastership of Giuliano Di Bernardo, the aforementioned Dignity Order is the operational apparatus of the Order of the Illuminati, which, under Di Bernardo's leadership, works in an exclusive and unique way reviving and updating the Illuminati movements of the 18th century, not as an atavistic imitation of Weishaupt's Bavarian Illuminati (nor as the various theatrical and bogus "neo-Illuminati" organizations that have sprung up from time to time around the world), but as a modern and philosophically informed esoteric-initiatory society for the material and moral development of humanity, thus reinvigorating the tradition of the "Illuminati."

In response to my request, Giuliano Di Bernardo wrote and sent me the following text, in which he summarizes his reasoning regarding the Order of the Illuminati, of which he is the Grand Master:

> Space travel and the deciphering of the human genome are just some of the scientific breakthroughs of our time. These have led to important technological applications, which have had a profound impact on our daily existence. For example, it is now possible to travel anywhere on the planet in a short amount of time; we have mobile phones that allow us to communicate with others wherever they are; through television we get real-time updates on events taking place in all corners of the earth. The consequence of all this is "globalization." Many see it as the cause of many of the evils that afflict humanity today and hope we can return to the way things were previously. Unfortunately, this cannot be the case. The more discoveries science makes, the more globalization becomes a necessity.
>
> In an increasingly globalized world, there is no longer any place for traditional Freemasonry with its national territorial restrictions. The challenges imposed by global society can no longer be resolved within individual States, but require a broader vision involving international strategies that go beyond the narrow confines of individual nations. The same is true of Freemasonry, which is a mirror of society. If Freemasonry is to keep pace with the times and be involved in the improvement of humanity, it needs to become international. I understood the difficulties that would come with this transformation, but there was no alternative: Freemasonry must either renew itself or it will disappear.
>
> Traditional Freemasonry was characterized by a lack of "universality." To be truly universal, Freemasonry must exclude no one. Over the course of its history, however, it has excluded important swathes of society such as women. While it is true that, in previous centuries, women were considered merely for their reproductive capacity, now that they have been

emancipated, they can no longer be excluded. Plato understood this when he stated that he saw no difference in the social activities of man and woman. The exclusion of women from Freemasonry today means forgoing a fundamental resource for Masonic activities in society. Is this how we want to meet the challenges of science?

Freemasonry imposes other important exclusions. English Freemasonry, for example, requires Masons to declare a religious faith. This means that atheists are excluded. In order to be consistent, most scientists who do not believe in a deity would also be excluded. On the other hand, some European Freemasonries exclude those who profess a religious faith, which contradicts English Freemasonry. Lastly, some Freemasonries admit women, but they are considered "irregular." Confusion reigns supreme.

To achieve true universality, the various Freemasonries should review their doctrines. Is that possible? What makes Freemasonry special is its esoteric and initiatory foundation. This means that it differs from all other conceptions of life and man precisely because it has chosen this basis and has fostered it, as I explained in *Filosofia della Massoneria*. If this defining aspect of Freemasonry ceases to exist, then Freemasonry is no longer Freemasonry.

Does Freemasonry still have an esoteric foundation? This is the question that lies at the very heart of the problem. Lack of internationalism and universality concerns the so-called "profane" aspects of Freemasonry. However, Freemasonry is not just a profane society. Its esoteric foundations are specific to it. The "profane" and the "esoteric" must be brought together, with priority given to the latter. Is the esoteric still a priority for today's Masons? All the analysis and reflections on Freemasonry conducted thus far have referred to its "profane" foundation. Nothing has been said of the "esoteric" foundation that is specific to it.

Formally speaking, all existing Freemasonries observe the esoteric rules: Masons gather in the "Temple" (sacred place), wear aprons and jewels, recite rituals, initiate, pass, and raise candidates to the Higher Degrees. Everything seems to be in order. But it is not. The truth is that all this is devoid of any authentically esoteric meaning. Rituality, in all its manifestations, is today an unconscious repetition of actions. Those who perform them do not necessarily know their meaning. This makes for a breeding ground for profound contradictions and anomalies. For example, consider my election to Grand Master of the Grand Orient of Italy. The run-off between Tiberi and me took place within the consecrated Temple, in the performance of ritual. Yet, in that very place, plots of all kinds were being made to take over power. It was all "profane." Why do such things in the sacredness of the Temple? Why mix it with ritual? The answer is simple: none of those present understood the initiatory foundation of Freemasonry. How can they be Masons? The truth is that they are not.

Freemasonry arose out of the esoteric tradition of humanity, which began in Greece in the sixth century B.C.E. with the mystery religion of Orpheus and incorporates its doctrine and rituality. After the Second World

War, however, as Italian Freemasonry emerged from the exile imposed by fascism, representatives of American Freemasonry became the bearers of the "democratization" of Freemasonry, bestowing on it a political role. Italian Freemasonry thus had no chance of rebuilding itself. Nothing remains of the glorious Freemasonry that existed before the advent of fascism. This is how the "counter-initiation" of Freemasonry was born, developing later into the likes of the P2 Lodge. Counter-initiation upturned the Masonic pyramid, with the tip pointing downwards and the base upwards.

The esoteric foundation that is specific to Freemasonry is particularly expressed through the Ceremonies of Initiation, Passing, and Raising, which are marked by Ritual. Ritual, therefore, is essential to the "perfecting" stages of the Mason. But what ritual? It is immediately clear that there is no one single ritual valid for all Freemasonries around the world. Of the few important rituals, the most important is the Emulation Rite used in the United Grand Lodge of England and in the Freemasonries it recognizes. This Ritual, however, was taken from the Bible. It can be said that it presents the same religious characteristics as the Bible. Thus, it is a religious ritual, as the fathers of the Catholic Church maintain. Moreover, the Emulation Ritual is in stark contrast with the "Declaration" issued on 21 June 1985 by the United Grand Lodge of England, entitled "Freemasonry and Religion," which states in no uncertain terms that Freemasonry is not a religion. If Freemasonry is not a religion, then why does it use a religious ritual? The answer is simple: there are no philosophers in the United Grand Lodge of England.

While maintaining the Masonic conception of man, in order to go beyond current Freemasonry, it is necessary to reform it along the lines proposed here. This is exactly what I did when I founded the Order of the Illuminati and the Dignity Order. My vision of a universal and harmonic human society, which I had tried unsuccessfully to implement in the Grand Orient of Italy and in the Regular Grand Lodge of Italy, was waiting to enter human history. What would it look like?

I began to review the various esoteric societies that had come and gone throughout the millennia of human existence, to find the right historical "raiments" for my vision of life and man. I was drawn to the Order of the Illuminati, an Order about which I had considerable knowledge. The Illuminati expressed true universality because they considered man for his innate qualities, irrespective of gender, skin colour, language, religion, and culture. I made my decision. I would awaken Italy and the world to the Order of the Illuminati. I was qualified to do so, as I had been Grand Master of a regular Freemasonic Body that had bestowed initiatory authority on me.

The Order of the Illuminati would take account of the difficulties faced by Freemasonry today. First, it must restore the proper traditional relationship between its initiatory and profane identities. Since the main consequence of the deterioration of Freemasonry has been that the pyramid

has been upturned, the Order of the Illuminati will have to return it to its upright position. Hence, even in its profane identity, power must descend from the top downwards. The initiatory and profane pyramids must be positioned in the same manner and be overlapping.

A characteristic shared by the Order of the Illuminati and Freemasonry is the awareness that their respective followers must have of their principles, doctrines, esotericism, symbolism, history, and projects. Unfortunately, Freemasonry has admitted men who are unaware of such matters. This is another important reason why it has deteriorated. The Illuminati, on the other hand, must have such an awareness in order to educate humanity.

All regenerations of the Illuminati have certain characteristics in common. The first is *universality*. The Illuminati are human beings who have light in their consciousness, irrespective of their identity in terms of gender, race, religion, language, and culture. They, and only they, will form the "International Order of the Illuminati." Their sphere of competence is the world. Their projects involve the planet.

The second characteristic is *wisdom*. The Illuminati are wise because they follow the path of initiatory perfection. They have attained the highest levels in all areas of human knowledge. They are similar to the Philosophers of Plato's Republic, who governed public affairs with wisdom and justice.

The third characteristic is *authority*. Knowledge and wisdom give man the authority to govern. Those who exercise this authority must be charismatic and far-sighted and must have a Project that will guide humanity towards wellbeing and happiness.

The fourth characteristic is *power*. Authority requires the necessary power so that the Project can be realized. The success of human enterprise is guaranteed where power is exercised wisely.

The fifth characteristic is the *Enlightened One*. When the "globalization" process is complete, the problem that will arise will concern how to govern humanity. A choice will have to be made between various possible forms of government: "democracy" (majority rule) and "tyranny" (rule by one). Currently, there is a tendency, particularly in the West, to spread democracy, considered to be the best form of government, to all regions of the planet. However, there is no shortage of examples of its degeneration and negative consequences, causing us to doubt that democracy can govern the world in the future. The alternative is tyranny (or monarchy or dictatorship). Tyranny, however, evokes the disastrous world wars that shaped the twentieth century, thanks to tyrants such as Hitler and Stalin. With good reason, humanity tries to forget that dark part of its history. Therefore, negative reactions to the concept of rule by one are understandable. To guide the future of humanity, however, we need to seek the best possible government rationally rather than emotionally. The tyrant referred to here is not an emulation of Hitler or Stalin, but rather an expression of the Community of the Illuminati, governing the world wisely, leading humanity towards wellbeing and happiness.

This is the vision I had when I revived the Illuminati, first in Italy and then in other countries. The articles of association are dated 1 August 2002. The first seat was in Rome, at 33 Piazza di Spagna. In its profane history, the Order of the Illuminati devised plans for the material and moral improvement of humanity. The completion of these projects involved those who have direct and concrete knowledge of societal differentiation, inequality, and stratification. They are the ones who have the necessary skills to carry out the projects conceived by the Illuminati. Those who devise the plans for improvement are known as "speculative"; those who complete them in society are termed "operative."

To avoid misunderstandings, I have assembled the operative Illuminati into an Order that I have called "Dignity," whose purpose is the defense of human dignity. Therefore, Dignity is the operative part of the Illuminati. Dignity is an international esoteric Order that can be represented as a pyramid headed by the Grand Master. I am the founder and Grand Master of Dignity. The institutional purpose of the Order is the defense of human dignity, that is, the moral nobility to which man belongs on account of his intrinsic qualities and his very nature. The notion of "dignity," therefore, refers to man as such and expresses a universal characteristic that is found in all men, regardless of gender, age, race, religion, language, and culture. Therefore, it is a constituent element of man, in that, if man loses his dignity, then he is no longer a man.

Precisely because dignity is an irrepressible condition of man and is an expression of his nature, it is found in all conceptions of man, from the religious to the secular. However, in the world we live in, dignity is despised and humiliated. Many men and women are forced to live without dignity. It is for this reason that humanity is losing the ideals that have always sustained it. Faith in God is on the wane, and humanity seems to be lost in the mists of materialism and utilitarianism. Since the notion of "dignity" is universal, its scope is unlimited. Therefore, the countless qualities that characterize human nature fall within its sphere.

An Order that proposes the defense of human dignity must choose the most essential qualities and values that define it. This choice must take account of the particular historical and circumstantial conditions affecting humanity. In the world in which we live, defense of human dignity primarily means defending ethnic minorities, women, the weak, and the persecuted. For the Order of Dignity, defense of these aspects of human dignity is the most important institutional purpose, to be achieved with the highest moral and intellectual commitment.

Since Dignity is an international Order, I established in Austria, under Austrian law, the "International Association of Dignity," registered on 4 July 2012, with headquarters in Vienna. Its task is to govern national Orders. Dignity was founded in Italy and is spreading to other countries in the world.

The Order of the Illuminati has both mythical and historical origins. Their mythical origins date back to antiquity when they comprise all the

manifestations of the sacred devoted to the light. Of particular importance were the Orphic and Pythagorean sects. The most illustrious precedent was the *Illuminati of Bavaria.* Although the historical circumstances that gave rise to this Order were very different from those of today, its origin, development, and demise may stand as a model for those seeking to understand the reasons why the Illuminati should return. The main difference between the Illuminati of the past and those of today is that, whereas the former wanted to animate a closed and stable world almost entirely dominated by the Jesuits, the latter instead want to give stability and openness to a world that seems excessively changeable. Adam Weishaupt founded the Order of the Illuminati on 1 May 1776 in Bavaria. Its declared purpose was to bring its members to the highest degree of morality and virtue, to free their minds from prejudice and superstition, and to reform the world by defeating the evils that afflict it. As regards Freemasonry, Weishaupt maintained that his Order should remain distinct from it because the mysteries of Freemasonry were too puerile and too easily accessible to public opinion. The consequence was that the grades and rituals of the Illuminati were different from those of the Freemasons. Moreover, the Order of the Illuminati, being a specific interpretation of the millenary esoteric tradition that starts with Orphism in the sixth century B.C.E., presents some common characteristics with other initiatory societies, such as the Rosicrucian Movement and Freemasonry.

Figure 1-17: The coat of arms of the Dignity Order, which is the operational apparatus of the Order of the Illuminati under the Grand Mastership of Professor Giuliano Di Bernardo.

I have the honor and pleasure to be a personal and institutional associate and assistant of Giuliano Di Bernardo both in the Dignity Order and in its inner circle, that is, in the Illuminati Executive Council which functions as the steering committee of the Dignity Order (Fig. 1-18, 1-19, and 1-20). His wisdom guides my work in Freemasonry, and the training that I have received from the Order of the Illuminati under the Grand Mastership of Giuliano Di Bernardo has played a key role in the way that I have articulated the Modern and Perfecting Rite of Symbolic Masonry.

Figure 1-18: The Illuminati Diploma of Dr. Nicolas Laos, issued, signed and sealed by Grand Master Giuliano Di Bernardo, stating the following:
"The Grand Master Giuliano Di Bernaardo Officially Grants Enlightenment to the Most Illustrious Gentleman Nicolas Laos and Confers Upon Him the Power and the Privileges of the Illuminati." The operational apparatus of the Illuminati is the Dignity Order (www.dignityorder.com).

Figure 1-19: The official certificate with which Grand Master Giuliano Di Bernardo conferred upon Dr. Nicolas Laos "the Power and the Privileges to form in Greece the National Headquarters of the Illuminati." The operational apparatus of the Illuminati is the Dignity Order (www.dignityorder.com).

Figure 1-20: The Grand Master of the Order of the Illuminati the Most Eminent Brother Professor Giuliano Di Bernardo (sitting in his private office in his residence in Trento, Italy) and Dr. Nicolas Laos (standing next to him), February 2023.

Giuliano Di Bernardo has defined the world-concenception and the philosophical anthropology of the Order of the Illuminati, and he has authored the rituals of the Order of the Illuminati. I am fully and uncompromisingly committed to his Illuminati Order and to the advices and guidelines that he has personally given to me in our private meetings. I have created the Modern and Perfecting Rite of Symbolic Masonry, which is officially and exactly exposed and explained in this volume, in order to provide people with a way of practicing the degrees of Symbolic Masonry and of the Ancient and Accepted Scottish Rite in line with the fundamental ethos of Giuliano Di Bernardo's Order of the Illuminati and with the results of his and my own philosophical research.

Figure 1-21: On the left, the Grand Master and Sovereign Grand Commander 33º of the Italy-based Gran Loggia Iniziatica Esoterica the Most Venerable and Most Puissant Sister Ginevra Di Nicola, who has installed me as the Grand Master and Sovereign Grand Commander 33º of the Gran Loggia Iniziatica Esoterica di Grecia, and, on the right, the Illuminati Grand Master the Most Eminent Brother Giuliano Di Bernardo, who has conferred upon me the Power and the Privileges of the Illuminati; during a meeting that took place between them in April 2023.

Let us recall the tragedy *Prometheus Bound*, which was composed by Aeschylus in the fifth century B.C.E. Aeschylus was not only one of the most prominent ancient Greek authors of Greek tragedy but also an initiate

of the Eleusinian Mysteries and a brave fighter, since he fought to defend Athens against the invading Persian army at the Battle of Marathon (490 B.C.E.). Aristotle, in his *Nicomachean Ethics* (111a8–10), writes that Aeschylus was once accused of improperly revealing secrets of the Eleusinian Mysteries, but he was ultimately acquitted.

At the outset of the play *Prometheus Bound*, Prometheus is seen accompanied by two faithful proxies of Zeus (the "father" of the gods). The names of these two proxies of Zeus are Kratos (literally meaning State) and Bia (literally meaning Force), and they personify brute power and callous violence, respectively. Kratos states that the punishment meted out to Prometheus is due to the fact that he stole fire from the gods and revealed the secret of how it is produced to humanity. Moreover, Kratos added that the punitive measure taken will compel Prometheus to understand and submit to the sovereignty of Zeus.

We must understand that Prometheus did not give fire to humans simply in order to enable them to cook their bread and vegetables. Fire is a precondition of technology, and technology is the means by which Prometheus decided to save humanity. Aeschylus reveals that fire is a key underpinning of civilization. But fire is not enough: in order for a technologically advanced civilization to exist, a psychological type of fire is also necessary: hope, the belief that life is not merely a path to death. In particular, in Aeschylus's *Prometheus Bound* (250–260), Prometheus says that he caused humans to cease keeping their eyes fixed on their future deaths, put hope in their hearts, and gave them fire, enabling them to master many arts. Thus, Prometheus defines the preconditions of a technologically advanced civilization: energy, and a sense of perspective and progress. Without historical time—or without the production of history by humanity and without the creative integration of humanity into history—no civilization can be developed. And without civilization, specifically, without technology and institutions, the humankind will be eliminated, will fall into Hades. In Aeschylus's *Prometheus Bound*, Zeus symbolizes blind constraining forces and the realm of necessity. Such forces may be natural or social. Promethean consciousness, equipped with knowledge, rational will, and technology, revolts against both natural and social necessities. **This Promethean tradition invites us to a dynamic and creative way of understanding and practicing Freemasonry.**

The term "individual" can be conceptualized in the following two ways: (i) as an empirical, rational, thinking, and willing individual which is an undifferentiated sample of the human species; (ii) as a moral, autonomous, and hence (essentially) non-social being. When we affirm the unity of the human species at the level of morality, we are "citizens of the world," and

we maintain a level of universality that is similar to that of the exact sciences. From that perspective, in essence, there are onlyindividuals and the human species, which is often called the "society of the human species," and there are no other significant particular socio-historical entities (particular collective identities become insignificant or merely "private affairs"). However, when humans start appreciating their differences, a different world emerges: in that case, confirming Jean-Jacques Rousseau's arguments, people maintain that humans become humans only through their participation in a particular integral society, in a *specific* society, and, from that perspective, the "society of the human species" becomes an ideational abstraction, as Rousseau had once argued in the context of his communication with Diderot.[11]

Apart from the categories of "individualism" and "collectivism," there is a third category: that of "personhood," which means an individual-in-communion. In the event that personhood is constituted by an individual's relationship with a universal reason, such an individual becomes socialized by the very fact that such an individual's consciousness has been expanded in order to carry a universal reason, a reason that transcends any particular individual and refers to humanity. Therefore, both modern cosmopolitan individualists (who recognize and acknowledge only the significance of the individual's identity) and Rousseau (who emphasizes the significance of particular collective identities) often ignore that a person whose personhood is constituted by one's relationship with a universal reason (put slightly differently, a person who is specifically an existential-otherness-in-communion-with-a-universal-reason) can carry the entire human species within one's soul and, hence, think and act globally for the benefit of the society of the human species without losing or diminishing one's particularity (namely, one's particular collective identity, such as ethnic/cultural self-identity).

A scientifically rigorous study of the history of civilization in general, and of the history of politics in particular, enables one to identify and analyze progressive patterns of development of moral consciousness. As Michele Moody-Adams has pointed out, "moral progress in belief involves deepening our grasp of existing moral concepts, while moral progress in practices involves realizing deepened moral understandings in behavior or social institutions."[12] Allen Buchanan's and Russell Powell's typology of moral progress is particularly helpful in order to identify and analyze progressive patterns of development of moral consciousness.[13]

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[11] Vaughan, *The Political Writings of Jean Jacques Rousseau*.
[12] Moody-Adams, "The Idea of Moral Progress."
[13] Buchanan and Powell, *The Evolution of Moral Progress*.

As regards moral concepts themselves, the understanding of the virtues, moral reasoning, and moral motivation, world history exhibits the following progressive patterns: (i) an increasing *specification* of morality, in the sense that moral rules tend to be increasingly differentiated from religion and law; (ii) an increasing *internalization* of morality, in the sense that, as Immanuel Kant has pointedly argued, moral consciousness evaluates actions not only on the basis of their consequences, but also on the basis of the agent's motives; (iii) an increasing *individualization* of morality, in the sense that, in addition to group rights, individual rights are increasingly emphasized, esteemed, and protected, especially as people mature psycho-spiritually; (iv) an increasing *expansion* of morality, in the sense that human rights (namely, "rights we have simply because we exist as human beings—they are not granted by any state," according to the United Nations' basic definition of "human rights") are increasingly emphasized, esteemed, and protected, especially as modernity is consolidated and develops further.

As regards the understanding of moral standing, moral statuses, and justice, world history exhibits the following progressive pattern: humanity's increasing desire and efforts to create and impose new institutions in order to achieve higher levels of justice. The abolition of slavery and the development of international law on the basis of the International Bill of Human Rights are two characteristic cases in point.

As regards the proper moralization of humanity and the understanding of the nature of morality, world history exhibits the following progressive pattern: several manifestations of inhuman and degrading treatment that were considered to be normal in previous societies (for instance, child abuse, torture, gender discrimination, various forms of political and spiritual despotism, etc.) are unacceptable to and morally condemned by the modern human being. Even though, in the contemporary world, human rights abuses continue to take place, the authorities that are responsible for or involved in such types of immoral behavior try to find justifications for them, and they usually do not dare to commit human rights abuses in a blatant way.

However, the identification of the above progressive patterns of development of moral consciousness can only partially refute the arguments of moral skepticism. The history of humanity is characterized by both cases of moral progress and cases of moral setback. In fact, one can discern whole segments of historical space-time that are overwhelmed by morally negative and unacceptable situations, such as those caused by capitalist oligarchies during the "Long Depression" (1873–96) and the "Great Depression" (1929–39), the twentieth-century fascist/Nazi regimes,

the twentieth-century regimes of bureaucratic socialism (state capitalism), etc. Therefore, neither the viewpoint that is focused on progressive patterns of development of moral consciousness nor the viewpoint that is focused on the instability of moral consciousness and on cases of moral setback can lead to a comprehensive and rigorous way of understanding the dynamics of moral consciousness.

In order to obtain a comprehensive and rigorous way of understanding the dynamics of moral consciousness, one must extricate oneself from both the intellectual shackles of moral progressivism (specifically the viewpoint that is focused on progressive patterns of development of moral consciousness) and the intellectual shackles of moral skepticism (specifically the viewpoint that is focused on the instability of moral consciousness and on cases of moral setback), and search for those structural elements of moral consciousness that enable one to argue that moral consciousness is characterized by structural stability. By maintaining that one should inquire into the structural stability of the operation of moral consciousness, I mean that one should inquire into the qualitative features of moral consciousness that are recurrent. In particular, if we inquire into the contents of moral values, then we realize that, in different segments of historical space-time, different values were placed at the apex of the corresponding "moral pyramid."

For instance, as regards the prevailing moral criterion, the study of the history of the European and the modern American civilizations implies the following: in early Antiquity, the prevailing moral criterion was bravery, and the corresponding anthropological ideal was a hero; in classical Antiquity, the prevailing moral criterion was education, and the corresponding anthropological ideal was a wise person or a philosopher; in late Antiquity, the prevailing moral criterion was sanctity (in the sense of psychic beauty); in the Middle Ages, the prevailing moral criterion was chivalry (with its integrated religious, moral, and social code); in seventeenth-century French society, the prevailing moral criterion was honesty (paving the way for the conception of the modern nation-state as the social space in which "honesty" is manifested and becomes meaningful and, thus, underpinning nationalism and the French notion of the "human of the State" ("homme d'État")); in nineteenth-century British society, the prevailing moral criterion was social success, and the corresponding anthropological ideal was a person who complies with the Victorian model of social discipline and control (underpinning Great Britain's capitalist system and imperialist policy); in nineteenth-century American society, the prevailing moral criterion was individual success—namely, success that originates from and is based on an individual's own actions, thoughts,

and will—and the corresponding anthropological ideal was a "self-made individual" (in particular, the phrase "self-made man" was coined on 2 February 1842 by Henry Clay Sr. in the United States Senate to describe individuals whose success was an entirely individual achievement, and, by the mid-1950s, "success" in the U.S.A. generally implied "business success," underpinning the United States' capitalist system and neo-imperialist policy).

Additionally, nationalism has significantly contributed to the relativization of many people's perceptions of morality and rationality. The age of nationalism in its most precise sense is usually dated from the eighteenth century, and it is intimately related to the American and the French revolutions. However, the European system of nation-states had already emerged from the European wars of religion, which began in the sixteenth century (after the Protestant Reformation). In particular, the aforementioned system was a system of sovereign princes whose cultural rivalries were kept in check by the principle "whosoever's territory, his religion" ("cuius regio eius religio"), and whose political rivalries were kept in check by a system that is known as the "balance of power."[14]

The system of the balance of power was weakened but not destroyed by the revolutions that broke out in the eighteenth and the nineteenth centuries, and it was restored by the so-called Holy Alliance after the defeat of Napoleon Bonaparte in 1815. Nationalism and its states-system were internationalized in the aftermath of the First World War. In the context of *Realpolitik*, the principles of national sovereignty and national self-determination have been systematically used and invoked by nationalists in order to counter classical ontology and, especially, its quest for universal values and principles, as well as in order to equip national bourgeois elites and state bureaucracies with ultimate authority on moral questions and with powerful means for the conduct of psychological operations (for instance, the rhetoric about "patriotism" and "national security" has often served as a pretext for the violation of human rights and liberties by national governments and for the development of the industry of war).

Even though different values were placed at the apex of the corresponding "moral pyramid" in different segments of historical space-time, certain values, such as "veracity," "uprightness," "accountability," "strength," and "perseverance" seem to have been exerting indisputable moral authority over humanity throughout its known history, irrespective of the particular ways in which they are interpreted by different human communities. In

[14]Kissinger, *Diplomacy*.

addition, if we inquire into the forms of moral values, and if we approach morality in a formalist way, then we realize that humanity makes a fundamental distinction between "good" (perceived as moral positivity) and "evil" (perceived as moral negativity) in every segment of historical space-time. Therefore, by inquiring into the contents of moral values and into the forms through which moral values are manifested, we realize that moral consciousness has some recurrent qualitative features (meaning that it is characterized by structural stability).

It goes without saying that the social system exerts *significant influence* over moral consciousness, and the latter internalizes and reflects social values. But the social system *does not create* moral consciousness itself, and moral consciousness can always judge and change the established system of values instead of passively conforming to it. Hence, moral consciousness seems to be an innate attribute of the human being. In particular, through a combination of sentiments, volition, and reason, moral consciousness obtains a conception of "goodness," and it determines the conditions under which goodness can be historically objectified and, thus, become historically meaningful.

The German philosopher Ernst Bloch (1885–1977) has pointedly observed that the human being (in spite of the gloomier littérateurs and in spite of many psychoanalysts' tendency to emphasize humanity's inner litter) is a hoping animal.[15] At the most fundamental level, humanity expresses its urge to hope by being unsatisfied and by wishing to envisage an alternative (specifically, better) state of the world. At the highest level, humanity's urge to hope is expressed through a strategic existential vision, or a philosophical utopia: an ideal type of perfection that human beings seek or try to realize and guides human action like an intellectual Sun.

In contrast to defective forms of utopia, which are intellectually constrained to the building of particular bureaucracies, a strategic existential vision, or a philosophical utopia, is intrinsically transcendent in the sense that it is not a particular historical goal itself, but it is an ideal type that guides historical action and creativity "from above," and, therefore, it underpins a continuous process of progress and a world of endless possibilities. Such utopias have given rise to the greatest achievements of spirituality, philosophy, science, technology, art, political life, business life, etc., and they underpin the endless progress of civilization and of moral consciousness. Hence, Bloch has argued that we

[15]Bloch, *The Spirit of Utopia*.

should always be able to imagine that things could be different and, indeed, better.[16]

The transition from the natural sciences to the social sciences is an upward-moving process known as "emergentism." The specific nature of sociology clearly emerges when we examine it from an ontological point of view. In his book *Le Regole dell'Azione Sociale* (Milano: Il Saggiatore, 1983), Giuliano Di Bernardo (specifically, in the chapter entitled "La fondazione del sociale") shows that social reality is constructed by humanity through "constitutive rules." In particular, Di Bernardo (ibid) maintains that, based on constitutive rules, language, and the collective self, social reality has a dual ontology: one that is "visible," observable, made up of objects from the external world, such as houses, monuments, and money; the other is "invisible," made up, for instance, of housing regulations, the aesthetics of monuments, and the significance of money.

The term "ideology" was first formulated by the French aristocrat and philosopher Destutt de Tracy in 1796, to denote a systematic critical and (intellectually) therapeutic (or corrective) study of the sensory bases of ideas.[17] In the most general way, we can define "ideology" as a pattern (recurring module) of symbolically invested beliefs and expressions that determine the way in which consciousness perceives, interprets, and evaluates the world in order, according to its intentionality, to formulate, mobilize, direct, organize, and justify specific ways or guiding axes of action and to reject others. In this light, ideology provides a coherent system of ideas that forms the basis of the historical action of the bearer of the corresponding ideology. Therefore, ideology prevents intellectual squinting and ensures the stability of the strategic orientation of thought and action. Furthermore, the ideological way of thinking underpins rigorous political critique, because a genuine ideologue criticizes systems themselves instead of merely linking political criticism to particular political actors, such as particular politicians, particular businesspersons, particular state bureaucrats, etc.

On the other hand, ideology may not always function as a coherent system of ideas that ensures the stability of the strategic orientation of thought and action, but it may degenerate into a system of ideas that overemphasizes, in an obsessive and arbitrary way, their importance in the constitution and transformation of reality. In particular, in its *degenerate version*, the term "ideology" denotes a deficit of realism that results from self-delusion or the interest of the ideologue—in this case, the representative of a

[16] Bloch, *The Principle of Hope*.

[17] Lichtheim, *The Concept of Ideology, and Other Essays*.

prejudiced social type or collective subject acting as a propagandist (whether or not one is aware of this situation). Hence, the degenerate version of ideology can be called obsession, which is a psychiatric condition, or "false consciousness" in Karl Marx's terminology.
In the present book, I argue that concepts are not mere reflections of some relations that exist in the world, but they are also a peculiar factor in these relations. Ideas are facts, while concepts are social constructions that give meaning to ideological systems. Ideological systems are collective products that embody, to a greater or lesser extent, widespread concepts. As I mentioned above, ideologies (in their *non-degenerate version*) are not an illusory consciousness, but they are political thought, which is part of social and political reality.

The interplay between ideology and politics

Before inquiring into the study of different ideologies, it is useful to recall the following historical milestones:
1944:

- July: Bretton Woods Conference, creation of the International Monetary Fund (IMF) and the World Bank. In the wake of the First and the Second World Wars, a new Euroatlantic monetary system was formally established at the Bretton Woods Conference held in 1944. Agreement was reached amongst the major Euroatlantic rulers on a monetary system in which only one currency, specifically, the U.S. dollar, would be redeemable in gold (at the rate of $35 per ounce of gold). All other currencies in the world would have their value determined in relation to the U.S. dollar. Secondly, only governments, through their Central Banks, could redeem dollars for gold, whereas ordinary people, who would be required to use paper currencies, could not redeem any currency for gold. Each member-state of the International Monetary Fund (IMF) would be required to deposit 25% of all gold reserves that the state possessed with the IMF. The major political reasons why the use of gold as money was prohibited were the following: (i) To prevent the undermining of the new paper-money monetary regime by the use of gold as money. (ii) To ensure the leading role of the U.S. financial system, primarily of the U.S. Federal Reserve, as the major custodian of the gold of many nation-states, which had stored their national gold reserves in the U.S.A. (iii) Once the member-states of the IMF had deposited 25% of their gold reserves with the IMF (and, ultimately, with the U.S.A.) and member-states had

begun to take IMF loans that were secured by that gold, it would then be possible to encourage them to store increasing portions of their national gold reserves with the IMF. If they held on to their gold, they could not use it in any way that would benefit them. Thus, the explicit prohibition of the use of gold as money paved the way for the U.S.A. to become the custodian of most of the gold reserves of the world and to give rise to a new global financial oligarchy.

- August: Dumbarton Oaks Conference, creation of the United Nations (UN).
- October: The Percentages agreement, a secret informal agreement between Churchill and Stalin during the Fourth Moscow Conference; it gave the percentage division of control over Eastern European countries, including the Balkans, dividing them into spheres of influence.

1945:

- February: Yalta Conference between Stalin, Churchill, and Roosevelt.
- April: Roosevelt dies.
- May: Germany surrenders.
- April–June: San Francisco Conference; UN Organization Charter.
- July: First test of A-bomb; Potsdam Conference between Truman, Churchill/Atlee, and Stalin.
- August: Hiroshima and Nagasaki destroyed by U.S. A-bombs; U.S.S.R. enters war in Asia.
- September: Japan surrenders.
- December: in the Shinto Directive, U.S. military leaders introduced the term “State Shinto” in order to differentiate Japan’s traditional state ideology from traditional Shinto practices. The decree established Shinto as a religion, and banned further ideological uses of Shinto by the Japanese State.

1946:

- March: Churchill’s Iron Curtain speech. Additionally: resumption of Greek civil war (it ended in August 1949 with the defeat of the communist guerillas). The role of Greece as a geostrategic protectorate of the U.S.A. and, generally, as a geopolitical flank of the Anglo-American establishment in the Eastern Mediterranean was consolidated.
- December: Resumption of First Indochina War (it ended in July 1954). Immediately after the end of World War II, the major Western powers were struggling to regain control of their former

colonies. Asia was a region for which France, the United Kingdom, and the United States reached a mutual agreement for their spheres of influence. In September 1946, France attempted to reassert its hegemony in Indochina and other former colonies where it had been humiliated by the Axis forces. With the help of the United Kingdom, and reinforced by the policy of the thirty-third U.S. President Harry S. Truman against the independence movements in Indochina, the French returned to Indochina and began their disastrous nine-year war against the Viet Minh, the fighters of the Vietnamese independence movement. The U.S. helped France to reconquer Indochina in exchange for France's support for the U.S. Marshall Plan and for France's anti-communist operations in Europe. By the time France was defeated at Dien Bien Phu in 1954, the U.S. had already spent $3.5 billion, representing 75% of the cost of the French war in Indochina. Nevertheless, the French faced serious financial problems in financing their covert war operations against the Vietnamese. Hence, in 1951, the French secret service SDECE (Service de Documentation Extérieure et du Contre-Espionnage) and the Service d'Action branch of its covert operations took over the huge opium (heroin) trade in French Indochina.[18] Known as the "Opium Monopoly," the opium trade was originally organized and developed by the French in the 1880s to finance their colonial rule in Indochina.[19] The Service d'Action branch of the French secret service SDECE had, in fact, given the code name "Operation X" to the covert war that France was waging in Indochina, financing it with the proceeds from the opium trade.[20]

1947:

- February: Great Britain announced its intent to terminate the Mandate for Palestine, referring the matter of the future of Palestine to the United Nations.
- March: Truman Doctrine announced (with the primary goal of containing Soviet geopolitical expansion during the Cold War).
- June: Marshall Plan announced (an American initiative to provide foreign aid to Western Europe in accordance with the Truman Doctrine).
- October: Creation of Cominform (Communist Information Bureau) by the Soviet Union.

[18] McCoy, *The Politics of Heroin in Southeast Asia*.
[19] Ibid, pp. 73–75.
[20] Ibid, pp. 99–100.

- November: United Nations Partition Plan for Palestine (UN General Assembly Resolution 181): UN General Assembly votes for partition of Palestine into Jewish and Arab states with Jerusalem under UN trusteeship. UN Partition Plan accepted by Jews but rejected by Arabs.

1948:

- February: *Coup* by Czech Communist Party.
- March: Partial blockage of Berlin begins.
- May: Fighting between Arabs and Jews in Palestine; British Mandate comes to an end; the Jewish provisional government under David Ben-Gurion proclaims the State of Israel.
- June: Berlin airlift begins; Yugoslavia ousted from Cominform (as a result of a prolonged political confrontation between the Yugoslav leader Josip Broz Tito and the Soviet leader Joseph Stalin).
- November: Truman re-elected U.S. president.

1949:

- January: The Council of Mutual Economic Assistance, known as CMEA or, more commonly, "Comecon," founded by the U.S.S.R., Bulgaria, Czechoslovakia, Hungary, Poland, and Romania as a regional, political, and economic organization that aimed to consolidate the Soviet Bloc. It was officially dissolved in 1991, following the collapse of communist regimes in Eastern Europe.
- April: North Atlantic Treaty signed in Washington. This treaty forms the legal basis of, and is implemented by, the North Atlantic Treaty Organization (NATO). The first Secretary General of NATO (from 1952 to 1957) was the British general and diplomat Hastings Lionel Ismay, 1st Baron Ismay, who is credited as having been the first person to say that the purpose of NATO was "to keep the Russians out, the Americans in, and the Germans down," a statement that has since become a common way to summarize the founding geostrategic rationale of NATO.
- May: End of the Berlin blockade; the State of Israel admitted to the United Nations.
- August: U.S.S.R. explodes first A-bomb. In the U.S.A., the hardliners in the Truman administration, led by Paul Nitze, Dean Acheson (Secretary of State), and George Kenan, sought an "appropriate response."
- September: Federal Republic of Germany, commonly known as West Germany, comes into existence.

- October: People's Republic of China proclaimed; German People's Republic (commonly known as East Germany) proclaimed.

1955:

- May: Warsaw Treaty Organization, officially the Treaty of Friendship, Cooperation and Mutual Assistance, but more commonly known as the Warsaw Pact, between the Soviet Union and seven other Eastern Bloc socialist republics of Central and Eastern Europe is signed. It was the military complement to the Comecon, and it was created in reaction to the integration of West Germany into NATO. In February 1991, the Warsaw Pact was declared disbanded.
- November: Resumption of Vietnam War, often referred to as the Second Indochina War (it ended in April 1975 with the Liberation of Saigon by North Vietnamese). In the aftermath of the First Indochina War, at the International Geneva Conference on 21 July 1954, the new socialist French government and the Viet Minh made an agreement that effectively gave the Viet Minh control of North Vietnam above the 17th parallel, thus creating a socialist Vietnamese state, while South Vietnam continued to be under the rule of the Vietnamese Emperor Bao Dai. The agreement was denounced by the U.S.A. and the "State of Vietnam" (a puppet governmental entity in Southeast Asia that existed from 1949 until 1955, first as a member of the French Union and later as a puppet state of the U.S.A. and France). A year later, Bao Dai was deposed by his Prime Minister Ngo Dinh Diem, who proclaimed the Republic of Vietnam (South Vietnam). Soon an insurgency, backed by North Vietnam, developed against Diem's government. The conflict gradually escalated into the Vietnam War. North Vietnam was supported by China, the Soviet Union, and other communist allies, whereas South Vietnam was supported by the U.S.A. and other anti-communist allies. Diem, a Roman Catholic bourgeois supported by the U.S.A., represented Vietnamese nationalism, promoting an anti-communist and anti-colonialist "third way," opposed to both Bao Dai (who was subservient to the French imperialism) and the North Vietnamese communist leader Ho Chi Minh. However, Diem's favoritism towards Roman Catholics and persecution of South Vietnam's Buddhist majority led to the "Buddhist crisis" of 1963. Thus, Diem's regime lost favor with the U.S.A. and the leadership of the Army of the Republic of Vietnam. On 1 November 1963, South Vietnam's leading generals launched a successful *coup d'état* with assistance from the CIA, and, thus,

the South Vietnamese general Du'o'ng Van Minh succeeded Diem as president (Diem was assassinated on the orders of Du'o'ng Van Minh). The Vietnam War ended on 30 April 1975, when Saigon, the capital of South Vietnam, was captured by the forces of North Vietnam. The event led to the formal reunification of Vietnam into the Socialist Republic of Vietnam.

1957:

Treaty of Rome signed (by Belgium, France, Italy, Luxemburg, the Netherlands, and West Germany). This treaty forms the legal basis of, and is implemented by, the European Economic Community (EEC), which aimed to bring about economic integration among its members. In essence, it was created as an economic apparatus of and complement to NATO. In 1993, the EEC (according to the Maastricht Treaty concluded in 1992 between the then twelve member-states of the "European Communities") was transformed into the European Union (EU). In essence, the EEC/EU project is a neo-imperial Frankish project that was created with three main purposes: firstly, in order to prevent the historic rivalries between the French-Frankish elites and the German-Frankish elites from turning into another European war (thus giving the Frankish elites—characteristically represented by the former French President Charles de Gaulle and the former German Chancellor Konrad Adenauer—a great geopolitical advantage over Continental Europe); secondly, to integrate the industrial development of the Carolingian Europe (primarily, of France and Germany) into a wider Euroatlantic institutional framework; and, thirdly, to combat socialism.

1971–73:

The transition from the Bretton Woods System to the Petro-Dollar System:

- 1971: The U.S. President Richard Nixon decided to break up Bretton Woods, proceeding with the unilateral cancellation of the direct international convertibility of the U.S. dollar to gold. In fact, the first significant challenge to the Bretton Woods System that emerged within the first twenty-five years of its existence came from France, where the Bretton Woods System was denounced as "America's exorbitant privilege." In addition to its lack of integrity, the Bretton Woods System was increasingly functioning in a manner that was unfairly advantageous to the U.S.A., and France had serious objections to such an unfair monetary regime. In the early 1960s, the French began to make efforts to restore gold's

central role in the international monetary system. Thus, the French started redeeming U.S. dollars for gold at $35 an ounce. In 1965, the French President Charles de Gaulle officially announced his intention to exchange France's U.S. dollar reserves for gold at the official exchange rate ($35 an ounce). The Vietnam War provided France with an additional reason for challenging the Bretton Woods System. Like all other IMF member-states, the U.S.A. had to deposit 25% of its gold reserves with the IMF. In doing so, it provided the rest of the IMF member-states with information about the size of the U.S. gold reserves. In fact, the U.S.A. was under a legal obligation to redeem U.S. dollars for gold at $35 an ounce. Therefore, it was illegal and immoral for the U.S.A. to issue more paper U.S. dollars than could be redeemed for U.S. gold. But that is precisely what the U.S. Government did in order to finance the Vietnam War. France responded to this illegal and immoral conduct by demanding a devaluation of the U.S. dollar through a change in the price of gold. By 1966, non-U.S. central banks held $14 billion, while the U.S.A. had only $13.2 billion in gold reserve. Of those reserves, only $3.2 billion was able to cover foreign holdings, because the rest was covering domestic holdings. In May 1971, West Germany left the Bretton Woods System. In August 1971, the U.S. dollar dropped in value against European currencies, and Switzerland left the Bretton Woods System. Finally, on 13 August 1971, U.S. President Richard Nixon directed Treasury Secretary John Bowden Connally Jr. to suspend, with certain exceptions, the convertibility of the U.S. dollar into gold or other reserve assets, ordering the "gold window" to be closed. As a result, foreign governments could no longer exchange their U.S. dollars for gold.

- 1973: The establishment of the petro-dollar monetary system. Within just two years after the collapse of the Bretton Woods System, the U.S. managed to take advantage of the October 1973 Arab-Israeli War and the oil crisis in order to replace a monetary system hitherto based on gold, or superficially based on gold (namely, Bretton Woods), with a new one based on oil. The U.S. Secretary of State Henry Kissinger successfully negotiated with Saudi Arabia's King Faisal an agreement requiring oil to be sold for only U.S. dollars. In addition, Saudi Arabia persuaded the other Arabs to join in the agreement. This was the birth of the petro-dollar monetary system, in which oil functions as gold to the advantage of the U.S. dollar and the U.S. financial establishment.

Thus, an opportunity was created for the U.S. Federal Reserve to create as much money as it wanted and to then feed that money into the banking system, paving the way for the so-called "casino capitalism" and transforming war into a strategic industry of the U.S. economy (for a systematic study of this issue, see: Susan Strange, *Casino Capitalism*, with a new introduction by Matthew Watson, Manchester: Manchester University Press, 2016).

1982–86:

- In 1982, a few months before he briefly became Brezhnev's successor, Yuri Andropov (1914–84), a KGB chief, recognized that Soviet socialism had been compromised and undermined by internal corruption, organizational flaws, and the imperfect way in which the reform of people's consciousness was sought. Therefore, he declared that a long and complex transitional period would be necessary before the completion of the socialist transformation of society, admitted the possibility of "the non-coincidence of the interests of various social groups," and conceded that specific mechanisms should be established "to record, compare and reconcile various interests."
- In 1986, oil prices plummeted by nearly 70% due to Saudi Arabia dramatically increasing its oil production. Thus, Soviet oil revenues, on which the Soviet government depended in order to prop up the rest of the Soviet Union's economy, plummeted by almost 75%. The new Soviet leader Mikhail Gorbachev responded by implementing reforms that proved too much for the system, eventually culminating in the collapse of the Soviet Union in 1991.

The most important ways in which the ideological phenomenon manifested itself in the context of modernity are the following:

- *The ideology of bourgeois-liberalism.* Feudalism, the dominant system in medieval Europe, was a system characterized by a rigid social stratification, according to which everyone had a rigidly instituted position within an "organic whole." The major constituent components were the class of the feudal lords, the class of the serfs, and the Church, whose major social role was to maintain a balance between the feudal lords and the serfs through religion. By the late Middle Ages, the bourgeois class (a social class of professionals who were neither feudal lords nor serfs) deprecated the political, economic, and spiritual despotism of the feudal system, revolted against it, and proclaimed that the social

position of an individual should not be determined by feudal institutions and instead should be freely determined by individual action and by the interaction between individuals in the context of a free and fair society. The origins of modern liberalism can be seen most clearly in the thinking and politics linked to the English Revolution of 1688—specifically, in British empiricism and constitutionalism. The principles of constitutionalism, religious toleration, and commercial activity which were promoted by the English Revolution of 1688 became a standard for European and American liberals in the eighteenth century.[21] The first group to use the "liberal" label in a political context was the Spanish political party of the "Liberales," which persistently fought for the implementation of the 1812 Constitution, and, in 1820, it overthrew the Spanish monarchy as part of the "Trienio Liberal." Arguably, the most important bourgeois revolution in the modern era is the French Revolution of 1789, whose major motto was "Liberty, Equality, Fraternity." Politics and philosophy proclaim the rights of the individual, who is understood as the protagonist of historical events. The liberal ideology underpinned the Industrial Revolution. Changes in technology made possible the application of new technologies, which created a sense of optimism about the future. Economic theorists invented "capitalism," which is understood as a corollary of liberalism, and, indeed, it has produced unprecedented economic wealth. However, the elite of the bourgeois class conceived capitalism as the embodiment of human freedom in the domain of economics. For this reason, after the displacement of feudalism by capitalism, the liberty and the rights of the human individual were gradually displaced by and subordinated to the liberty and the rights of the capital itself and the capitalist elite. Moreover, the rationalism of traditional liberalism and the original bourgeois class was gradually substituted and dominated by the selfish expediencies and passions of the capitalists, transforming the rationalist underpinnings and principles of classical political economy into contradictory and empirically insignificant assertions.

In the late nineteenth and the early twentieth centuries, the liberal justification of state intervention expanded in Britain in the context of the new social liberalism promoted by Thomas H. Green (1836–82) and Leonard T. Hobhouse (1864–1929), and according to the

[21] Duverger, *Introduction à la Politique*.

new political economy developed by John M. Keynes (1883–1946). Keynes rejected both "state socialism" and "economic anarchy," and he argued that poverty and "the economic struggle between classes and nations" (which could produce war) could be overcome by social intervention. In the United States of America, social liberalism was promoted by so-called "progressive" political currents, which were expressed by such writers as Herbert D. Croly (1869–1930) and John Dewey (1859–1952). During the same period, in continental Europe, social liberals and "progressive" thinkers were tracing their origins back to Rhenish industrialists, such as Friedrich Harkort (1793–1880), known as the "Father of the Ruhr," and some of them seemed more akin to socialism than to original liberal individualist doctrines, thus opening the way to a synthesis between liberalism and socialism (e.g., the "Gotha Program," namely, the party platform adopted by the nascent German Social Democratic Party at its initial party congress, held in the town of Gotha in 1875, Eduard Bernstein's social democratic theory, etc.). In Britain and in continental Europe, the new liberalism, which was prone to a synthesis between liberal principles and social democratic pursuits, was not, at least at first, unreservedly accepted by the official liberal parties, but, on the other hand, social democratic parties rushed to harbor it. In the United States, since the "Great Depression" of the 1930s, the "progressives" (namely, left-wing liberals) have captured the label "liberal," whereas the older liberals (right-wing liberals) have been known as "conservatives." From the conservatives' viewpoint, the new liberals had conceded too much to socialism, whereas, from the new liberals' viewpoint, the conservatives were politically fixated on outdated liberal policies that no longer served the ends of liberalism.

During and after World War II, conservative liberalism was reinforced, and its popularity increased as a result of the Anglo-American elite's ideological war against the Nazis and the Soviets. In the 1950s, some liberal intellectuals, such as Friedrich A. von Hayek (1899–1992) and Karl R. Popper (1902–94), even argued that ideologies were insignificant, that liberalism was a universal truth (not merely one among the many ideologies), that liberalism was naturally superior to its rivals, and that normal liberal politics consisted merely in the antagonism among interest groups, not in ideological disputes. However, this arrogant liberal rhetoric was shattered by the return of ideological polarization in the 1960s and

the 1970s. The failure of social liberalism's management to extend the post-war economic boom into the last quarter of the twentieth century contributed to the political reinforcement of conservative liberalism, of which the British Prime Minister Margaret Thatcher and the U.S. President Ronald Reagan were characteristic representatives. The political debate between social liberalism (revitalized by such social democrats as the French President François Mitterrand and Jacques Delors, the eighth President of the European Commission) and conservative liberalism continued until the end of the Cold War. After the end of the Cold War, and as a result of the intensification of the process of globalization, especially in the fields of finance and communication, both social liberalism and conservative liberalism differ from each other mainly with regard to tactics and methodological issues. However, at the dawn of the twenty-first century, a major conflict arose regarding the architecture of the world order.

Aristotle defined politics as an "architectural science" whose purpose is to coordinate all the scientific disciplines that study humanity (sociology, economics, psychology, etc.). After the dissolution of the Soviet Union (as a consequence of an agreement between the last Soviet leader Mikhail Gorbachev and U.S. President Ronald Reagan), the liberal establishment in the West thought that it now had the historic opportunity and possibility to establish a new world order which would predominantly express and serve the Western liberal establishment's interests and outlook on life. Nevertheless, gradually, post-Soviet Russia, China, and India not only awakened politically, but also, creating a *de facto* "Empire of the East," put up a strong resistance against the imperialist policy of the liberal coalition of the West. In China, International Relations theory is based on the state's official Marxist/Leninist/Maoist legacy and on such traditional Chinese treatises on strategy and politics as Sun Tzu's *The Art of War*, Zhan Guo Ce's *Annals of the Warring States*, and San Guo Zhi's *History of the Three Kingdoms*. In India, International Relations theory is strongly influenced by development studies; and Rajni Kothari's work is an exciting contribution to Indian political scholarship (in his studies of politics in India and, generally, in the Third World, Kothari locates the basis of international action at the level of the individual, identifies the power of culture at the basic level of international action, and analyzes the manner in which the spread of freedom, power, and action takes place laterally among

individuals, then at village level, and among villages and beyond). In post-Soviet Russia, International Relations theory emphasizes the "civilizational identity" of states, the institution of an international multilogue (versus the established Anglo-American liberal monologue), norms regarding Great Power behavior, and the ruling Russian elites' intention to establish a multipolar world order (as an alternative to U.S./Euroatlantic hegemony). On 24 March 2023, the *Razvedchik* news magazine published an article written by the Russian Foreign Minister Sergey Lavrov, who explicitly supported "the rise of a multipolar architecture"; and, on 7 April 2023, during his official trip to Ankara, Sergey Lavrov stated that "what should be discussed are what the principles of the new world order, which we all need, will be based on." During the 2020s, Russia tends to explicitly and methodically pursue a creative synthesis between the traditional "Russian soul" and the technocratic, scientific, and technological achievements of the Soviet Union as well as to articulate an alternative political proposal to the liberal oligarchy of the West. This orientation of the Russian ruling elites, China's practical philosophy, the rapport between Russian President Vladimir Putin and Chinese President Xi Jinping, and, generally, the political awakening and the assertive foreign policy of the "Empire of the East" (Russia–China–India) are shaping historical conditions conductive to the emergence of a model of global governance and a politology that are consistent with the strategic vision that I advocate in this book and is promoted by the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.

- *The ideology of socialism.* By the middle of the nineteenth century, the European peoples realized that capitalism had displaced feudalism, but, instead of ushering in liberty, equality, and fraternity among the people, capitalism tends to replace the authoritarian and exploitative relationship between the feudal lords and the serfs with a new authoritarian and exploitative relationship, that between the capitalist class and the proletariat (working-class). Therefore, socialism emerged as a criticism of and a revolt against capitalism, just as the bourgeois ideology had previously emerged as a criticism of and a revolt against feudalism. In fact, the term "socialism" first appeared in 1832 in *Le Globe*, a liberal French newspaper of the French philosopher and political economist Pierre Leroux. By the 1840s, socialism had already become the object of rigorous social-scientific analysis by the German economist and

sociologist Lorenz von Stein. Moreover, the English socialist intellectual and activist Thomas Hodgskin (1787–1869) articulated a thorough critical analysis of capitalism and of the labor class under capitalism, and his writings exerted a significant influence on subsequent generations of socialists, including Karl Marx. In particular, from the perspective of Thomas Hodgskin, socialism signifies an attempt to create a rational and fair market, in the context of which production and exchange are based on the labor theory of value (freed from exploitative institutions) as part of natural right, which endows moral consciousness, the freedom of the individual, social justice, and social autonomy with ontological underpinnings (in accordance with Thomas Hodgskin's deism). In addition, the Russian-Soviet philosopher and scientist Alexander Bogdanov, one of the acknowledged founders of the science of planning and organizational theory, has argued that socialism is meaningless without a "universal organizational science" capable of combining and coordinating all the individual disciplines.

- *The ideology of nationalism*. The ideology of nationalism is a product of the synthesis between liberalism and romanticism. It is important to understand the difference between romanticism and rationalism. Rationalism gives primacy to thought vales (which are characterized by intellectual abstraction and universalism) over life values (which are characterized by sensationalism and particularity). Hence, rationalism underpins cosmopolitanism. By contrast, romanticism, giving primacy to life values over thought values, underpins communitarianism, and often cultivates a closed and herd mentality combined with an idealized naturalism. The ideology of nationalism is inextricably linked to Hegel's communitarianism. Opposing Kant, who is the paradigmatic representative of critical rationalism, Hegel maintains that the human individual exists truly as a conscious being only within the nation, and one can ascend to the universal spirit only if and to the extent that one's spiritual freedom is self-abolished and absorbed by the community. Hence, according to Hegel, the individuality of the citizen presupposes the individuality of the state. Furthermore, Hegel's account of international politics is set out in sections 321–340 of his book *Philosophy of Right*. According to Hegel, the state is absolutely sovereign, that is, there can be no higher authority than the state.

  A radical form of nationalism is known as fascism. Fascism opposes materialism, liberal democracy, and Marxism, and regards

them as different aspects of the same materialist evil. This revolt against materialism underpins the political alliance between anti-liberal and anti-bourgeois nationalism and an anti-Marxist, revolutionary variety of socialism. The fascist ideological synthesis highlighted the defects of a political culture inherited from the twentieth century and the French Revolution, and it rejected liberal individualism. The fascist ideological synthesis was clearly expressed in the 1910s in such publications as *Les Cahiers du Cercle Proudhon*, published by the Cercle Proudhon, a political group founded in France in 1911 by Georges Valois and Édouard Berth and inspired by the French philosopher Georges Sorel (1847–1922), who founded revolutionary syndicalism. The combination of Sorel's anti-Marxist socialism and the theories of organic, exclusive nationalism that were developed by the French nationalist politicians and writers Maurice Barrès (1862–1923) and Charles Maurras (1868–1952) and by the Italian nationalist politician and writer Enrico Corradini (1865–1931) gave rise to the fascist revolutionary movement.

During the Cold War, the West's liberal oligarchy instrumentalized and co-opted fascist forces in order to combat Marxism–Leninism and fight against the Soviet bloc. For instance, Lucian W. Pye (1921–2008), a political science professor at the Massachusetts Institute of Technology, focused his research work on the modernization of Third World nations and became regarded as one of the foremost practitioners and proponents of the concept of "political culture" and of political psychology. Pye served as adviser to U.S. President John F. Kennedy and played a key role in the formation of the liberal imperialist policy of the Kennedy administration, with McNamara as Secretary of Defense. In his book *Aspects of Political Development* (Boston: Little, Brown and Company, 1966), Pye argues that the Army is the most modernized institution in transitioning societies, and, in the same book, Pye argues that the acculturation process in the Army tends to focus on the acquisition of economically significant skills and that, in developing countries, the Armed Forces can provide the citizen with a sense of rights and duties and with an appreciation for political action.

By 1962, the White House and the U.S. Pentagon had devised a new strategy of counter-insurgency to combat communist guerillas. McNamara's liberal imperialist policy led to the creation of American Special Forces, such as the Green Berets, and secret

paramilitary operations throughout Asia and Latin America. The policy of McNamara's liberal imperialism often led to the support of fascist regimes (military juntas), such as Franco's regime in Spain (1936–1975) and the Greek military junta of 1967–1974. Moreover, when Brazil's generals overthrew the democratically elected President João Goulart in 1964 and proclaimed that their junta saved the nation from communism, U.S. President Lyndon B. Johnson promptly recognized the new military regime. According to this strategy of liberal imperialism, in 1965, U.S. President Lyndon B. Johnson dispatched 23,000 marines to the Dominican Republic (without even bothering to consult the U.S. Congress) in order to support and consolidate the fascist military *coup* that overthrew the legally elected President Juan Bosch and to suppress Bosch's constitutionalist forces. Johnson argued that the popular democratic movement in the Dominican Republic had fallen into the hands of communist conspirators.

The Nixon administration (1969–1974) supported fascist military regimes in Greece and Cyprus, sold jet aircraft to the fascist governments of South Africa and Portugal, engaged in difficult counter-insurgency campaigns in Africa, and violated United Nations sanctions against Rhodesia.[22] In 1970, U.S. President Richard Nixon and his National Security Adviser Henry A. Kissinger authorized the Central Intelligence Agency (CIA) to encourage a military *coup* in Chile in order to prevent the inauguration of the democratically elected socialist presidential candidate Salvador Allende, a loyal and lifelong liberal Freemason, but the plan was not successful.[23] The CIA provided funding for the mass strikes against Allende's government in 1972 and 1973 and for extensive black propaganda in the newspaper *El Mercurio* (ibid). CIA's first approach to stop Allende was called "Track I" approach, and CIA's second approach, the "Track II" approach, consisted of the encouragement of a military overthrow (ibid). On 11 September 1973, Allende died during a military *coup d'état* that was launched by Army Commander-in-Chief Augusto Pinochet, who was a fascist Freemason actively supported by the CIA (ibid). Augusto Pinochet's fascist regime, which lasted from 1973 to 1990, implemented a free-market oriented economic policy under the guidance of the neoliberal "Chicago Boys" (a group of Chilean

[22] Papandreou, *Democracy at Gunpoint*.
[23]Kornbluh, "CIA Acknowledges Ties to Pinochet's Repression."

economists, most of whom trained at the Department of Economics of the University of Chicago under Milton Friedman and Arnold Harberger), and many of Pinochet's officers were paid contacts of the CIA or the U.S. military.

In 1976, Henry A. Kissinger, who was Secretary of State under U.S. President Gerald R. Ford, took a similar line as he had towards Chile when Jorge R. Videla, a senior commander in the Argentine Army, came to power in a *coup d'état* that overthrew the elected government of Isabel Martínez de Perón. Through a process called the National Reorganization Process, Videla's fascist military regime consolidated its power by launching brutal reprisals and "disappearances" against political opponents. [24] The National Reorganization Process regime lasted from 1976 until 1983. Previously secret State Department documents, including some published by the Argentine newspaper *Clarín*, show that U.S. officials had prior knowledge of Videla's intentions and supported him. More than a week before Videla's *coup d'état*, U.S. Ambassador Robert Hill sent William Rogers, who was the U.S. Assistant Secretary of State for Latin America, a secret cable reporting that the commander of Argentine Navy, Admiral Emilio Massera, had requested that the U.S. embassy "indicate to him one or two reputable public relations firms in the U.S. which might handle the problem for a future military government" (ibid). While Rogers suggested delaying official recognition of Videla's dictatorial government, the U.S. Secretary of State Henry A. Kissinger ordered full U.S. support to Videla, and among those who implemented this policy in 1976 were Richard Cheney, who was the White House Chief of Staff, and Donald Rumsfeld, who was Defense Secretary (ibid). About twenty years later, Richard Cheney became the U.S. Secretary of Defense under President George H. W. Bush, and he also became the 46th Vice President of the United States from 2001 to 2009 under President George W. Bush. Donald Rumsfeld served again as Secretary of Defense from 2001 to 2006 under President George W. Bush.

The moral and philosophical legitimization of fascist regimes by liberal oligarchies and neoconservatives continued and was rigorously formulated in Jeane Kirkpatrick's celebrated *Commentary* essay "Dictatorships and Double Standards" (1979). Jeane Kirkpatrick, a Georgetown University political science

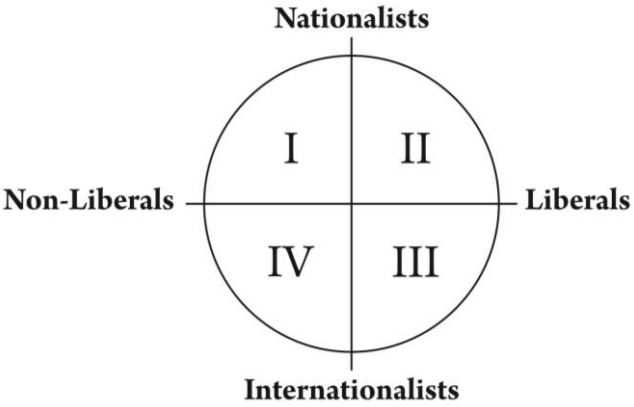
[24]Osorio, Novaro, and Dinges, eds, "On 30th Anniversary of Argentine Coup."

professor, served on Ronald Reagan's Cabinet on the National Security Council, the Foreign Intelligence Advisory Board, and the Defense Policy Review Board, and also served as the 16th U.S. Ambassador to the United Nations (1981–85). In her previous celebrated essay, Kirkpatrick formulated what is known as the "Kirkpatrick doctrine," according to which right-wing authoritarian regimes which were allies of the West were distinct from and better than left-wing "totalitarian" regimes, because pro-U.S. right-wing authoritarian regimes were putatively susceptible to an incremental transition to liberal democracy, and their enmity to communism served as the predominant piece of evidence for their political disposition.

At the beginning of the twenty-first century, faced with the challenges and the imperatives of a historical reality characterized by globalization and the enormous development of science and technology, but also by the awareness of the structural contradictions of capitalism, the inability of classical democracy to function effectively under conditions of increased social complexity, the collapse of the Soviet coalition, the historical inadequacy of nationalism, and the inability of any type of romantic communism to deal seriously and effectively with issues of organization and administration, the ideologies of liberalism, socialism, and nationalism found themselves in an existential crisis. The Modern and Perfecting Rite of Symbolic Masonry is imbued with a deep sense of historical responsibility, cultivates historical responsibility, and seeks to historicize Freemasonry, in contradistinction to pure mysticism. Therefore, **the Modern and Perfecting Rite of Symbolic Masonry educates its members in political and economic thought and in science, and it offers a sketch of a new integrated ideological proposal that consists of an aristocratic and scientifically rigorous conception of socialism that utilizes and endorses several aspects of Marxism–Leninism, but its roots can be traced to Plato's political thought, practical philosophy, cybernetics, and a universal ethic inspired by Buddha, Confucius, Orpheus, Socrates, and Kant.** Thus, instead of advocating for the dictatorship of the capital, the dictatorship of the proletariat, democratic illusions, or for postmodern grievance groups, we propose a model of government by what Socrates has called the "epaiontes" (i.e., "those with real understanding," the "genuine experts," "those who perceive things according to their nature"). With reference to the "circle of political theories" displayed in Figure 1-22, the ideology of our Order, the Modern and Perfecting Rite of Symbolic Masonry, belongs to Quadrant IV.

In Figure 1-22, we can see the four political categories, or political families, with regard to liberalism, anti-liberalism, nationalism, and internationalism. Quadrant I corresponds to non-liberal nationalists (e.g., fascism, neo-fascist movements, etc.). Quadrant II corresponds to liberal nationalists (e.g., modern British and U.S. conservative and republican political groups, French Gaullism, German and Italian Christian-Democratic political groups, anti-globalist socialist/social-democratic parties, etc.). It is worth mentioning that, in continental Europe, many nineteenth-century liberals were preoccupied with national unification, especially in Germany, Italy, the Central Europe, and Greece, and, therefore, they were less opposed to the unifying and centralizing power of the state than many of their British and American liberal contemporaries, who were more attracted to anti-statist *laissez-faire* economic doctrines and policies. Quadrant III corresponds to liberal internationalists (e.g., various liberal and liberal social-democratic political parties, U.S. Neoconservatives and Neoliberals, various secular libertarian activists and political parties, George Soros's Open Society Foundations, various postmodern grievance groups usually associated with ecological and LGBTQ rights movements, etc.). Quadrant IV corresponds to non-liberal internationalists. As regards the political history of the Western world during the 19th and the 20th centuries, the most important political force that belongs to Quadrant IV is Marxism–Leninism. However, the Modern and Perfecting Rite of Symbolic Masonry offers a new ideology that belongs to Quadrant IV; and, as I mentioned earlier, this ideology is an aristocratic and scientifically rigorous conception of socialism that utilizes and endorses several aspects of Marxism–Leninism, but its roots can be traced to Plato's political thought, practical philosophy, cybernetics, and a universal ethic inspired by Buddha, Confucius, Orpheus, Socrates, and Kant. This is my notion of "enlightened totalitarianism," which, as a matter of fact, has, in an eclectic and rational way, assimilated various elements of the thinking of previous (liberal and non-liberal) political theories.

Figure 1-22: The "circle of political theories" with regard to liberalism, anti-liberalism, nationalism, and internationalism.



CHAPTER 2

ENTERED APPRENTICE OF THE M∴P∴R∴S∴M∴

According to the Modern and Perfecting Rite of Symbolic Masonry, the Lodge room has the following typical layout:
The point of entry is in the middle of the West and facing East. The Venerable Master (V.M.) sits in the middle of the East and facing West. The Senior Overseer (S.O.) sits in the West at a convenient distance from the north (i.e., the left-hand) side of the door and facing East, leaving a comfortable distance between his/her seat and the western wall of the Lodge room. The Junior Overseer (J.O.) sits in the West at a convenient distance from the south (i.e., the right-hand) side of the door and facing East, leaving a comfortable distance between his/her seat and the western wall of the Lodge room. The seats of the S.O. and the J.O. are aligned with each other and symmetrical with respect to the point of entry. The rows of seats in the North and the South of the Lodge are called "Columns," and are so placed as not to obstruct the view of the Overseers, the Expert, the Inner Guard, and the Outher Guard, who sit in the West. There is a comfortable distance between the front row of each "Column" and the imaginary straight line connecting the corresponding Overseer's seat to the East of the Lodge.
The Venerable Master and the Overseers have pedestals on which there are columns: an Ionic Column stands on the northern part of the Venerable Master's pedestal (i.e., to his/her right), a Doric Column stands on the southern part of the Senior Overseer's pedestal (i.e., to his/her right), and a Corinthian Column stands on the southern part of the Junior Overseer's pedestal (i.e., to his/her right). Moreover, the V.M., the S.O., and the J.O. each have a desk lamp placed on their respective pedestals.
The west side of the Venerable Master's pedestal is adorned with a big gold jewel, which can be seen by the Brothers and the Sisters of the Columns and depicts Euclid's 47th Proposition suspended by a Square (Fig. 2-1).

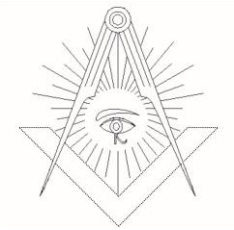
Figure 2-1: The jewel of the Venerable Master's pedestal according to the M∴P∴R∴S∴M∴

The 47th Proposition of Euclid, also known as the Pythagorean Theorem, states: in every right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. This theorem underpins the definition of the Euclidean metric (namely, the Euclidean distance function). Let $\mathbb{R}^2$ *be the usual Euclidean plane, where the set of all ordered pairs are real numbers (in the* xy*-coordinate system). In fact, Cartesianism (specifically, Descartes's analytic geometry), which is based on the algebraization of geometry, gave rise to the idea that numbers can be thought of as positions on the number line (thus, fusing geometry and arithmetic). Given the* xy*-coordinates of any two typical points of* $\mathbb{R}^2$*, say* $p = (x_1, y_1)$ *and* $q = (x_2, y_2)$*, the Euclidean distance between these points is given by*

$d_E(p, q) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Above the Venerable Master, there is a transparency upon which is a symbol consisting of a Square, a Pair of Compasses, and a rayed Eye (known as the "All-Seeing Eye"), and the Pair of Compasses is placed above the Square with the rayed Eye in the center (Fig. 2-2).

Figure 2-2: Square and Compasses with a rayed Eye.



The Orator sits in the East, next to the Venerable Master, on the south side of the Venerable Master's pedestal (i.e., to his/her left), both facing West.

The Director of Ceremonies (D.C.) is located in the South, and the Assistant Director of Ceremonies (A.D.C.) sits next to the Director of Ceremonies on the west side (i.e., to his/her left), both facing North. The Secretary and the Treasurer are located in the North, and the Treasurer sits next to the Secretary on the west side (i.e., to his/her right), both facing South. The Secretary has a pedestal, too, on which he/she writes in the Lodge's minute book. The Expert sits next to the Senior Overseer, on the south side of the Senior Overseer's pedestal (i.e., to his/her right), both facing East. The Director of Music (or Organist) sits in the southwestern corner of the Lodge room (facing East), and he/she handles a music device, playing music as indicated by the Ritual. The Inner Guard (I.G.) sits next to the Junior Overseer, on the north side of the Junior Overseer's pedestal (i.e., to his/her left), both facing East. When the Outer Guard (O.G.) is inside the Lodge room, he/she sits exactly next to the door, on the south (i.e., the right-hand) side of the door, his/her chair resting against the western wall of the Lodge room, and facing East (so that the seats of the J.O. and the I.G. do not block the Outer Guard's view of the East).

The wall of the East is orange-red, whereas the walls of the West, the North, and the South are light blue. The ceiling is dark blue, emblazoned with gold constellations. The "zone of the East" is elevated by five steps, and it is separated by a railing. The Venerable Master's pedestal in the East is elevated by two more steps. The only persons who have the right to sit in the zone of the East are the Venerable Master of the Lodge, the Orator of the Lodge, the Order's Grand Master, the Grand Officers of the Order's Grand Lodge, any Past Grand Officer of the Order's Grand Lodge, and invited dignitaries.

The pedestals of the Senior Overseer and the Junior Overseer are elevated by two steps. In front of the east side of the Senior Overseer's pedestal, a Perfect Ashlar is placed on the floor in such a way that it does not impede the movement of the members of the Lodge. In front of the east side of the Junior Overseer's pedestal, a Rough Ashlar is placed on the floor in such a way that it does not impede the movement of the members of the Lodge.The Secretary's pedestal is not elevated.

The central part of the floor of the Lodge is adorned with a Mosaic Pavement, which is a carpet comprising black and white squares (like a chessboard) and tessellated edges. Freemasons "square the Lodge" by walking around the Mosaic Pavement (and not on it); this practice symbolizes accuracy, perfection, and dignity.

The Altar of Truth (a double cubed altar) is placed at the center of the Mosaic Pavement, and it is a parallelepiped pedestal table on which there are three pillars. On the northern part of the Altar of Truth stands a pillar

with a terrestrial ball (depicting the Earth) placed on top. On the southern part of the Altar of Truth stands a pillar with a celestial ball (depicting the Zodiac Circle) placed on top. Between these two pillars, stands the pillar of the Blazing Star, or Glory, a pillar on top of which is placed a gold Pentagram (Pentalpha) bearing the inscription "G" in green. In front of the west side of the Altar of Truth, a kneeling stool is placed on the Mosaic Pavement so that one who kneels before the Altar of Truth faces East.
Unless the Ritual indicates otherwise, everyone moves in the Lodge only by walking clockwise on the part of the floor that is not covered by the Mosaic Pavement. Moreover, unless the Ritual indicates otherwise, whenever the Director of Ceremonies, the Assistant Director of Ceremonies, and the Expert move in the Lodge, they hold their respective wands in the right hand.
Before the Opening Ceremony, the Outer Guard waits outside the door of the Lodge.
The Entered Apprentices occupy the first row(s) of chairs located in the North (facing South), while the Fellow Crafts occupy the back row(s) of chairs located in the North (facing South). The Master Masons occupy the rows of chairs located in the South (facing North). Those persons who have the right to sit in the East, when present in a Lodge meeting, sit to the right of the Lodge's Venerable Master and to the left of the Lodge's Orator, in the East (facing West).
Finally, before the Initiation Ceremony, the Lodge room should be discreetly scented by burning aloeswood/agarwood incense sticks or essential oil.

Opening Ceremony

V.M.: Brother/Sister Senior Overseer, are you a Free and Accepted Mason?
S.O.: I am, according to my Brothers' and Sisters' judgment and due to my work as a conscientious and diligent member of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.
V.M.: What is the first duty of the Overseers in the Lodge?
S.O.: It is to safeguard the privacy and the exclusivity of our labors.
V.M.: Brother/Sister Junior Overseer, make sure that the Lodge is working according to our rules of privacy and exclusivity.
J.O.: Brother/Sister Inner Guard, act in accordance with the duties arising from your Office.
The Inner Guard, holding his/her sword in his/her left hand, pointing upward, opens the door of the Lodge and instructs the Outer Guard (who

was standing outside the door of the Lodge, guarding the porch of the Lodge) to take his/her seat in the Lodge. The Outer Guard, holding his/her sword in his/her right hand, pointing downward, enters the Lodge and takes his/her seat in it. The Inner Guard closes and locks the door of the Lodge, takes the key and puts it on the Junior Overseer's pedestal, and says to the Junior Overseer: "Brother/Sister Junior Overseer, we are internally and externally safe." When this is done, the Inner Guard (holding his/her sword in his/her left hand, pointing upward) takes his/her seat in the Lodge.

J.O.: The Lodge is working according to our rules of privacy and exclusivity.

S.O.: Venerable Master, the Lodge is working according to our rules of privacy and exclusivity.

At this point, the Inner Guard, who was holding his/her sword in his/her left hand, pointing upward, and the Outer Guard, who was holding his/her sword in his/her right hand, pointing downward, put their swords in their scabbards.

V.M. gavels once, repeated by S.O. and J.O.: To order, Brothers and Sisters, in the First Degree.

All rise and take Step with Entered Apprentice Sign. The Brothers and the Sisters of the Columns stand facing the central axis of the Lodge. However, instead of giving the Entered Apprentice Sign, the Venerable Master and the two Overseers are standing holding their gavels with their right hand on the chest.

V.M.: Brother/Sister Senior Overseer, what is the second duty of the Overseers in the Lodge?

S.O.: It is to make sure that all the Brothers and all the Sisters are in order.

V.M.: Are they all in order?

Each Overseer holds the gavel with his/her right hand on the chest, leaves his/her pedestal and (moving clockwise) walks through his/her respective Column without going up to the zone of the East, and retraces his/her steps. Both Overseers ensure the exact execution of the Order Sign (in the First Degree). When this is done, they resume their regular seats (always moving clockwise, and they remain standing).

J.O.: Brother/Sister Senior Overseer, all the Brothers and all the Sisters who are lined up in front of the South Column are in order.

S.O.: All the Brothers and all the Sisters who are lined up in front of the North Column are in order. Venerable Master, all the Brothers and all the Sisters who adorn the Columns are in order.

The Overseers put their gavels on their respective pedestals, and they stand with the Order Sign (in the First Degree), as do the other attendees.

V.M. gavels once: The same is true of the East.
The Venerable Master puts his/her gavel on his/her pedestal, and stands with the Order Sign (in the First Degree), as do the other attendees.
V.M.: Brother/Sister Senior Overseer, how many hours are there in a Lodge of Entered Apprentices?
S.O.: Five.
V.M.: Define and explain them.
S.O.: Twelve is before the Lodge is opened and when the Lodge is closed. Noon is when the Venerable Master is about to open the Lodge. High noon is when the Lodge is duly opened. Midnight is when the Venerable Master is about to close the Lodge. High midnight is when the Lodge is closed, the profane are allowed to approach, and our Brothers and Sisters work secretly among the profane. This is the peculiar way in which the consciousness of our Initiates refers to time, and it is due to the intentionality, or the referentiality, of their consciousness, or due to the fact that their consciousness is the consciousness of its contents, which thus become experiences for it.
V.M.: Is this the only way in which our consciousness refers to time?
S.O.: No, because consciousness is not passive. In fact, as the German philosopher Edmund Husserl has taught, consciousness not only treats the presence of experiences within itself in a critical way, but also causes their presence, as it is implied by the term "intentionality." Intentionality is not only the ability to refer to something, but also the ability to cause something. Given that, as the French philosopher Henri Bergson has taught, intentionality consists of both the ability to refer and the ability to cause, we realize that the term "intentionality" expresses the dynamism of consciousness, and the dynamism of consciousness manifests itself in the manner in which consciousness intervenes in temporality, restructures time, and, thus, underpins the transition from physical time to historical time.
V.M.: What time is it, Brother/Sister Junior Overseer?
J.O.: It is noon.
V.M.: Since it is the hour at which we must open this Lodge of Entered Apprentices, Brothers and Sisters, working to the Glory of the Enlightened Humanity, I declare the Lodge duly open *(all cut Sign)* for the purposes of our Masonic Order in the First Degree.
V.M. gives E.A.-gavel: three equal distinct knocks.
S.O. gives E.A.-gavel.
J.O. gives E.A.-gavel.
The D.C. (holding his/her wand in the right hand) goes to the V.M.'s Pedestal, opens the Order's Book of Constitutions and arranges a Square

and a Pair of Compasses, placing them on top of the open Book of Constitutions. The Order's Ritual is an integral part of the Order's Book of Constitutions, because the major goal of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry is to preserve, protect, defend, teach, and propagate the Modern and Perfecting Rite of Symbolic Masonry internationally. The Book of Constitutions always remains placed on the Venerable Master's pedestal, so that he/she can read it; the points of the Pair of Compasses are directed towards the Venerable Master and hidden by the arms of the Square (the angle of the Square is directed towards the Venerable Master, while the arms of the Square are directed towards the Columns of the Lodge). The Book of Constitutions is the rule book of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry that regulates all the Lodges and all the Freemasons under its jurisdiction. The Venerable Master sits when the aforementioned tasks have been completed. All the Brothers and all the Sisters take their seats when the Venerable Master sits. Thereupon, the Lodge works in accordance with its agenda.

Initiation Ceremony

Before the Opening Ceremony, the Venerable Master orders the Assistant Director of Ceremonies to prepare the Candidate both physically and mentally. Hence, during the Opening Ceremony, the Assistant Director of Ceremonies (holding his/her wand in the right hand) remains outside the Lodge. In particular, before the Brothers and the Sisters start entering the Lodge, the Assistant Director of Ceremonies places the Candidate in the Chamber of Reflection, where the Candidate remains alone during the Opening Ceremony. As long as the Candidate is in the Chamber of Reflection, the Assistant Director of Ceremonies guards the closed door of the Chamber of Reflection from the outside.

The Chamber of Reflection is a small darkened room within the anteroom of the Lodge, and it is built according to the ratio 1:2. The door of the Chamber of Reflection is in the West. The interior of the Chamber of Reflection is decorated as follows: The floor, the walls, and the ceiling of the Chamber of Reflection are painted an opaque black. The West wall is emblazoned with the inscription "Virtue and Silence" (in the middle, above the door). On the North wall are painted: a human skeleton, the acronym V.I.T.R.I.O.L. (meaning "Visita Interiora Terrae, Rectificandoque, Invenies Occultum Lapidem," or "visit the interior of the earth, and purifying it, you will find the hidden stone," which is another way of saying that, by turning inward, knowing, analyzing, and purifying

yourself, you will find the truth), and the alchemical symbols of Sulfur, Mercury, and Salt (which symbolize the active principle, the passive principle, and their synthesis, respectively).

Sulfur (The Active Principle): 

Mercury (The Passive Principle): 

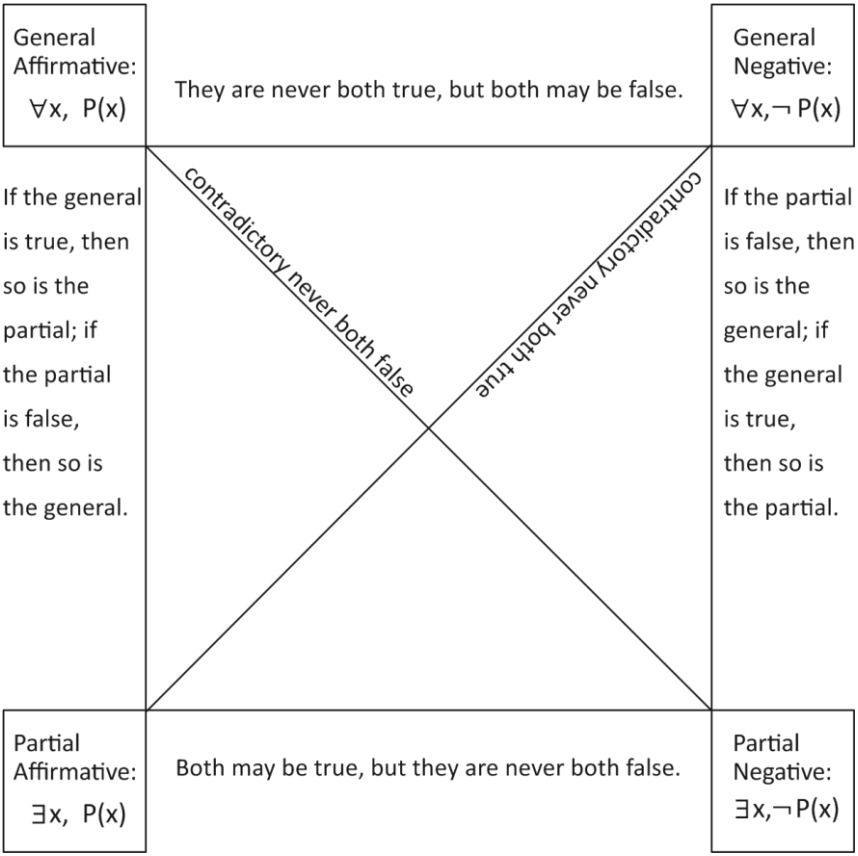
Salt (The Synthesis between the Active Principle and the Passive Principle): 

The South wall is painted with a Sickle, a Hammer, a Pencil, a hand grasping a poignard pointing downward, and the following sentences: "If the mainspring of popular government in peacetime is virtue, the mainspring of popular government during a revolution is both virtue and terror; virtue, without which terror is baneful; terror, without which virtue is powerless. Terror is nothing more than speedy, severe, and inflexible justice; it is thus an emanation of virtue" (Maximilien Robespierre, "On Political Morality," speech to the Convention, delivered on 5 February 1794). The East wall is emblazoned with a Rooster crowing and the following sentences: "You shall see the light" (in the middle, above the Rooster) and "Vigilance and Perseverance" (in the middle under the Rooster). In the center of the Chamber of Reflection, there is a table and a chair directed towards the South. On the table, there are a desk lamp that illuminates the room, a table clock, a Five Petaled Rose, and a picture of Aristotle's logical square. Aristotle's logical square is a diagram representing the relations between the four basic categorical propositions (Fig. 2-3).

Logic (the science of correct reasoning) is a necessary underpinning of every philosophical and scientific endeavor. The first systematization of logic is due to the ancient Greek philosopher and scientist Aristotle. For this reason, the phrase "Aristotelian logic" is still commonly used. Aristotle's works on logic were grouped together by ancient commentators under the title Organon ("Instrument"). In particular, the Organon comprises the following logical treatises of Aristotle: (i) Categories, (ii) On Interpretation, (iii) Prior Analytics, (iv) Posterior Analytics, (v) Topics, and (vi) On Sophistical Refutations. The title Organon, meaning instrument, implies that logic is an instrument and a method used by

philosophy and science, and, according to both Aristotle and the later Peripatetics, the ultimate purpose of correct reasoning is to create correct social relationships and to enable people to correctly communicate the results of philosophical and scientific research to each other. In the third century B.C.E., the Greek Stoic philosopher and logician Chrysippus founded a propositional calculus, studying implication, conjunction, and disjunction. In the mid-nineteenth century, the (largely self-taught) English mathematician, philosopher, and logician George Boole put logic within a rigorous mathematical setting, thus giving rise to what has been known as "Boolean algebra."

Figure 2-3: Aristotle's Logical Square.



In the aforementioned diagram, the following notation is used:

- $P(x)$: *predicate letter (meaning that x (an object) has property P),*
- $\exists$: *"there exists,"*
- $\forall$: *universal quantification ("for every"),*
- $\neg$: *negation ("not").*

Finally, before the Initiation Ceremony, the Chamber of Reflection should be discreetly scented by burning olibanum incense sticks or essential oil. After the Lodge is opened, the Venerable Master informs the members of the Lodge that the purpose of the meeting is to initiate a profane into the

Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry. The Director of Ceremonies is instructed by the Venerable Master to make sure that the Candidate is properly prepared for initiation and lead him/her in front of the door of the Lodge.
The Inner Guard takes the key from the Junior Overseer's pedestal, lets the Director of Ceremonies leave the Lodge, closes and locks the door, puts the key back on the Junior Overseer's pedestal, and resumes his/her seat. The Director of Ceremonies and the Assistant Director of Ceremonies lead the Candidate outside the Chamber of Reflection and prepare him/her as follows: they divest him/her of all metals and money, they put a black double blindfold over his/her eyes, they clothe him/her in a magenta cloak, and they place a white cable-tow with a running noose about his/her neck. The outer blindfold is thicker and denser than the inner blindfold. Both the outer blindfold and the inner blindfold are black, but the outer blindfold does not allow the Candidate to see anything, whereas the inner blindfold allows the Candidate to have a very dim view of the environment. When this is done, the Director of Ceremonies holds the loose end of the cable-tow, takes the Candidate by the right arm, and leads him/her forward to the door of the Lodge. The Assistant Director of Ceremonies follows behind the Director of Ceremonies. Once there, the Director of Ceremonies instructs the Candidate to knock on the door with two equal distinct knocks. Thereupon the Assistant Director of Ceremonies says: "Dare to know" (source: Immanuel Kant, from his essay "What Is the Enlightenment?"). The Director of Ceremonies then instructs the Candidate to repeat the same knocks on the door, and the Assistant Director of Ceremonies says: "Have the courage to use your own understanding" (source: ibid). The Director of Ceremonies then instructs the Candidate to repeat the same knocks on the door for a third time, and the Assistant Director of Ceremonies says: "Enlightenment is man's emergence from his self-imposed ignorance" (source: ibid).
V.M.: Who knocks at the door of the Lodge as a profane?
Then the Inner Guard, having taken the key from the Junior Overseer's pedestal, goes to the door of the Lodge and opens it.
D.C. (standing outside the door of the Lodge and speaking in a loud voice, so that he/she can be heard in the East): Mr./Ms. . . . *(name and surname of the Candidate)*, a free person of sound judgment and good report, who desires to become a Freemason and, specifically, to apprentice himself/herself to a Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.
The D.C. taps once on the floor with his/her wand.

V.M. gavels once: The assessment report that has been prepared by the competent committee of our Lodge for the evaluation of candidates states that Mr./Ms. . . . *(name and surname of the Candidate)* is free of irrationality and superstition, and has removed the veils of glamour from his/her mind. Brother/Sister Inner Guard, let the Candidate enter and be received in due form.

The Expert (holding his/her wand in the right hand) leaves his/her seat, takes a Pair of Compasses from the S.O., and stands inside the Lodge, facing the Candidate, about three feet from the door of the Lodge. The I.G. steps back while the D.C., with the Candidate, enters the Lodge, followed by the A.D.C. The Inner Guard closes and locks the door, holds the key and stands at the left hand of the Expert, both facing the Candidate. Next, the Candidate, guided by the D.C. and followed by the A.D.C., advances one or two steps inside the Lodge, when he/she is stopped by the Expert, who presents one point of a Pair of Compasses to the Candidate's left breast.

Expert: Mr./Ms. . . . *(name and surname of the Candidate),* on entering this Lodge for the first time, I receive you on the point of a Pair of Compasses pressing your left breast, in order to teach you as well as to warn you that our Order will be systematically monitoring and assessing every aspect of your character and behavior throughout your apprenticeship with us.

After this is done, the Expert gives the Pair of Compasses to the I.G., and the latter leaves the key and the Pair of Compasses on the Junior Overseer's pedestal, and resumes his/her seat. The D.C. now leaves the Candidate and the loose end of the cable-tow in the hands of the Expert, and the D.C. and the A.D.C. resume their seats. The Expert takes the Candidate by the right arm and holds the loose end of the cable-tow. Both the Expert and the Candidate stand facing the Venerable Master.

V.M.: Mr./Ms. . . . *(name and surname of the Candidate),* I am the Venerable Master of this Lodge, and I congratulate you on having achieved to cross the gates of this Freemasonic Institution.

The exact origin of the term "Freemasonry" is not known. In 1391, at Oxford, there is reference to a "master of free stones," and there are also Masons called "mason layers." The term "Freemason" is known to have been in use in 1374. The prefix "free" was used to describe those who worked in and sculptured and carved free stone, which was a fine-grained sandstone or limestone lending itself to easy carving and sculpturing and suitable for window and door frames, vaultings, capitals, and other ornamentation used largely in Gothic Architecture. Moreover, the prefix "free" was used to indicate a free man, especially in Scotland. This was

meant to refer to a man who had completed his indentures and was free to work on his own. The first recorded use of the word Lodge in a Masonic context can be traced back to the building of a Cistercian Monastery at Vale Royal near Chester in 1278. In that period of the history of Masonry, the Lodge was a hut in which Masons conversed, organized their work, took their midday meals, and, on certain occasions, they could even sleep. By the mid-fourteenth century C.E., there were elaborate rules governing the behavior of Masons of the Lodge attached to York Minster.

In ancient times, Masonry became a true science and art by the Greeks. In 1940, C. F. C. Hawkes, assistant Keeper of Antiquities in the British Museum, published his seminal book entitled *The Prehistoric Foundations of Europe to the Mycenaean Age*, in which he argued that the megalithic architecture, the use of copper, the building of walls around cities, and the erection of ancient edifices are ancient Greek achievements. The distinguished and highly influential American Freemason, researcher, and historiographer Manly P. Hall, in his seminal book entitled *The Secret Teachings of All Ages: An Encyclopedic Outline of Masonic, Hermetic, Qabbalistic and Rosicrucian Symbolical Philosophy*, originally published in 1928, argues that the "Dionysian Architects" constituted an ancient secret society whose principles and doctrines were similar to those of modern Freemasonry. According to Hall, the Dionysian Architects constituted an organization of builders bound together by their secret knowledge of the relationship between the exoteric and the esoteric aspects of architecture. They were supposedly employed by King Solomon in the building of the Temple of Jerusalem, although they were not Jews, and they were followers of Bacchus and Dionysus. Moreover, according to Hall, the Dionysian Architects erected many of the great monuments of Antiquity, possessed a secret code for communicating among themselves and for marking their stones, had special convocations and sacred feasts, and Hiram Abiff, the Architect of King Solomon's Temple, was an initiate of this society.

In the eighteenth century, the Grand Lodge of London and Westminster was formed under the direction of the Rev. Dr. John Theophilus Desaguliers, who emerged as one of the most prominent pedagogues of Hanoverian England. In the 1720s, the Rev. Dr. John Theophilus Desaguliers began the English Masonic tradition of treating Hiram Abiff as the major Freemasonic symbol of devotion to duty. In 1 Kings 7:13–14, Hiram is described as the son of a widow from the tribe of Naphtali who was the son of a Tyrian bronze worker, contracted by Solomon to cast the bronze furnishings and ornate decorations for the Jerusalem Temple. In the original Hebrew version of 2 Chronicles 2:13,

the name "Huram Abi" is mentioned. The Masonic term "Hiram Abiff" derives from the Hebrew term "Huram Abi," and the term "Abi" or "Abiff" is a Hebrew term of respect literally meaning "father," and, in the English Freemasonic context, it means "master craftsman."

The motif that is known as the Seal of Solomon, or Hexalpha (two interlaced equilateral triangles), was also one of the Dionysian Architects' symbols. The "Hexalpha" probably emerged as a symbol of harmonious duality and, particularly, of the ten primary contrasting qualities of Pythagoras: the limited and unlimited, odd and even, male and female, one and the many, right and left, rest and motion, straight and curved, light and darkness, good and bad, as well as the square and the oblong. In the context of Pythagorean philosophy, the Hexalpha and the Icosahedron represent the union of complementary forces.

With regard to the ancient Roman Colleges, in his aforementioned book, Manly P. Hall argues that the Roman *Collegia* of skilled architects were a subdivision of the greater Ionian body, and that it is most likely that the Dionysian Architects exerted profound influence on early Islamic culture, too. Indeed, it is a historical fact that ancient Roman art was derived from and relied heavily on ancient Greek art. According to M. P. Hall, during their expeditions in the East, the medieval Knights Templar came in touch with the architectural heritage of the Dionysian Artificers and introduced many of the Dionysian Artificers' symbols and doctrines into medieval Europe.

Sir David Brewster, in his book *The History of Freemasonry*, published in 1804, argues that the Masons sent to King Solomon from Tyre were members of the Dionysian fraternity. In the eleventh century B.C.E., the inhabitants of the ancient Greek city of Athens thought that their country was too small and the soil of poor quality. They sailed to Asia Minor and drove out the inhabitants of a piece of land that they called Ionia, and where they built many towns. They took with them their knowledge of the sciences and arts, and they introduced the original Mysteries of Pallas and Dionysus, the Roman Bacchus, into Ionia.

John A. Weisse, M.D., in his book *The Obelisk and Freemasonry*, published in 1880, argues that the Dionysian Artificers made their appearance no later than 1000 B.C.E. They enjoyed particular privileges and immunities, possessed secret means of recognition, and were bound together by special secret ties. In his previous book, Weisse argues that it has been claimed that King Solomon of Israel, at the instance of King Hiram of Tyre, employed the Dionysian Artificers at the Jerusalem Temple and Palace, and that they were also employed at the construction of the Temple of Diana at Ephesus. Additionally, according to Weisse

(ibid), from the Dionysian Artificers sprang the guilds of the travelling Masons known in the Middle Ages.

In medieval times, Freemasonry was a closed Trade Guild. In those years, the Masters of the Craft were in possession of secret knowledge about the weights of various types of stone per solid cubit, their capacity, the effects of grains, the methods used to raise stones to their required setting, the construction of machines to achieve this, and many other matters connected with the Craft of Masonry. Thus, in the Middle Ages, a Master of the Craft was justified in claiming that he was a Master of Arts and Sciences. With the decay of the Guilds, this knowledge ceased to retain its secret nature, and it became more generally available through the textbooks and tables of the Architectural and Engineering Professions, which have now taken the place of the Guild of Masons. However, the journey of the stones has provided a symbolism of the advance of the men who wrought the stones. In the context of Symbolic Masonry, the stone and its worker became identified in the Lodge symbolic ceremonies. In all the different rites of Symbolic Masonry, the Candidate represents a "living stone" that is being wrought from the rough, in which it is received from the Quarry to a state of perfection. Thus, even though there are various different Freemasonic Rites practiced around the world, the most general definition of Symbolic Masonry is that it is a system of spiritual and social development based on the symbolic use of Operative Masons' tools, habits, methods of work, and structures of organization.

Historians are now certain that Scottish operative Lodges began to admit non-operative members as "Accepted" in the seventeenth century—that is, honorary or gentlemen, Masons—and that the Accepted Masons had gained the ascendancy by the eighteenth century. The Lodges in which the Accepted Masons had gained the ascendancy became symbolic, or "speculative," Lodges, while others continued practicing Operative Masonry. In addition, historical researchers have found several references to the Scottish stonemasons' Lodges having a Mason's Word and secret modes of recognition, thus establishing a system of mutual trust among travelling Operative Masons.

In an enumeration of the guilds entitled to representation in the Common Council of London in 1370, guild No. 17 was the "Company of Freemasons," and guild No. 34 was the "Company of Masons." The Company of Masons appears to have been of greater numerical strength than the Company of Freemasons since it had four representatives, as opposed to two for the other. Albert Mackey's *History of Freemasonry* suggests that the Freemasons formed a smaller and more select society, but this is pure speculation because there are no historical facts about the

special social status of Freemasons. However, the previous list establishes the existence of two separate guilds. Ultimately, these two guilds were merged, taking a coat of arms which displayed three white castles with black doors and windows on a black field, together with a silver or scalloped chevron and on it a black pair of compasses. Furthermore, town mason guilds frequently united with, or formed parts of, guilds of other workers employed in the building trades, and the rules laid down for practical guidance of members of the Craft corresponded in the main with similar rules laid down in other craft guilds of that period.

In the British Museum, there is an old document from around 1390 C.E. that gives some rules and regulations, or, as they are known among Freemasons, "Charges." This document is called the *Regius Poem*, and it is admitted to be the oldest genuine record of the Craft of Masonry known. The aforementioned Charges indicate that Freemasonry was not only a technical activity, but also an organization that safeguarded and promoted a specific ethos. Thus, from the outset, Freemasonry has also been a cultural institution. The fact that Freemasonry has a cultural core, which transcends the technical activities of Operative Masons, enabled and encouraged the survival of Freemasonry as a symbolic system, and indeed a cultural phenomenon, after the decay of the guild system.

In order to understand the transition from Operative Masonry to Symbolic Masonry, you must realize that Symbolic Masons do not intend to build any material, stone temple, but they try to erect a symbolic temple whose major pillars are Freedom, Reason, Justice, and Love.

Having crossed the gates of this Freemasonic Institution, are you prepared to go forward?

Candidate (prompted by the Expert): I am prepared.

V.M.: Mr./Ms. . . . *(name and surname of the Candidate)*, is it still your sincere desire to be initiated into this Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry?

Candidate (prompted by the Expert): It is.

V.M.: As a trial of your courage, your fidelity, and your power of endurance, this Lodge enjoins upon you a series of esoteric journeys. Go forth as an advocate and defender of the ethos of our Order.

The Director of Music starts playing Johann Sebastian Bach's "Little" Fugue in G Minor. The Director of Ceremonies places a small seesaw in the South, and he/she takes the Rough Ashlar from the Junior Overseer's pedestal and places it in the North. When these are done, he/she resumes his/her seat. The Expert, holding the blindfolded Candidate by the right arm and the loose end of the cable-tow, guides him/her during the esoteric journeys. The first journey consists of a walk through the Lodge (without

stepping on the Mosaic Pavement), through the North, the East, the South, and back to the West. At the end of the journey, when the Candidate is back in the West, the Director of Music stops playing music. During the first esoteric journey, there are two stumbling blocks, one in the North and another in the South. The stumbling block in the North is the Rough Ashlar that the Director of Ceremonies has placed there. The Candidate is guided against the Rough Ashlar by the Expert, who makes sure that the Candidate remains upright and safe. When this is done, the Director of Ceremonies takes the Rough Ashlar from the North, places it back in its original position (in front of the east side of the Junior Overseer's pedestal), and resumes his/her seat. The stumbling block in the South is a small seesaw on which the Candidate walks (from one edge to the other) with the assistance of the Expert, who supports the Candidate to remain upright and safe. When this is done, the Director of Ceremonies removes the seesaw, placing it behind the chairs that exist in the South, and resumes his/her seat. When the first esoteric journey is completed, the Expert and the Candidate stand in the West, at the left hand of the J.O., and they face towards the East.

J.O.: Venerable Master, the Candidate has completed his/her first esoteric journey.

V.M.: What happened to the Candidate during his/her first esoteric journey?

J.O.: The Candidate tripped over the rough ashlar, but his/her handler has saved him/her from falling. Moreover, with the assistance of his/her handler, the Candidate managed to walk on unstable ground without falling. The Candidate has completed his/her first esoteric journey successfully, and he/she has made significant progress in Wisdom.

V.M.: Brother/Sister Expert, guide the Candidate through his/her second esoteric journey.

The Director of Music starts playing Johannes Brahms's Symphony No. 3 in F Major, Op. 90-4, Allegro. The Expert, holding the blindfolded Candidate by the right arm and the loose end of the cable-tow, guides him/her through the second esoteric journey. The second journey consists again of a walk through the Lodge, through the North, the East, the South, and back to the West. At the end of the journey, when the Candidate is back in the West, the Director of Music stops playing music. During the second esoteric journey, in the places where there were previously stumbling blocks, the Candidate is stopped by a strong pressure against his/her chest and right shoulder. In particular, in the North, the Secretary stops the Candidate by pressing against his/her chest and right shoulder, but the Expert pushes the Candidate through, and the latter continues

his/her journey; and, in the South, the Director of Ceremonies stops the Candidate by pressing against his/her chest and right shoulder, but again the Expert pushes the Candidate through, and the latter continues his/her journey and returns to the West. When the second esoteric journey is completed, the Expert and the Candidate stand in the West, at the left hand of the J.O., and they face towards the East.

J.O.: Venerable Master, the Candidate has completed his/her second esoteric journey.

V.M.: What happened to the Candidate during his/her second esoteric journey?

J.O.: Opposing forces tried to halt his/her progress, but the Candidate continued his/her journey with the support of his/her guide. The Candidate has completed his/her second esoteric journey successfully, and he/she has made significant progress in Strength.

V.M.: Those who are committed to fulfilling what is recommended in these esoteric journeys feel sorrow for the fact that many of our fellow humans are enslaved to imperfect images of God created by humans and to spiritual insignificance. We must work in order to enlighten humans, and we must use our privileges in order to correct mistakes. Mr./Ms. . . . *(name and surname of the Candidate),* are you willing to persist in your desire to continue on the Path to the Light?

Candidate (prompted by the Expert): I am.

V.M.: Mr./Ms. . . . *(name and surname of the Candidate),* are you willing to take the Oath of an Entered Apprentice Mason according to our Order's Ritual and Constitution?

Candidate (prompted by the Expert): I am.

The J.O. approaches the Candidate and hands him/her the "Bitter Chalice" (namely, a large cup filled with a bitter drink), which the Candidate must drink to the bottom. When this is done, the J.O. removes the outer of the two blindfolds from the Candidate, and then, taking the chalice and the Candidate's outer blindfold with him/her, the J.O. resumes his/her seat.

V.M.: Brother/Sister Expert, guide the Candidate through his/her third esoteric journey.

The Director of Music starts playing Ludwig van Beethoven's Grosse Fuge, Op. 133. The Expert, holding the blindfolded Candidate by the right arm and the loose end of the cable-tow, guides him/her through the third esoteric journey. The third journey consists again of a walk through the Lodge, through the North, the East, the South, and back to the West. At the end of the journey, when the Candidate is back in the West, the Director of Music stops playing music. During the third esoteric journey, there are no

more obstacles on the Candidate's path. When the third esoteric journey is completed, the Expert and the Candidate stand in the West, at the left hand of the J.O., and they face towards the East.

J.O.: Venerable Master, the Candidate has completed his/her third esoteric journey.

V.M.: What happened to the Candidate during his/her third esoteric journey?

J.O.: The Candidate continued his/her esoteric journey by harmonizing himself/herself with our Order's rhythm, and he/she has made significant progress in Beauty.

V.M.: Mr./Ms. . . . *(name and surname of the Candidate)*, is it still your sincere desire to be initiated into this Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry?

Candidate (prompted by the Expert): It is.

All rise silently and (with the exception of the Expert) constantly point their outstretched right index finger at the Candidate's heart, and they remain in that position.

V.M.: Brother/Sister Junior Overseer, remove the Candidate's blindfold.

The order is executed: the J.O. approaches the Candidate and removes the latter's remaining, inner blindfold. All attendees (with the exception of the Expert), pointing their outstretched right index finger at the Candidate's heart, say together, referring to the Candidate: "It is up to you." Then the J.O. puts the inner blindfold back over the Candidate's eyes. When this is done, the J.O. returns to his/her seat, and all Brothers and Sisters (with the exception of the Expert) give the Sign of an Entered Apprentice Freemason and retain it—that is, they are "in Order." The Expert conducts the Candidate directly to the kneeling stool, which is placed in front of the west side of the Altar of Truth, and then the Expert releases the Candidate's right arm. The Expert assists the Candidate to kneel on both knees (instructing him/her in a whisper) and to place both hands on the Altar of Truth. The Expert continues to hold the loose end of the cable-tow and stand at the right hand of the Candidate. The Expert assists the Candidate to repeat the Oath, which is read by the Venerable Master.

V.M. (addressing the Candidate): Repeat your name at length and say after me: I . . . *(name and surname of the Candidate)*, in the presence of the Universal Cause, often referred to as the Great Architect of the Universe, and in the presence of this Lodge, promise to promote the purpose of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry to the best of my ability by word, writing, deed, and example. I pledge obedience to the Constitution and the Regulations of the Autonomous Order of the Modern and Perfecting Rite of Symbolic

Masonry and the Decrees of the Grand Lodge. I promise to faithfully fulfill the obligations imposed on me by or pursuant to this Order's Laws and Decrees and, in view of the nature of this Order as a private exclusive membership association, to maintain this Order's policy of secrecy. Being aware that this Order works both overtly and covertly and that it maintains both overt and covert rituals and teachings, I swear and promise all these without equivocation or mental reservation and with no hope of escaping the penalty that our Order reserves for the traitors of the First Degree.

After the aforementioned Solemn Obligation, the attendees cut the Sign of an Entered Apprentice, and they form the Fraternal Chain: all Brothers and Sisters, with the exception of the Expert, give their right neighbor the left hand, and their left neighbor the right hand, so that the Candidate is included in this Fraternal Chain between the two Overseers. The Fraternal Chain remains.

V.M. (addressing the Candidate): To become strong, one must understand the depths of the darkness within and how to raise oneself up. Our Matter is luminous and in itself shining and clear, but it is only to be found in the darkest dwelling. Dear Candidate, given your present situation, what is your predominant desire?

Candidate (prompted by the Expert): Light.

V.M.: Brother/Sister Expert, let that blessing be restored to the Candidate.

The Expert removes the cable-tow from the neck of the Candidate, and then he/she removes the remaining, inner blindfold from the eyes of the Candidate. The Expert places the cable-tow and the blindfold on the Altar of Truth.

V.M. (addressing the Candidate): Having been restored to the blessing of light, I present to you what we consider the three great symbolic lights of Freemasonry: the Square, the Pair of Compasses, and the letter G. The Square is to test and regulate our actions, the Pair of Compasses is to keep us in due bounds with humanity in general and our Order in particular, and the letter G stands for the Greek words "Gnosis" and "Geometry." Rise, newly obliged Brother/Sister among Brothers and Sisters of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.

The Candidate rises with the assistance of the Expert. All the attendees cut the Fraternal Chain, and, with the exception of the Expert, they resume their seats. The Expert turns left and, holding the Candidate's right hand, conducts the Candidate to the north side of the Venerable Master's pedestal, so that they both are in a position parallel to the pedestal and about two feet from it, facing South towards the Venerable Master; and the Expert releases the Candidate's hand.

V.M. (addressing the Candidate): Brother/Sister . . . *(name and surname of the Candidate)*, I now present to you the three lesser symbolic lights of Freemasonry: they are situated East, South, and West, and they are meant to represent the Sun, the Moon, and the Venerable Master of the Lodge. The Sun governs the day and is symbolically associated with Gold, which is a symbol of the Mind; the Moon governs the night and is symbolically associated with Silver, which is a symbol of the Body; and the Venerable Master governs his/her Lodge.

According to our Order's Ritual, the predominant colors of Symbolic Masonry are white, black, light blue, dark blue, orange-red, yellow-gold, and green. White is the combination of all colors, and it symbolizes purity and emanation of light. Black is the absence of color, and it symbolizes putrefaction and absorption of light. When alchemists maintain that, by time and force, the Black Dragon of putrefaction can become fashioned into the White Swan of purity, they refer to a significant mental change. Light blue symbolizes the breaking down of old imperfect forms, the overcoming of egotism, the destruction of one's attachments to material possessions, and a further breaking down of artificial psychological structures by total immersion in the unconscious. Orange-red symbolizes a combination of moral development and courage, the elimination of inner impurities, the "rediscovery" of our essence, and the reclaiming of visionary "gold." Dark blue symbolizes the purification of the self through introspection in order to free oneself from irrational passions and sentiments and to ensure that no impurities derived from the conscious or the subconscious mind inhibit the spiritual, the historical, and the political progress of humanity. Yellow-gold symbolizes intellect, knowledge, calmness, peace, and gladness. Green, being the almost universal color of leaves and growing shoots of all vegetation, symbolizes hope, progress, evolution, and regeneration. In our Lodges, the wall of the East is painted orange-red, the walls of the West, the North, and the South are painted light blue, and the ceiling is painted dark blue and emblazoned with gold constellations. The pillar of the Blazing Star, or Glory, which stands on the Altar of Truth, is a pillar on top of which is placed a gold Pentagram at the center of which is placed the letter G painted green.

The working tools of an Entered Apprentice are the 24-inch Gauge, the common Gavel, and the Chisel. The 24-inch Gauge is a measurement tool that refers us to the twenty-four hours of the day, which we should use wisely. The common Gavel symbolizes the force of moral consciousness. The Chisel is a symbol of the advantages of education, which underpin true nobility.

I shall now proceed to entrust you with the marks by which, in our Order, the Entered Apprentices are known to each other and distinguished from the other people. In general, all Squares, Levels, and Perpendiculars are the truest and most proper Signs by means of which genuine Freemasons can identify each other. Therefore, your body, being considered a symbol of your mind, must be perfectly erect *(the Expert assists the Candidate to comply),* and your feet, symbolizing the rectitude of your actions, must form a Square *(the Expert instructs the Candidate in whisper to unite his/her heels so that they form a Square).* You will now take a short pace towards me with your left foot, bringing the right heel into the hollow of the left foot *(the Expert assists the Candidate to comply).* That is the first regular step in Freemasonry, and it is in this position that the Sign, the Token, and the Word of the First Degree are communicated. *(The V.M. rises, faces the Candidate, and takes Step).*

Place the right hand in front of the throat with the fingers together and the thumb extended in the form of a square to the left of the trachea. *(The V.M. illustrates and ensures that the Candidate copies; the Expert assists the Candidate).* In this position, one is "in Order" in the First Degree. The Sign is given by drawing the hand horizontally across to the right shoulder and letting it drop to the side with the arm at full length. *(The V.M. illustrates and ensures that the Candidate copies; the Expert assists the Candidate).* This Sign is formed of the Square, the Level, and the Perpendicular, and it alludes to the symbolic penalty of the First Degree, which implies that a Freemason would rather have had his/her throat cut across than profane his/her initiation.

The Grip or Token is given as follows *(the V.M. takes the Candidate's right hand and adjusts the grip by placing the Candidate's thumb in position before placing his/her own; the Expert assists the Candidate):* by three equal, distinct pressures of the thumb on the first joint of the hand, since deductive reasoning consists of three terms, the major and the minor propositions and the conclusion. This Grip or Token demands a Word. In our Order, the Word of the First Degree is communicated as follows: the first Brother/Sister says "Ask," the second Brother/Sister responds by saying "Seek," and then they both say "Knock."

Finally, in our Order, there are also other methods of mutual recognition which will be properly communicated to you by the Secretary after the end of this ceremony; and the Secretary will guide you through our Order's private channels and protocols of communication.

The V.M. places the Candidate's right hand in the left hand of the Expert and sits. The Expert turns right, controlling the Candidate by holding his/her right hand, regains the floor of the Lodge, turns left, instructs the

Candidate in whisper to step off with the left foot, and passes in front of the Venerable Master's pedestal to the southeastern corner, where they halt for a moment in order to emphasize the practice of squaring the Lodge. Then the Expert leads the Candidate to the southwestern corner, where they halt for a moment in order to emphasize the practice of squaring the Lodge. When this is done, the Expert leads the Candidate to the south side of the Junior Overseer's pedestal, where they stand parallel to the pedestal (facing it) and a convenient distance from it. The Expert releases the hand of the Candidate. Afterwards, the Expert taps once on the floor with his/her wand.

Expert: Brother/Sister Junior Overseer, I present to you Brother/Sister . . . *(name and surname of the Candidate).*

J.O.: Brother/Sister . . . *(name and surname of the Candidate)*, advance to me as a Freemason.

The Expert instructs the Candidate in whisper to take Step only and ensures that the Candidate does not give Sign at this stage.

J.O.: What is that?

Candidate (prompted by the Expert): The first regular step in Freemasonry.

J.O.: Have you learned anything else?

Candidate (prompted by the Expert): I have.

The Expert instructs the Candidate in whisper to give the Entered Apprentice's Sign and cut it.

J.O.: What is that?

Candidate (prompted by the Expert): The Sign of an Entered Apprentice Freemason.

J.O.: Have you anything to communicate?

Candidate (prompted by the Expert): I have.

J.O. rises, faces the Candidate, takes Step, and offers hand. The Expert places the Candidate's right hand in that of the J.O., and, with the left hand, he/she adjusts the grip from above. The J.O. gives the grip after the Expert has adjusted the Candidate's right thumb, and he/she retains the grip.

J.O.: What is this?

Candidate (prompted by the Expert): The Grip or Token of an Entered Apprentice Freemason.

J.O.: What does it demand?

Candidate (prompted by the Expert): A Word.

J.O.: Give me that Word.

Candidate (prompted by the Expert): Ask.

J.O.: Seek.

J.O. and Candidate (prompted by the Expert): Knock.
J.O.: Pass.
The J.O. replaces the Candidate's right hand in the left hand of the Expert and remains standing. The Expert takes the Candidate by the right hand to the south side of the Senior Overseer's pedestal, and he/she places the Candidate's right hand in the Senior Overseer's left hand and, standing on the right side of the Candidate, he/she ensures that they are both facing East. Then the S.O. rises, takes Step, and gives the Entered Apprentice's Sign, which he/she retains.
S.O.: Venerable Master, I present to you Brother/Sister . . . *(name and surname of the Candidate)* in order to be invested with the distinguishing badge of an Entered Apprentice Freemason.
V.M.: Brother/Sister Senior Overseer, I delegate you to invest him/her with the distinguishing badge of an Entered Apprentice Freemason.
The Expert divests the Candidate of his/her magenta cloak, and places it on the Senior Overseer's pedestal. Then the S.O. cuts the Sign, releases the hand of the Candidate, and, with the Candidate facing him/her, he/she invests the Candidate with the apron and the sash of an Entered Apprentice Freemason (the Expert assists as necessary). The sash is worn over the right shoulder and under the left arm. Moreover, the S.O. gives the Candidate a pair of white gloves and asks him/her to wear them. Finally, the S.O. places the distinctive breast jewel of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry on the Candidate's left breast (this breast jewel, suspended by a white ribbon and consisting of a gold Square by which is suspended a gold square shape within which is inscribed a gold circle within which is inscribed a gold equilateral triangle, is worn by every member of this Order, irrespective of one's Masonic degree and administrative rank). Thereupon, the Expert takes the Candidate by the right hand and turns him/her towards the East, so that they both stand facing East, and then the Expert releases the Candidate's right hand.
V.M.: Brother/Sister . . . *(name and surname of the Candidate)*, the distinguishing badge of an Entered Apprentice Freemason is a white apron, symbolizing labor, innocence, and the bond of friendship. It is sixteen inches wide and fourteen inches deep. In general, the Masonic apron has a triangular flap pointing down in order to indicate that wisdom has entered a Freemason's mind and controls him/her from the inside. In our Order, Entered Apprentice Freemasons are also invested with a white sash, five inches wide, edged with light blue braid, and embroidered with a gold rayed square shape having the number 1 in orange-red at its center. The sash is worn over the right shoulder and under the left arm. In our

Order, the distinctive symbol of an Entered Apprentice Freemason is a square shape as it is defined in Euclidean geometry: a regular quadrilateral. In other words, in Euclidean geometry, a square is defined as a quadrilateral such that all sides have equal length and every interior angle is a right angle (equal to 90 degrees). In sacred geometry, the square symbolizes orientation, direction, as well as law and order. The four sides of a square symbolize the four dimensions of the physical world: the three dimensions of physical space and the dimension of time. In addition, all Symbolic Masons always wear white gloves at Lodge meetings. White gloves symbolize the qualitative difference between the acts of a profane and one who has been truly purified and enlightened by our Craft. I congratulate you on being accepted to this Masonic Order. The continuity of this Order depends on obedience. You entered this Lodge in a symbolic state of blindness and poverty, and, by your obedience during this initiation ceremony, you have symbolically escaped the dangers of stabbing and strangulation. On your entrance into the Lodge, the Expert presented one point of a Pair of Compasses to your left breast, implying that, if you had attempted to rush forward in a disobedient way, you would have caused your death by stabbing, since the Expert would have done his/her duty. Moreover, there was a cable-tow with a running noose about your neck, implying that any attempt of retreat would have proved fatal, too. However, there was a third danger, which continues to hang over you, and it is symbolized by the Sign of an Entered Apprentice Freemason. You should not be afraid of this danger, because today you became a sworn member of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, and this bond is binding for life. You are now at liberty to retire in order to restore yourself to your personal comforts. When you return to the Lodge, you will listen to a Charge that explains important aspects of our noble institution.

The S.O. and the J.O. resume their seats. The Expert takes the Candidate by the right hand and, moving counter-clockwise, conducts him/her to the door. The I.G. takes the key from the Junior Overseer's pedestal, goes to the door in front of the Expert and opens it, closing and locking it after the Expert and the Candidate have gone out. Then, the I.G. puts the key back on the Junior Overseer's pedestal and resumes his/her seat. The Venerable Master announces that he/she suspends the labors of the Lodge for a ten-minute break, and gavels once. The I.G. gives the Entered Apprentice's knocks on the door of the Lodge (namely, three equal distinct knocks), from the inside, and then resumes his/her seat. In this way, the Expert, who is outside the Lodge with the Candidate, understands that the Lodge has suspended its labors for a break. Outside the Lodge, the Expert

returns to the Candidate his/her money and his/her metal objects, which were withheld before the Candidate entered the Lodge room. The Candidate continues to wear the regalia of an Entered Apprentice. After a ten-minute break, the Venerable Master announces that the Lodge resumes its labors, and gavels once.

J.O.: Brother/Sister Inner Guard, invite the Expert and the Candidate to return to the Lodge.

The I.G. takes the key from the Junior Overseer's pedestal, goes to the door and receives the Expert and the Candidate. The I.G. then closes and locks the door, puts the key back on the Junior Overseer's pedestal, and resumes his/her seat. The Expert conducts the Candidate by the right hand to the south side of the Senior Overseer's pedestal, and they both face towards the East. The Expert releases the hand of the Candidate.

Expert: Brother/Sister . . . *(name and surname of the Candidate),* salute the Venerable Master as a Freemason.

The Candidate takes Step, gives the Entered Apprentice's Sign, and cuts it.

V.M.: Brother/Sister Expert, you will place our newly initiated Brother/Sister at the northeastern part of the Lodge. Brother/Sister . . . *(name and surname of the Candidate),* according to an old Masonic tradition, at the erection of all stately and superb edifices, the first or foundation stone is laid at the northeastern corner of the building, and, you, being newly initiated into Freemasonry, are placed at the northeastern part of the Lodge in order to symbolically represent that stone. May you always honor your initiation.

The Expert takes the right hand of the Candidate and leads him/her up North to the northeastern part of the Lodge, and instructs him/her to sit there. When this is done, the Expert leaves the Candidate sitting on a chair at the northeastern part of the Lodge, and resumes his/her regular seat in the Lodge. Everyone is sitting while the Charge is delivered.

Charge:

Orator: The Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry is an association of Masonic Lodges maintaining and practicing the Modern and Perfecting Rite of Symbolic Masonry, which consists of three degrees, namely, those of the Entered Apprentice, the Fellow Craft, and the Master Mason, and of a system of thirty appendant degrees controlled by the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry. The overarching governing body of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry is called the Grand

Lodge of the Order, and it is presided over by the Order's Grand Master. In our Order, the Grand Master of the Grand Lodge is the *ex officio* Sovereign Grand Commander of the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for our Grand Lodge, the Deputy Grand Master of the Grand Lodge is the *ex officio* Pro Sovereign Grand Commander of the aforementioned Supreme Council, the Assistant Grand Master of the Grand Lodge is the *ex officio* Lieutenant Grand Commander of the aforementioned Supreme Council, the Senior Grand Overseer of the Grand Lodge is the *ex officio* Grand Chancellor of the aforementioned Supreme Council, the Junior Grand Overseer of the Grand Lodge is the *ex officio* Grand Chamberlain of the aforementioned Supreme Council, the Grand Orator of the Grand Lodge is the *ex officio* Grand Librarian and Grand Recorder of the aforementioned Supreme Council, the Grand Director of Ceremonies of the Grand Lodge is the *ex officio* Grand Master of Ceremonies of the aforementioned Supreme Council, the Grand Secretary of the Grand Lodge is the *ex officio* Grand Secretary General and Grand Keeper of the Seal of the aforementioned Supreme Council, the Grand Treasurer of the Grand Lodge is the *ex officio* Grand Treasurer of the aforementioned Supreme Council, the Grand Expert of the Grand Lodge is the *ex officio* Grand Captain of the Guard of the aforementioned Supreme Council, the Grand Inner Guard of the Grand Lodge is the *ex officio* Grand Standard Bearer of the aforementioned Supreme Council, and the Grand Outer Guard of the Grand Lodge is the *ex officio* Grand Sword Bearer of the aforementioned Supreme Council. Therefore, the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry has under its authority all the degrees from the first to the thirty-third; and, according to the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, the degrees of the Ancient and Accepted Scottish Rite, 4º–33º, are not superior to the three degrees of Symbolic Masonry, but they are educational supplements to the third degree of Symbolic Masonry and degrees with which we honor members of our Order who have worked in an excellent manner.

With regard to "autonomy," there are two types of societies: autonomous societies and heteronomous societies. In heteronomous societies, people unquestionably conform to the rules, values, goals, and ways of life that the instituted social structure imposes on them and with which it spiritually molds them. Any society whose laws are supposed to have been given by God Himself is a heteronomous society. In such a society where the deity, or the absolute being, is conceived of not only as the most abstract source of the meaning of beings and things in the world but also

as a transcendent legislator who has given particular laws, the question whether the established legal system is just cannot be posed; for such people, this question is meaningless, because they believe that their laws constitute a covenant between them and God Himself. This mentality characterizes all pre-modern traditional Middle Eastern and Far Eastern societies, pre-Colombian American societies, primitive races, and the traditional medieval European societies.

On the other hand, in an autonomous civilization, the fundamental issues of social life are essentially contested, and reality is approached with reference to reason and a dialectical conception of history. The most characteristic examples of autonomous societies are the ancient Athenian city-state from the eighth century B.C.E. to the fifth century B.C.E., the classical Roman republic, the civilization of the Italian Renaissance, and several other Western European societies from the Late Middle Ages onwards. These societies subjected the fundamental issues of social organization to rational analysis and assessment, gave birth to a movement of political awakening and participation that potentially included everybody, and allowed their citizens to question traditional representations of the world and traditional ideas about what has value and what has not, about what is just and what is not, etc.

One of the most important characteristics of the citizens of an autonomous society is the fact that they pose themselves such questions as: "how should society be instituted?", "how should we think?", "what is truth and how can we achieve it?", etc. An autonomous society chooses its values not in order to be enslaved to those values or to a tradition, but because its values are characterized by rationality, and they are susceptible to critical thinking. In other words, an autonomous society lacks neither tradition, nor laws, nor values, rather it accepts and defends its tradition, laws, and values on rational grounds and after critical thinking. An autonomous society highlights and protects the ability and the right to criticize neither as an end in itself nor as a pleasant habit, but as an expression of personal and social autonomy. Therefore, an autonomous society is characterized by a critical spirit and not by a spirit of criticism. A "critical spirit," which is based on rational accountability and freedom, should never be confused with a "spirit of criticism," which is characterized by egoism, arbitrary selfishness, and irrationality.

The symbolic meaning of a Masonic Lodge was expounded by the distinguished Scottish author, editor, and Freemason William Preston in his book entitled *Illustrations of Masonry*, originally published in 1772. Even though that book remains a useful Masonic guide and compendium, it expresses and reflects the ethos and the needs of a particular

Freemasonic movement. This, in turn, expresses and reflects the ethos and the needs of the ruling British elites of the eighteenth and the nineteenth centuries. The Modern and Perfecting Rite of Symbolic Masonry embodies a substantive and radical revision of the rituals, the catechisms, and the constitutions of the Freemasonic institutions that were established in the British Isles during the eighteenth and the nineteenth centuries. In particular, the Modern and Perfecting Rite of Symbolic Masonry embodies a Freemasonic Reformation whose four major purposes are the following: firstly, to endow Symbolic Masonry with a rigorous method of philosophical studies; secondly, to endow Symbolic Masonry with an explicit and rigorous system of philosophical anthropology; thirdly, to extricate Masonic thought from the intellectual shackles of atavism, superficial ritualism, superstition, vainglory, daydreaming, and petty-bourgeois mentalities; and, fourthly, to orient Freemasonry towards a rational, cosmopolitan, and humanistic vision untainted by particular international-political actors' methods of selfish expediency.

The word "philosophy" derives from the Greek word "philosophīa" ("φιλοσοφία"). The ancient Greek word "philosophīa" is composed of two other ancient Greek words: "phileīn" ("φιλεῖν") and "sophīa" ("σοφία"). The word "phileīn" means "to love," "to endorse," and "to be wont to do (something)"; the word "sophīa" means "wisdom." Thus, according to the etymology of the ancient Greek word "philosophīa," philosophy means love for, pursuit of, and devotion to wisdom. By the term "wisdom," we may mean a set of dispositions, skills, and policies on the basis of which one can deliberate about the relationship between consciousness and the objects to which consciousness refers, as well as about what matters and has value in life, and act accordingly.

The basic organizational units of our Order are termed "Lodges," and each has a Venerable Master at its head. The overarching governing body of our Order is called the "Grand Lodge of the Modern and Perfecting Rite of Symbolic Masonry." According to our Order's Constitution, no person may be a member of two Lodges at the same time, and no person may hold two Offices in a Lodge or in the Grand Lodge at one time. Furthermore, according to our Order's Constitution, twenty-one Master Masons may apply to the Grand Lodge for a "Warrant" to form a subordinate Lodge or organization.

Endorsing particular aspects of William Preston's *Illustrations of Masonry* and following the philosophical strategy of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, we interpret the system of a Masonic Lodge as a symbol of the human mind. The human mind is the foundation and the major focal point of

philosophy. In general, by the term "mind," we mean a system of faculties or powers that constitutes an ontological attribute of a living organism. According to the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, we can define and explain the correspondences between the Officers of a Symbolic Lodge and the major mental faculties, or the major functional aspects of the mind, as follows:

The Venerable Master symbolizes consciousness. Basically, consciousness can be construed as an existential state that allows one to develop the functions that are necessary in order to know both one's existential environment as well as the events that take place around oneself and within oneself. Furthermore, consciousness has all the attributes of a being itself, and it can be considered as the synopsis of the human being and the core of the mind. The jewel of the Venerable Master of the Symbolic Lodge is a gold Square. The Venerable Master is equipped with a gavel. The gavel is a symbol of authority. The Venerable Master directs all the business of the Lodge, and he/she presides over ritual and ceremonies.

In our Order, the Venerable Master, the Senior Overseer, and the Junior Overseer of each Lodge are elected by secret ballot for a two-year term as follows: every even year, any Master Mason in good standing is able to register with the Order's Grand Secretariat as a candidate for the Office of the Venerable Master of his/her Lodge. The Council of the Conservators of our Order vets all registered candidates, and, according to its by-laws, it approves three to run in the election in each Lodge. All Fellow Crafts and Master Masons of the corresponding Lodge, provided that they are in good standing, have the right to vote in order to elect their Lodge's Venerable Master. Of the three candidates, the first in votes is installed as the Venerable Master, the second in votes is installed as the Senior Overseer, and the third in votes is installed as the Junior Overseer. In the event of a tie, the result will be determined according to the order in which the candidates were raised to the degree of a Master Mason—in this case, the older members enjoy an advantage over the younger ones.

The Senior Overseer symbolizes thinking. Thinking is based on symbols, which represent various objects and events, and it is a complex mental faculty characterized by the creation and the manipulation of symbols, their meanings, and their mutual relations. In the context of the communication between conscious beings, symbols are forms that express commonly accepted intentions and actions, and they can be organized into particular systems that are called codes. These codes underpin the activity and the behavior of conscious beings, and, therefore, a society of conscious beings is an inter-subjective and conscious "continuum." The

elements of such a code are called signs. Each sign is associated with a meaning in relation to the entire code to which it belongs, as well as in relation to its acceptance by each and every conscious being that uses the corresponding code. The jewel of the Senior Overseer is a gold Level. The Senior Overseer is equipped with a gavel, and he/she is the second in command of the Lodge. The Senior Overseer is in charge of the Lodge when it is in labor, provides the Venerable Master with assessment reports about the state of the Lodge and the performance and the ethos of its members, and he/she replaces the Venerable Master whenever the latter is absent.

The Junior Overseer symbolizes perception. Perception is a process whereby a living organism organizes and interprets sensory-sensuous data by relating them to the results of previous experiences. In other words, perception is not static, but a developing attribute of living organisms; it is active in the sense that it affects the raw material of scattered and crude sensory-sensuous data in order to organize and interpret them; and it is completed with the reconstruction of the present (present sensory-sensuous data) by means of the past (data originating from previous experiences). Therefore, perception is intimately related to memory and judgment. Furthermore, there are two kinds of perception: "external perception," which is caused as a result of external, physical or social stimuli, and "internal perception," which is caused as a result of internal stimuli related to the awareness of one's ideas, thoughts, and desires. The jewel of the Junior Overseer is a gold Plumb Rule. The Junior Overseer is equipped with a gavel, and he/she is the third in command of the Lodge. The Junior Overseer is responsible for the Brothers and the Sisters when the Lodge is at ease or refreshment, he/she assists the Senior Overseer in preparing reports to the Venerable Master, and he/she, too, may open the Lodge if the Venerable Master is unable to attend the meeting.

The Venerable Master, the Senior Overseer, and the Junior Overseer constitute the Supreme Triangle of the Lodge. For this reason, they have the distinction of wearing special caps. Their caps are of light blue velvet, three inches deep with a gold triple tau at the front:

The tau is the nineteenth letter of the Greek alphabet. An old traditional method of joining the stones was based on the use of brass tees, which were let into the stone and run-in with molten lead. The appropriate jointer for joining two stones was a double tee, whereas the appropriate jointer for

joining three stones was a triple tee, or triple tau. Moreover, the triple tau that is placed on these caps resembles a letter T resting on the traverse beam of the letter H, and so it is also said to mean "Theca ubi res pretiosa deponitur," meaning "a place where the precious thing is concealed," as well as "Clavis ad Thesaurum," meaning "a key to the treasure."

The Venerable Master and the two Overseers wear light blue gauntlets fringed with gold and adorned with the emblems of their respective Offices embroidered in gold. Since the gauntlets protect the wrist, they symbolize power, authority, and protection.

The Orator symbolizes judgment. Judgment is one's ability to compare and contrast ideas or events, to perceive their relations with other ideas or events, and to extract correct conclusions through comparison and contrast. The Jewel of the Orator is a gold Pentagram—more specifically, an upright five-pointed star polygon, which is also one of the most important Pythagorean symbols. The Pentagram contains ten points: the five points of the star and the five vertices of the inner pentagon.

No. 1 symbolizes the generative principle of Nature, and the Unity of the Source of Creation, typified by the point within a circle.

No. 2 symbolizes the active and passive qualities of Nature.

No. 3 symbolizes deductive reasoning (since, in deductive reasoning, there is a first premise, then a second premise, and, finally, an inference).

No. 4 symbolizes the four classical elements, or states of matter, which make up the physical world: water, air, earth, and fire. In modern physics, the classical element of fire corresponds to a state of matter that is called plasma.

No. 5 symbolizes the five basic senses: touch, sight, hearing, smell, and taste.

No. 6 is considered an epitome of Nature, as it presents to our view the three spatial dimensions, the temporal dimension, the principle of action, and the principle of reaction.

No. 7 symbolizes the seven liberal arts and sciences: Grammar, Logic, Rhetoric, Arithmetic, Geometry, Music, and Astronomy.

No. 8, being equal to two to the third power, is the first cube, and it symbolizes equality.

No. 9, being the last and biggest of the single-digit integers, symbolizes completion.

No. 10 is symbolically associated with the Pythagorean Tetractys, a triangular figure consisting of ten points arranged in four rows: one, two, three, and four points in each row. The Pythagorean musical system was based on the Tetractys, since the rows can be read as the

ratios of 4:3 (perfect fourth), 3:2 (perfect fifth), and 2:1 (octave), forming the basic intervals of the Pythagorean scales.

The Orator is responsible for the strict observance of the Order's Constitution and Ritual by the Lodge, and he/she is in charge of Masonic Education. Thus, he/she is the facilitator of Masonic law, Masonic knowledge, and Masonic philosophy. The Orator is directly appointed by the Council of the Conservators of our Order and serves at the latter's discretion. The Orator of a Lodge is not allowed to hold any other Lodge Office at one and the same time.

The Director of Ceremonies and the Assistant Director of Ceremonies symbolize imagination. Imagination is a mental faculty that enables one to form mental images, representations, that do not (directly) derive from the senses. Imagination is not subject to the principle of reality, as the latter is formed by the established institutions, and it develops because consciousness cannot conceive the absolute in an objective way. Therefore, imagination endows the things that it conceives with new significances, and it reorganizes them into new historical forms, utilizing elements of its external existential conditions (such as latent social trends and changes) which have not already crystallized into formally established institutions. From the aforementioned perspective, imagination can be considered as a kind of visionary perception. The jewel of the Director of Ceremonies consists of two gold rods in saltire, tied by a gold ribbon. The jewel of the Assistant Director of Ceremonies consists of two gold rods in saltire, surmounted by a gold bar bearing the word "ASSISTANT." The Director of Ceremonies and the Assistant Director of Ceremonies always hold their respective wands in the right hand. Their wands are symbols of inspiration and organization. The Director of Ceremonies' wand is a piece of wood that is approximately six feet in length, with the jewel of the Director of Ceremonies attached to the top. Similarly, the Assistant Director of Ceremonies' wand is a piece of wood that is approximately six feet in length, with the jewel of the Assistant Director of Ceremonies attached to the top. The principal duty of the Director of Ceremonies is to ensure that Masonic ceremonies are conducted smoothly, sincerely, and with decorum, according to the Order's Ritual. To that end, it is usual for the Director of Ceremonies to lead and direct rehearsals of the Ritual. The Assistant Director of Ceremonies, as indicated by the name of his/her Office, is an assistant to the Director of Ceremonies. Both the Director of Ceremonies and the Assistant Director of Ceremonies are appointed by the Venerable Master and serve at the Venerable Master's discretion.

The Secretary symbolizes memory. Memory is one's ability to preserve the past within oneself—or, equivalently, the function whereby one retains

and accordingly mobilizes preexisting impressions. The jewel of the Secretary consists of two gold pens in saltire, tied by a gold ribbon. The Secretary's duties require him/her to handle all Masonic correspondence to the members, minutes of Lodge meetings, petitions of new candidates, continuous Lodge member count, and several other administrative duties determined by the Venerable Master and the Grand Lodge. The Secretary is appointed by the Venerable Master and serves at the Venerable Master's discretion.

The Treasurer symbolizes volition, or will. Volition, or will, is one's ability to make decisions and implement them kinetically. Conscious free will, in particular, may not initiate human action, either due to physical-biological factors or due to unconscious factors, but it can decide whether to allow a voluntary process to reach its conclusion since it determines motor actions. The jewel of the Treasurer is a gold Key. The Treasurer is responsible for all financial transactions. He/she is appointed by the Venerable Master and serves at the Venerable Master's discretion.

The Expert symbolizes orientation. Orientation is a specific sense that helps one to verify one's position in space and time.The jewel of the Expert consists of two gold Chisels in saltire. The Expert always holds his/her respective wand in the right hand. The Expert's wand symbolizes guidance and control, and it is a piece of wood that is approximately six feet in length, with the jewel of the Expert attached to the top. The principal duties of the Expert are to conduct the candidates around the Lodge and instruct them according to the Ritual, to attend to the Venerable Master as needed, and to carry the Venerable Master's orders to the Senior Overseer and to the Junior Overseer. The Expert is appointed by the Venerable Master and serves at the Venerable Master's discretion.

The Director of Music symbolizes emotion or affect. Emotion or affect is the mental faculty that determines one's mood. In general, one's capability to feel joy or sorrow as well as the intensity, duration, and stability of one's feelings depend on the proper functioning of emotion. When an emotion is endowed with a judgment, namely, when consciousness judges emotions, then an emotion becomes a "sentiment." The jewel of the Director of Music is a gold Lyre. The Director of Music provides music for Masonic ceremonies according to the Order's Ritual. He/she is appointed by the Venerable Master and serves at the Venerable Master's discretion.

The Inner Guard symbolizes attention. Attention is a mental faculty that focuses conscious functions on particular stimuli in a selective way, and it operates as a link between perception and consciousness. The jewel of the Inner Guard consists of two gold Swords in saltire. Over his/her

apron, the Inner Guard wears a sword belt, from which is suspended a scabbard in which he/she carries his/her sword on the left hip. The duties of the Inner Guard are to report to the Venerable Master when Brothers and Sisters claim admission, to admit Masons and other visitors according to the Order's rules, to receive the candidates according to the Ritual, and to obey the commands of the Junior Overseer. The Inner Guard is appointed by the Venerable Master and serves at the Venerable Master's discretion.

The Outer Guard symbolizes association. Association refers to a phenomenon in which an idea that is present in consciousness attracts other relevant ideas to it in a way that is automatic and independent of one's will. The jewel of the Outer Guard is a gold sword. Over his/her apron, the Outer Guard wears a sword belt, from which is suspended a scabbard in which he/she carries his/her sword on the left hip. The Outer Guard's principal duty is to ensure that only those who are duly qualified are allowed to enter the Lodge room. The Outer Guard is appointed by the Venerable Master and serves at the Venerable Master's discretion.

During their term of office, the Lodge Officers wear the standard regalia of a Master Mason of our Order and a light blue collarette with gold edging, from which is suspended the corresponding Office's jewel. When a Venerable Master ceases to hold this Office, he/she is called a Past Master. All Past Masters continue to wear the same collarette and the same collarette jewel as the active Venerable Masters, but only the latter have the distinction of wearing the corresponding cap and the corresponding gauntlets. Every other Past Lodge Officer, given that he/she does not hold any Lodge Office, wears only the standard regalia of a Master Mason of our Order.

In our Order, at the entrance of the Lodge, there are two magnificent Pillars. That on the left is called "Id," for which reason it bears the inscription "I," and that on the right is called "Superego," for which reason it bears the inscription "S"; and the door of the Lodge is called "Ego," for which reason it bears the inscription "E." According to the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, the Masonic Lodge room symbolizes Freud's structural model of the human mind. In the 1900s, the Austrian psychiatrist Sigmund Freud, who is the acknowledged founder of psychoanalysis, articulated a structural model of the mind, according to which the elements that structure the "mental apparatus" are ordered in the "mental space" and constitute three vertically superimposed apartments that he called the "id," the "ego," and the "superego," and he defined them as follows:

"Id": It consists of impulses, instinctive urges, and everything connected with the major biological needs of the human being. Instinct is a highly formalized behavioral code that reflects the logic of organic nature. The "id" does not have discretion (the capacity to distinguish between "right" and "wrong"). Instead, it is motivated by the pleasure principle, which wants to immediately gratify all impulses. It is innate, and it remains unaffected by experience, for which reason it is not subject to any moral or sociogenic constraint. In our Order, the pillar that stands on the left-hand (that is, the north) side of the entrance of the Lodge bears the inscription "I," and it symbolizes the mental "apartment" that is called the "id."

Freud has identified and analyzed two fundamental instincts: the first is "eros," or life instinct, and the second is "thanatos," namely, the death instinct and source of destructive urges. Moreover, Freud has used the term "libido" in order to refer to instinctive psychological or psychic energy associated with sexual urges and, generally, with all constructive human activity. Thus, "libido" is opposed by "thanatos," and their interaction underpins all the variations of human activity.

The "id" resides in the unconscious mind, which, as the French psychoanalyst Jacques Lacan has pointed out, implies the existence of a type of knowledge that does not know itself. This knowledge is essentially the unconscious non-cognition that shapes the initial conditions of psychoanalytic theory and practice. Through its structure, it shapes the matrix of all forms of desire, having a structure similar to language—specifically, it is a series of signifiers. Based on the linguistic structuralism of the Swiss linguist and philosopher Ferdinand de Saussure, Lacan argues that the "signifier" is the symbol, sound, or image that represents an underlying concept or meaning, the "signified" is the concept of the thing to which the signifier refers, "the signifier represents the subject for another signifier," and the unconscious is structured like "language" in the sense of the relations of signifiers to each other. At a certain stage, unconscious knowledge is placed in the "other," it exists actually, and the signifiers that constitute it are differentially defined and characterized by a dual form. Within this ambiguous relativity, the subject cannot be identified unless there is a particular signifier that exceptionally defines itself and, in this way, provides knowledge with coherence.

In general, psychoanalysis, especially in the light of Lacan's work, leads us to the awareness that the definition of "discourse" starts from the following fundamental questions, which are of concern to any willing subject: who is the "other," what does he/she want from me, and why does the encounter with him/her fail? A "discourse" is a form of answer to the

aforementioned questions, and, of course, each answer has a critical relation to other possible answers. But how can discourse express the unconscious without denying or rejecting it? To answer this question, Lacan formulated the theory of the four discourses, which are an attempt to generalize the existential structures of the individual. In particular, according to Lacan, a discourse is not only an utterance and an utterer, but it is also a situation which is structurally determined by the following four questions: (i) Who is the bearer of the "discourse"? (ii) Who is the "other" to whom it is addressed? (iii) What effect does the "discourse" have on him/her? (iv) What is the "uttered"? The aforementioned four questions define four positions in which Lacan locates the signifying structure of the "thing" in the unconscious: the fragment of the primary object (Mother), the split subject (behind which lie the death drive and the phallus), the Name of the Father (the symbolically absent father), and the Idea of the Ego (the actual present father).

"Ego": It is the administrative center of personality, and its creation and gradual development take place under the influence of accumulated experience. It contains all functions of consciousness as well as unconscious functions, such as the defense mechanisms of the ego. In particular, the ego's consideration of reality is conscious, but the ego may also keep censored or forbidden desires hidden by unconsciously repressing them by means of the defense mechanisms of the ego. The ego is motivated by the principle of reality, it has discretion, and it develops rational thinking in order to weigh pleasure against its consequences. The renowned Swiss psychiatrist and psychoanalyst Carl Jung has pointedly argued that "the attainment of consciousness was . . . the magical weapon which gave man victory over the earth, and which we hope will give him a still greater victory over himself" (Carl Jung, *The Collected Works of Carl Jung*, vol. 10: *Civilization in Transition*, edited and translated by Gerhard Adler and R. F. C. Hull, Princeton, N.J.: Princeton University Press, 1978, par. 289). In our Order, the door of the Lodge bears the inscription "E," and it symbolizes the mental "apartment" that is called the "ego."

"Superego": It consists of two components: firstly, the ego ideal, or ideal self, or the rules and standards one should adhere to; secondly, moral consciousness, which is the consciousness of existence itself when it operates as a judge. The "superego" is the moral compass of personality, and its development is determined by influences exerted on the human being by the parents, persons of authority, and one's microsocial environment. In our Order, the pillar that stands on the right-hand (that is, the south) side of the entrance of the Lodge bears the inscription "S," and it symbolizes the mental "apartment" that is called the "superego."

Firstly, we strive to maintain the rule of consciousness over the unconscious by analyzing the secondary, unconscious mind, which is formed by wishes and desires that are repressed by social norms, reason, and common sense, which are adaptation mechanisms to reality. Secondly, we inquire into the meaning of consciousness, reason, reality, truth, and morality themselves, on which the concept of normality is based, and we aim to achieve transcendence, meaning to go beyond the ordinary state of normality and its limitations. Thus, we aim to experience an existential state that can be summed up by the concepts of extraordinariness and excellence.

The Chamber of Reflection symbolizes the womb and, generally, a state of darkness prior to being brought to a new light in the Lodge.

The form of a Symbolic Lodge is a regular parallelepipedon, and its length, symbolically speaking, extends from East to West; its breadth, symbolically speaking, extends from North to South; and it is, symbolically speaking, as deep as the earth and as high as the heavens. These dimensions of a Lodge symbolize the universality of Symbolic Masonry.

A Symbolic Lodge is situated due East and West, because the material Sun, which is a *sine qua non* source of material life and a natural symbol of Reason, rises in the East and sets in the West.

Masonic Lodges are supported by three great pillars, which are called Wisdom, Strength, and Beauty, and they are symbolized by the following three noble Architectural Orders: the Ionic, the Doric, and the Corinthian, respectively.

The ceiling of a Masonic Lodge symbolically corresponds to the Sky and the existential horizons of humanity. The members of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry pursue a course of celestial ascension with the assistance of a philosophical ladder composed of many staves or rounds, which point out as many manifestations of existence. However, the three principal manifestations of existence are the following: instinct, experience, and rational understanding or intellect.

At the level of instinct, conscious activity is minimized, and every instinctive action of existence overlays or ignores every conscious activity. Instinct is a condensed logical manifestation whose correctness has been confirmed by the application of the problem-solving method that is known as “trial and error” by an unlimited number of generations, and it reflects the logic of organic nature. Thus, instinctive action has the character of an *a priori* integrated process. No obstacle to the affirmation of instinctive behavior can change the intrinsic logic of instinct itself. However, an obstacle to the affirmation of instinctive behavior can modify the manner

in which existence adapts to each situation. In fact, adaptation is based on the method of "trial and error."

At the level of experience, the intentionality of consciousness is expressed through the functioning of the senses, which are oriented towards the world and with which they connect existence. Experience is a conscious state that is part of the receptive aspects of existence. At the level of experience, consciousness is passive.

At the level of rational understanding, corresponding to the mental process used in thinking and perceiving, consciousness plays an active role, manifesting in reason. In his seminal book *Critique of Pure Reason*, Immanuel Kant, who was arguably the most important representative of the European Enlightenment, explained that "reason" is an *a priori* (that is, pre-experiential) structure, in the context of which various categories are interrelated, and, whenever they are adequately activated, they can connect isolated empirical data with each other, thus making possible the formulation of synthetic judgments through which one can creatively transcend the level of experience and ascend to the level of rational understanding.

Intimately related to the different levels at which the intentionality of consciousness is expressed are the different degrees and forms of knowledge. By the term "knowledge," we mean: (i) the mental action through which an object is recognized as an object of consciousness; (ii) the mental action through which consciousness conceives the substance of its object; (iii) the object whose image or idea is contained in consciousness; and (iv) that conscious content which is identified with the substance of the object of knowledge. Therefore, the term "knowledge" can be construed as a firm consideration of an object as something that corresponds to reality.

The four basic forms of knowledge, namely, the four basic relations between consciousness and any object of consciousness, are the following:

Belief: The term "belief" has two meanings: firstly, it means that one accepts something as real, even though the claim about its reality is neither based on experience nor on logical proof; secondly, it means that one has merely formed an opinion by remaining focused on the appearance of things. In the latter case, the term "belief" is synonymous with the term "dōxa," which, as Plato maintains in his *Republic*, means the acquaintance with an object that can be provided by an unstable appearance. It is contrasted with "epistēme," which, as Plato maintains in his *Republic*, means a firm and stable intellectual grasp.

Empirical knowledge: Empirical knowledge is a form of knowledge that is provided by the senses. Its object may be inside or outside us so

that, depending on the position of its object, empirical knowledge is distinguished into internal experience and external experience. The major pioneers and representatives of the philosophical "school" of empiricism are Francis Bacon, David Hume, John Locke, and Robert Boyle. Experience is a form of conscious knowledge that is superior to belief and inferior to logical knowledge. In fact, empiricism has the following defects: (i) the epistemological warrant offered by empiricism is very narrow, because it is based on direct observation. Therefore, it rules out any consideration of (unobservable) things such as social structures, or even social facts (which, according to the French sociologist Émile Durkheim, refer to those shared social concepts and understandings, such as crime, which he argued that should be treated as "things"). Hence, a strict variety of empiricism actually offers a very narrow-minded and restrictive understanding of "reality." (ii) Empiricism does not allow us to talk about "causes," since these are unobservable. In the context of empiricism, causation is reduced to mere correlation, and our inquiry is therefore limited to that of "prediction" and cannot involve "explanation." (iii) The kind of pure unvarnished perception requested by empiricists is impossible.

Logical knowledge: Logical knowledge is a form of knowledge that derives from the rational faculty of consciousness, and it is characterized by indisputable and logically grounded truths (judgments about the reality of things). Logic may be defined as a theory of true propositions, or, equivalently, as a theory of correct reasoning. Any relation between concepts is formulated by means of propositions. According to Aristotle's *Organon*, the backbone of any science is a set of propositions, so that, starting from the very primitive principles and causes, one can proceed to learn the rest. Aristotle's logic is focused on the notion of deduction (syllogism), which was defined by Aristotle, in the first book of his work entitled *Prior Analytics*, as follows: "A deduction is speech (*logos*) in which, certain things having been supposed, something different from those supposed results of necessity because of their being so"; each of the things "supposed" is a premise of the argument, and what "results of necessity" is the conclusion.

By the term "concept," we mean the set of all predicates of a thing (or of a set of conspecific things) that express the substance of the given thing (or of the given set of conspecific things). In the scholarly discipline of logic, the "intension" of a concept is the set of all predicates of the given concept, or the set of all those elements due to which and by means of which the given concept can be known and distinguished from every other concept. In other words, the intension of a concept is its formal definition.

For instance, the properties of the three angles and the three sides of a geometric figure constitute the intension of the concept of a triangle. Moreover, in the scholarly discipline of logic, "extension" indicates a concept's range of applicability by naming the particular objects that it denotes. In other words, the extension of a concept encompasses all those things to which the given concept refers. For instance, the extension of the concept of a tree consists of all particular trees; the extension of the concept of a human being consists of all particular humans, etc.

By the term "genus" (plural: "genera"), we mean a concept whose extension includes other concepts, known as "species" or "kinds," which fall within it. In other words, "genera" are concepts with an extension bigger than that of other concepts, whereas "species" or "kinds" are concepts with an extension smaller than that of other concepts. For instance, the concept of a geometric figure is a genus with regard to the concept of a triangle, whereas the concept of a triangle, which appertains to the concept of a geometric figure, is a kind with regard to the concept of a geometric figure.

Through the process of "abstraction," we decrease the intension of concepts and increase their extension. Thus, due to abstraction, the concept of a human being can be gradually generalized into the following concepts: "vertebrate," "mammal," "animal," "living being," and "being"; "being" is the most general concept, in the sense that its intension is minimum and its extension is maximum. "Being," to which every other concept is reducible, cannot be further analyzed into other concepts. Concepts of such general type, which are not susceptible to further analysis into simpler concepts, and to which other concepts are reducible, are called "categories." Aristotle, in his book *Categories*, attempted to enumerate the most general species, or kinds, into which beings in the world are divided. In particular, in *Categories* (1b25), Aristotle lists the following as the ten highest categories of things "said without any combination": "substance" (for instance, man, horse), "quantity" (for instance, four-foot, five-foot), "quality" (for instance, white, grammatical), "relation" (for instance, double, half), "place" (for instance, in the Lyceum, in the market-place), "date" (for instance, yesterday, last year), "posture" (for instance, is lying, is sitting), "state" (for instance, has shoes on, has armor on), "action" (for instance, cutting, burning), and "passion" (for instance, being cut, being burned).

The primary formation of the basic image—that is, of the mental representation—of a being by philosophizing consciousness is due to the presence of the human reality in the world, and, therefore, it is based on experience. However, gradually, the basic image of a being undergoes

further processing by consciousness. As a result of its processing by consciousness, the basic image of a being discards its most specific traits and its accidental properties, and it is projected onto a conscious construct, so that it is replaced by the most abstract representation of the given being. For instance, the fundamental problem of perspective, in both art and mathematics, consists in correctly representing a three-dimensional picture or situation in a two-dimensional picture of it. In this way, consciousness facilitates and underpins the conception of and the functional interconnection between the most abstract aspect of a being and the world, into whose functional presence the given being is integrated. In fact, science and philosophy are based on the awareness that consciousness is something different from the surrounding body of nature, and they stem from the fact that consciousness discerns similarities in a multiplicity of events, abstracts these from their settings, generalizes them, and therefrom deduces other relationships consistent with further experience.

Intuition: Intuition, like instinct, manifests itself as a direct and condensed logical conception of objects, and, simultaneously, as a system of accumulated experiences whose origin tends to become unconscious. By the term "intuition," we mean that consciousness conceives a truth (that is, it formulates a judgment about the reality of an object) according to a process of conscious processing that starts from a minimum empirical or logical datum and rises to a whole abstract system with which consciousness realizes that it is connected or to which consciousness realizes that it belongs. There are three different varieties of intuition: (i) sensuous or psychological intuition, (ii) logical intuition, and (iii) metaphysical intuition. A characteristic type of sensuous or psychological intuition is Henri Bergson's conception of intuition; a characteristic type of logical intuition is Edmund Husserl's conception of intuition; and a characteristic type of metaphysical intuition is the Neoplatonic concept of ecstasy.

In our Order, the letter G, which stands for "Gnosis" and "Geometry" and is placed at the center of the Blazing Star, symbolizes a kind of knowledge that combines experience, logic, and intuition.

The floor of a Masonic Lodge is a Mosaic Pavement, which, by reason of its being variegated and checkered, points out that the sensuous world and history are characterized by contradictions and diversity.

The Blazing Star, or Glory, in the center of a Masonic Lodge refers us to the knowledge of the reality of the world and the reality of consciousness. The philosophical method that is inherent in our Order's Ritual recognizes and analyzes both the reality of consciousness and the reality of the world. Hence, it recognizes and analyzes both the objective

and the subjective forces of history. According to our Order's Ritual and Catechisms, the use of the symbol of the Blazing Star, or Glory, is intended to remind and teach us that history is neither the realm of necessity alone nor the realm of freedom alone, but the realm of the dialectic between necessity and freedom, and that our Order aims to create an enlightened, rational, and humanistic elite, capable of building a better humanity in a better world according to the Ritual and the inner Plans of our Order.

The Indented or Tessellated Border, a chord forming "knots of true love" and surrounding the Mosaic Pavement, refers us to the Planets, the celestial spheres. These, in turn, refer us to the concept of a perfect order through their harmonious motions.

Our Order identifies itself as "Autonomous." It does so according to Immanuel Kant's formula of autonomy. According to Kant's *Metaphysics of Morals*, in the moral realm, "good will" means never universalizing a maxim that would not treat humanity, whether in your own person or in the person of any other, as an end in itself. This is Kant's formula of autonomy. Within the framework of Kantian teleology, morality and politology can be connected as follows: if all persons had good will, then they would treat all others as ends in themselves, thus giving rise to a "kingdom of ends." Even though this is humanity's moral duty, this is not actually the case because of the pathology of humanity's moral condition. Hence, it is of vital significance to develop and impose a rational system of morality, utilizing Kant's principle of the "categorical imperative." A categorical imperative is a command, or a rule, that applies in all situations and at all times. Kant's categorical imperative is that we should do what is rationally (that is, logically) right for human beings.

Kant's moral theory has two basic assumptions. Firstly, only human beings are capable of rationality. Hence, Kant maintains, it is imperative to protect humans' rationality and humans' ability to be rational, since to take away our ability to be rational means to take away our humanity, given that the principal distinctive attribute of humanity is rationality. Secondly, as rational beings, only humans are autonomous—that is, they can make volitional choices—whereas all other creatures on the planet pretty much react instinctively. Moreover, as Kant has pointedly argued, humans' autonomy should be protected.

In his *Groundwork of the Metaphysics of Morals*, Kant argues that two ethical imperatives should guide our actions. Firstly, we should do what is best for everyone equally. Secondly, we should do what serves the needs of each individual equally. Thus, the first formulation of Kant's categorical imperative is the following: we ought to act in such a way that

we can will that our maxim should become a universal law. In other words, if it doesn't logically make sense that your action (that you are regarding as ethical) could be made dispositive, then it is not the best ethical action. The second formulation of Kant's categorical imperative is the following: we should act in such a way that we always treat humanity (whether in our own persons or in the person of any other), never simply as a means, but always at the same time as an end. From the perspective of Kant's original assumptions, if you use people as a means to achieve your own goals, if you deceive people, you trump people's rationality instead of preserving it. Therefore, you commit an assault on people's humanity itself.

The Modern and Perfecting Rite of Symbolic Masonry is universal—in fact, more than that, it cultivates cosmopolitanism. Cosmopolitanism refuses to regard any established political structure as the source of ultimate value; according to cosmopolitanism, the source of ultimate value is the universal reason to which each and every human individual has access due to one's humanity itself. Moreover, cosmopolitanism is a genuinely universalist principle, whereas there are universalist principles that are not cosmopolitan. For instance, in the context of modernity, particularly under the influence of German romanticism and, generally, German idealism (namely, such philosophers as Johann Gottfried von Herder and Georg Wilhelm Friedrich Hegel), it is possible to hold the nation as a universal source of a particular system of values.

According to Immanuel Kant, the categorical imperative enjoins the creation of a lawful state in order that (at least a partial) realization of the good will can be achieved. On the other hand, Kant has pointedly argued that war is a direct threat to security and justice for two reasons: firstly, because it kills people and destroys property and, secondly, because it is a direct cause of despotism and tyranny. Hence, the same moral imperative that enjoins the institution of a lawful state requires the abolition of war. One way to abolish war would be to abolish nation-states and replace them with a world-state. But there are objections to a world-state based on the linguistic and religious differences between states, on the obvious point that national elites and bureaucracies will not agree to lose their sovereignty and other national privileges, as well as on the logical point that the institution of a global government would suffer by important inefficiencies due to the intrinsic complexities of a world-state, and, having to exercise power on a global scale, it would ultimately stumble between despotism and anarchy. On the basis of the aforementioned objections, one can correctly conclude that a world-state is neither achievable nor desirable.

In view of the foregoing, a system of international relations must be developed which will prevent war without requiring the elimination of nation-states. As Kant argued in his book *Perpetual Peace*, the creation of such a system of international relations is based on the principles of open diplomacy, non-aggression, self-determination, non-intervention, the delineation of lawful means of making war, and disarmament. Furthermore, in the second section of his *Perpetual Peace*, Kant proposes the following three definitive articles of a perpetual peace between states: Firstly, "ius civitatis," that is, a constitution based on the civil right of individuals within a nation-state. Secondly, "ius gentium," that is, a constitution based on the international right of states in their relationships with each other. Thirdly, "ius cosmopoliticum," that is, a constitution based on cosmopolitan right, in terms of which individuals and states can be regarded as citizens of a universal state of humanity, or rather a world society. Of course, there may exist states that refuse to endorse the aforementioned legal order, but the states that endorse the aforementioned legal order should forge an alliance in order to maintain the aforementioned legal order, to defend it against its enemies, and to promote it globally in a wise and lawful way without negating international freedom.

Finally, let me draw your attention to the theory of the cosmopolitanism of nations that was articulated in the nineteenth century by the great Italian politician, journalist, and revolutionary Giuseppe Mazzini, who played a major role in the unification of Italy. The starting point of Mazzini's political thought is that humans are created equal, which entitles them to certain basic, *a priori* liberties. Furthermore, according to Mazzini, such equality also implies a moral obligation to cooperate with others towards the common good. Therefore, in addition to liberal rights, humans also need political structures that underpin cooperation. Mazzini has pointed out that any human group, even any "mob," can share the same territory, language, and tradition. However, when these shared experiences are governed by a conscious decision and an efficient plan to cooperate with others in order to pursue the common good, they constitute the nucleus of a nation. Consequently, the nation is neither a natural given nor an end in itself, but a means to overcome the individual's isolation and to unite people on a larger scale in order to coordinate their actions towards the common good. By understanding the nation as a great socializing and rationalizing force, Mazzini proposed the substitution of the old alliances of monarchies and reactionary regimes with a "Holy Alliance of the Peoples."

The Principal Jewels of the Lodge are the following: the Square, the Level, the Plumb Rule, the Rough Ashlar, and the Perfect Ashlar. The Square is a symbol of reason and morality and, in our view, it teaches critical rationalism and moral rationalism. The Level is a symbol of equality and, in our view, it teaches to combine aspects of the following three theoretical systems: Kant's moral rationalism and cosmopolitan liberalism, Marx's dialectical materialism and cosmopolitan socialism, and Mazzini's cosmopolitanism of nations. The Plumb Rule is a symbol of inner freedom and rectitude of conduct. The Rough Ashlar is the object on which the Apprentices work and put their mark, trying to transform it into a Perfect Ashlar. The Perfect Ashlar is the object on which the Craftsmen try and adjust their Masonic jewels.

Closing Ceremony

V.M.: Brothers and Sisters, you will rise and assist me to close the Lodge.
All rise.
V.M.: Brothers and Sisters, join me in giving the Sign of an Entered Apprentice Freemason and speaking the Word.
All take Step, give the Entered Apprentice's Sign, say "Ask–Seek–Knock," and then they cut the Sign.
V.M.: Brothers and Sisters, essentially philanthropic, philosophical, and progressive, our Order has as its objects the search for truth, the systematic inquiry into reality, the study of philosophy and science, and the practice of solidarity. It works for the material, the moral, and the intellectual improvement of humanity. Freedom, Reason, Justice, and Love.
All say: Freedom, Reason, Justice, and Love.
V.M.: Go in peace, my Brothers and Sisters, continuing to work for our Order under the law of secrecy; never lose sight of your Mother Lodge; and may we rejoin whenever we are called.
The Lodge is closed in the First Degree.

Lectures for the Further Training of Entered Apprentice Freemasons

After the initiation of an Entered Apprentice, the following four lectures, articulated in the form of questions and answers between the Venerable Master, the Senior Overseer, the Junior Overseer, and the Orator, should be read and discussed in the next four meetings of the Lodge, one Lecture at each meeting, for the further training of the newly initiated Entered

Apprentices. In general, formalized Masonic lectures constitute an integral part of a Masonic initiatory journey.

Lecture I

V.M.: What is Freemasonry according to our Order?
S.O.: A peculiar system of intellectual and moral rectitude veiled in initiatory rituals and illustrated by symbols.
V.M.: What is the first objective of our work?
S.O.: To mold the "organs" associated with the following three functions: firstly, mental homeostasis, or the ability to maintain a relatively stable mental state that persists despite changes in the external world; secondly, the communication between conscious beings; thirdly, humanity's adaptation to environment.
W.M.: Which are these organs?
S.O.: Personality, character, and behavior. Personality is the set of all psychosomatic properties and functions by means of which a human being interacts with oneself and with one's environment. Intimately related to the term "personality" is the term "soul," because soul is the personal way in which one manifests the force of life. Character is the expressive organ of personality. Behavior is the executive organ of personality, and it consists of impulses and learning. By the term "impulse," we mean a sudden and compelling urge or desire to act. By the term "learning," we mean a function that enables a person to utilize experience and training and to acquire new types of behavior in order to ultimately supplement and expand one's innate capacity for adaptation and creativity.
W.M.: Summarize the basic characteristics of the personality of a normal person.
S.O.: The basic characteristics of the personality of a "normal person" can be summarized as follows:

i. A normal relation between optimism and pessimism. A normal person is fundamentally optimistic. However, a normal person's optimism is rational, it is not in conflict with the principle of reality, and it does not give rise to irrational expectations.
ii. A normal relation between a sense of dependence and a sense of independence. Depending on the conditions and the needs that prevail in one's environment, a normal person can adapt to both the role of a leader (namely, an independent actor) and the role of a subordinate (namely, a dependent actor). Furthermore, a normal person does not spurn the others' offers of help, and willingly undertakes to help others.

iii. Normal levels of organization and systematicness. Normal persons have the tendency to be organized and neat and to tackle problems in a systematic way without, however, being "fixated" on ("obsessed" with) these properties.Therefore, they do not allow these properties to clash with other desires, especially with those which underpin creativity.

iv. A normal sense of curiosity. A normal person utilizes curiosity to facilitate creative adaptation without harming others.

v. Normal sexual identity. Normal persons are reconciled and satisfied with their sexual identity, and they are free from fears and complexes pertaining to sex.

vi. A normal relation between competitiveness and cooperation. Normal persons can act autonomously and self-reliantly in order to achieve one's goals, but they are also capable of willingly and creatively cooperating with others.

vii. A normal attitude towards authority. Normal persons do not make *a priori* assumptions about authority, and, therefore, they neither *a priori* reject authority, nor do they *a priori* submit themselves to the governing authorities, being aware of the consequences of their choices.

viii. Normal ways of expressing and controlling emotions. Normal persons do not repress their emotions, but they control their behavior.

ix. Ability to make close and stable relationships.

x. Ability to establish a viable equilibrium between the pursuit of satisfaction and the pursuit of safety.

xi. Self-esteem combined with the awareness of one's own constraints and weaknesses, and the ability to appreciate, admire, and trust others.

xii. Capacity for responsible decision-making.

V.M: Explain the manner in which the human personality is shaped on the basis of innate capabilities and needs on the one hand and social-environmental requirements on the other.

S.O.: The German social psychologist, psychoanalyst, and philosopher Erich Fromm, in his books *Escape from Freedom* and *The Nature of Man*, argues that humanity is dominated by two kinds of needs. Firstly, as an animal organism, the human being has certain biological needs (namely, hunger, thirst, sleep, rest, need for self-preservation) which one must satisfy. Secondly, as a being with reason, consciousness, and imagination, the human being has certain social needs. The most important of these social needs are the following: (i) The need for contact and

communication, which is satisfied by interpersonal relationships based on "productive love," which means mutual relations of care, respect, and understanding. (ii) The need for improvement and perfection—that is, the need felt by humans to transcend their animal nature and to realize the intentionality of their consciousness. (iii) The need of belonging, which is satisfied by the brotherhood of man. (iv) The need for personal identity, in the sense that every human being wants to experience his/her individuality, his/her uniqueness, and this need is satisfied by personal creation. (v) The need for a philosophical and evaluative framework—that is, a stable and consistent way of perceiving, understanding, and interpreting the world.

According to Fromm, the individual who lives in a society lacking love and solidarity between people gradually abandons his/her quest for personal freedom and individualization. He/she adopts a social formula, hoping that this (through submission and conformity) will provide a solution to the problem of isolation and insecurity. Parents at first and then agents of systematic education cultivate in the child—at the expense of his/her individual character—a social character adapted specifically to the logic and imperatives of the established political-economic system in which they themselves were brought up. The role of this social character is to shape the strengths of the members of society in such a way that their behavior does not involve conscious personal decisions, thus giving rise to individuals who act "as they should" (that is, as "socially correct" individuals) and find satisfaction in acting according to the demands of society. In other words, Fromm believes that social character hinders the development of a free and productive personality that can act according to its own logic and will.

Fromm distinguishes the following non-productive types (namely, social types that personal subjugation takes on): (i) Submissive: easily compromised, obedient, and easily accepting oppression. (ii) Exploiter: finds satisfaction and security in the accumulation of wealth. (iii) Sadist: tends to torment others. (iv) Masochist: enjoys torturing himself/herself. (v) Mechanistic-automatic: faithfully performs his/her role, doing whatever the social system asks him/her to do. On the other hand, Fromm contends, healthy but rare is the type of productive love, which is ready for mutual care, mutual understanding, and mutual respect. This reciprocal relationship which, according to Fromm, defines mental health is realized through the conscious exercise of reason, love, and productive work.

V.M.: What is time?

S.O.: In hindsight, time is an order on the set of the states through which a being passes successively. In order to better understand time, we have to

clarify the difference between the concepts of eternity, duration, time, and dynamized time. Eternity is the characteristic mode of being of what we construe by the term "absolute being." According to Plotinus, eternity is not the whole time, but the everlasting moment of being always equal to itself, and, from this perspective, the dynamization of time consists in ecstasy (the moment of rapture and instant elevation to the utmost levels of being). Duration is the characteristic mode of existence of every being that continuously tries to preserve and affirm its own substance and discard any alien substance. Time is the characteristic mode of being of the world as the latter is perceived and organized by consciousness. Dynamized time is the characteristic mode of being of consciousness because, as the philosophers Edmund Husserl, Henri Bergson, and Martin Heidegger have explained, consciousness perceives the reality of the world and thinks of the reality of the world, while simultaneously having intentionality and will, according to which consciousness acts in order to restructure the reality of the world in terms of dynamized time.

According to the Swiss psychologist Jean Piaget, time is an "intellectual construction" that facilitates the activity of consciousness. However, far from becoming a prisoner of its own constructs, such as time, consciousness forms systems of dynamized time by means of which consciousness restructures temporality and, ultimately, affirms its freedom and imposes its intentionality on the world. By dynamizing time, consciousness ultimately rationalizes and manages the world, given that the dynamization of time allows consciousness to understand the world in a more intelligent and a more creative way. Whereas time is an "intellectual construction" that derives from the reference of consciousness to the world, the dynamization of time is an "intellectual construction" that derives from the intentionality of consciousness. In particular, dynamization can be construed as the dynamic expression of the intentionality of consciousness whenever the intentionality of consciousness comprises a continuously updated strategic plan of action, formed by consciousness for the sake of consciousness. Dynamization implies both a dynamic attitude of consciousness towards the world and a way in which consciousness manages to intensify its presence in the world. The presence of consciousness in the world restructures time and, therefore, calls for the study of dynamized time.

V.M.: What is space?

S.O.: Whereas time can be thought of as the set of all points through which reality passes successively, space can be thought of as the set of all points over which reality is extended simultaneously. In fact, the great German-Serbian philosopher and mathematician Gottfried Wilhelm von

Leibniz has emphasized the distinction between the notion of succession and the notion of co-existence. In his fifth letter to the English metaphysician and theologian Samuel Clarke, he defined the terms "place" and "space." By the term "place," we should understand that which is the same in different moments to different existent things whose relations of co-existence with other existents, which are supposed to continue fixed from one of those moments to the other, agree entirely together. By the term "space," we should understand that which results from places taken together. Furthermore, in his *Principles of Philosophy*, Descartes maintains that, just as abstract time is distinct from duration, abstract space is distinct from extension, since the latter is concrete. According to Descartes's *Principles of Philosophy*, the surface on which a body ends constitutes a set of boundary points: if this set is considered with regard to its external side vis-à-vis the given body, then it can be thought of as the extension of the given body. If the same set is considered with regard to its internal side vis-à-vis the given body, however, then it can be thought of as the place of the given body. According to the aforementioned argument, which is largely in agreement with Aristotle's physics, the boundaries of "place" and "extension" coincide with each other, but "place" and "extension" differ from each other with regard to the perspective from which their boundaries are considered. In general, space is an abstract generalization of extension, and, therefore, it has been studied from several perspectives.

The first scientifically rigorous perception of space was formulated by the ancient Greek geometers. Around 300 B.C.E., Euclid published the definitive treatment of Greek geometry and number theory in his thirteen-volume *Elements*, building on the experience and the achievements of previous Greek mathematicians. Specifically, he builds on the Pythagoreans for Books I–IV, VII, and IX, on Archytas for Book VIII, on Eudoxus for Books V, VI, and XII, and on Theaetetus for Books X and XIII. The axiomatic method used by Euclid is the prototype for the entire field of "pure mathematics," which is "pure" in the sense that we need only pure thought, no physical experiments, in order to verify that the statements are correct—that is, we need only to check the reasoning in the demonstrations.

According to Euclidean geometry, space is three-dimensional and isotropic; in other words, it has the same value when measured in different directions. This scientific conception of space clashes with several mythical and folk perceptions of space, according to which space is connected with a form of temporality and is unisotropic. For instance, according to several mythical and folk perceptions of space, the "upward"

and the "forward" directions are evaluated as superior to the "downward" and the "backward" directions. The Euclidean perception of space, combined with the concept of gravity, found its fullest expression in Isaac Newton's calculus and mechanics. The modern conceptions of space and time are largely dependent on Newton's arguments regarding their indisputable reality, their divisibility, and their correspondence to empirical observations.

In the fifth century C.E., the Greek philosopher Proclus criticized Euclid's parallel postulate, according to which, "if a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles." In particular, Proclus argued that Euclid's parallel postulate should be struck out of the axioms of geometry altogether because, actually, it is a theorem involving many difficulties. Proclus offered the example of a hyperbola that approaches its asymptotes as closely as one likes without ever meeting them, thus indicating that the opposite of Euclid's conclusion is at least conceivable. Consequently, according to Proclus, Euclid's parallel postulate should be treated as a theorem, which should be proved from the other axioms. In fact, the first known such attempt was made, without success, by the second-century C.E. Greek mathematician, astronomer, and geographer Claudius Ptolemy. For about seventeen centuries, some of the best mathematicians unsuccessfully tried to prove Euclid's parallel postulate, which is equivalent to the statement that, given a line l and a point P not on l, there exists a unique line through P that does not intersect l.

In 1824, the German mathematician and physicist Johann Carl Friedrich Gauss wrote to the German mathematician Franz Adolph Taurinus, who had attempted to inquire into the theory of the parallels, that the assumption that the angle sum of a triangle is less than 180^{o} leads to a new peculiar geometry. This is quite different from the Euclidean one, but thoroughly consistent. However, Gauss was afraid to publish his research work in non-Euclidean geometry because, as he wrote to another important German mathematician and physicist, Friedrich Wilhelm Bessel, on 27 January 1829, he feared "the howl from the Boeotians" (an allusion to prejudiced, obtuse persons) if he were to make public the results of his research work. The first mathematician to publish an account on non-Euclidean geometry was the Russian mathematician Nikolai Ivanovich Lobachevski, who initially called this geometry "imaginary" and, later, "pangeometry." In the 1830s, Lobachevski mentioned that, in order to establish the validity of his non-Euclidean geometry, he needed the aid of

experiments, such as astronomical observations, as in the case of other natural laws.

In Gaussian–Lobachevskian geometry, known as hyperbolic geometry, Euclid's parallel postulate is replaced by the so-called "hyperbolic axiom," according to which, for any given line *L* and point *P*not on *L*, in the plane containing both line *L* and point *P*, there exist at least two distinct lines through *P* that do not intersect *L*. In Euclidean geometry, the sum of the three interior angles of a triangle is always equal to π radians (180°, or a straight line). However, in hyperbolic geometry, the sum of the three interior angles of a triangle is always strictly less than π radians. The difference is referred to as the "defect."

The renowned German mathematician Bernhard Riemann, who was a student of Gauss, had the most profound insight in non-Euclidean geometry. In the 1850s, Riemann invented the concept of an abstract geometric surface that need not be embeddable in Euclidean three-dimensional space, and, on this surface, the "lines" can be interpreted as geodesics, and the intrinsic curvature of the surface can be precisely defined. A "geodesic" is the shortest path between two points on a curved surface, therefore a geodesic can be considered as the non-Euclidean equivalent of a Euclidean straight line. For instance, airplanes, wishing to minimize the time that they spend on the air, do not follow Euclidean straight lines, but they follow shortest curves known as geodesics. In other words, Riemannian geometry is geometry on the ellipsoid or on the sphere, thus it exists on surfaces that have constant positive curvature; Gaussian–Lobachevskian geometry exists on surfaces that have constant negative curvature; and Euclidean geometry exists on surfaces that have constant zero curvature. This is the way in which modern geometers construe the reality of non-Euclidean planes. Therefore, whereas hyperbolic triangles are "thin" triangles in the sense that their angle sum is strictly less than 180°, Riemannian triangles, namely, triangles on the ellipsoid or on the sphere, are "fat" triangles in the sense that their angle sum is strictly greater than 180°.

Riemannian geometry was used by Albert Einstein in order to formulate the general theory of relativity. According to Newtonian mechanics, which is formulated in the context of Euclidean geometry, assuming zero curvature, the natural trajectory of a physical body that is not acted upon by any external force is a straight line. According to the general theory of relativity, gravity manifests itself as space-time curvature. Therefore, what Newton has called natural straight-line trajectories should be generalized into curved paths known as geodesics, or great circle arcs.

The general theory of relativity explains the operation of gravity. A very simple way in which one can present Einstein's general theory of relativity is the following metaphor: imagine a big rubber sheet stretched nice and taut before your eyes. If you watch a little marble as it rolls across the surface of this rubber sheet, then you will realize that it follows a simple straight-line trajectory. But if you watch the movement of a heavy rock on this rubber sheet, then you will realize that now the rubber sheet is deformed, warped, curved. In contrast to the previous marble, this rock does not follow a straight-line trajectory, but it follows a curved trajectory along the curved surface of the rubber sheet. Einstein took this idea and applied it to the fabric of space. Originally, the fabric of space may look nice and flat, like the rubber sheet in the previous example. However, if the Sun appears, the fabric of space curves. Similarly, in the vicinity of the Earth, the fabric of space curves, and the Moon is kept in orbit around the Earth because it rolls along a valley in the curved environment that is created by the Earth's mass. This is the manner in which, according to Einstein, gravity is communicated from place to place: through warps and curves in the fabric of the space, more specifically through warps and curves in space-time. For instance, the Earth is kept in orbit around the Sun because it rolls along a valley in the curved environment that is created by the Sun's mas, and, similarly, as I mentioned before, the Moon is kept in orbit around the Earth because it rolls along a valley in the curved environment that is created by the Earth's mass. For this reason, the general theory of relativity is necessarily founded on Riemannian geometry.

It is worth mentioning that the general theory of relativity makes the following predictions: rays of light passing close to a star should be bent towards it, and physical processes should take place more slowly in regions of low gravitational potential than in regions of high gravitational potential.

Moreover, the German physicist Arnold Sommerfeld in 1909 and the Serbian-Croatian mathematician Vladimir Veričak in 1912 proved that the special theory of relativity, which explains how speed affects mass, time, and space, is intimately related to hyperbolic geometry. In particular, according to the special theory of relativity, a light wave always travels at the same speed. In other words, the speed of light is always constant but time is relative, depending on one's state of motion. Observers in relative motion experience time differently.

If Euclidean geometry is consistent, then non-Euclidean geometries, specifically Gaussian–Lobachevskian geometry and Riemannian geometry, are also consistent, since we can construct models for the latter

within Euclidean geometry. Conversely, if non-Euclidean geometries, specifically Gaussian–Lobachevskian geometry and Riemannian geometry, are consistent, then Euclidean geometry is also consistent, because the lines in non-Euclidean geometries (the "horocycles" on the "horosphere" in hyperbolic space and the "geodesics" in Riemannian space) form a model of the lines on the Euclidean plane. Thus, the aforementioned geometries are equally consistent. Geometry on the sphere is known as "embedded geometry," since the spherical surface is thought of as embedded in the three-dimensional space, whereas geometry on the plane, which is a two-dimensional continuum, is known as "intrinsic geometry" since the plane representation of the world uses only the two dimensions that are intrinsic to the surface of the sphere. For instance, aviation is based on geodesics, so it uses embedded geometry, whereas two-dimensional maps of the world use intrinsic geometry.

It goes without saying that civil engineers, architects, and real-estate developers use Euclidean geometry, because they are concerned with ordinary measurements that are not too large. Nevertheless, the representational accuracy of Euclidean geometry is less certain when one is concerned with the measurement of larger distances. According to Albert Einstein, space and time are inseparable, and the geometry of space-time is affected by matter, so that light rays are curved by the gravitational attraction of masses. Therefore, physicists have ceased to think of space as an empty Newtonian box whose contours are unaffected by the masses of heavy bodies. The great French mathematician and philosopher Henri Poincaré, in his book *Science and Hypothesis* (originally published in 1902), argues that the geometric axioms are neither synthetic *a priori* intuitions as Kant has assumed, nor experimental facts as Newton has assumed, but they are scientific conventions, and one's choice among all possible conventions is determined by experimental facts. In other words, according to Poincaré, the choice of a geometry is free in principle, depending on the actual context in which one works, but it is limited by the necessity of avoiding contradictions. Therefore, it must be logically consistent.

For instance, Euclidean geometry is the most convenient geometry for ordinary engineering, but it is not the most convenient geometry for the theory of relativity or for aviation. Moreover, computations show that the geometry of perspective spaces is non-Euclidean since, in perspective spaces, collinearity is preserved instead of parallelism, and angles are not invariant under translation and rotation. Moreover, the German-American mathematician Rudolf Karl Luneburg has argued that the most convenient

geometry in order to study the "visual space" (the space that we perceive through vision) is hyperbolic geometry.

Due to the dynamic presence of consciousness in the world, space depends not only on the concepts of somewhere and nowhere, which refer to the place of bodies, but also on the concept of here, which is determined by consciousness. This is the essence of the dynamization of space by consciousness. In view of the foregoing, the cohesive bond between temporal presence and spatial presence is manifested in history.

Consciousness expresses its dynamic reference to the world through the dynamization of space-time, and through its operation in the context of dynamized space-time. Dynamization underpins and vindicates the intentionality of the consciousness of a being that is governed not only by a biological program, but also by an evaluative one. By the term "evaluative program," I mean a program in terms of which consciousness structures and restructures itself and the world. In the context of an evaluative program, consciousness:

- determines the content of a scholarly discipline by tackling the philosophical problems (particularly, the ontological, epistemological, and ethical problems) that the corresponding object evokes;
- is committed to logic and reasonable explanations;
- is committed to history, which refers both to a methodology (specifically historiography) and to a way of understanding reality (in terms of the interventions of consciousness in the world);
- is committed to a system of values and to a moral criterion.

Thus, both the spatial and temporal modes of being are enriched with the possibility of experiencing and utilizing a dynamized mode of being. Consequently, space and time are not universal conditions, nor are they the conditions in which consciousness operates. Instead, consciousness creates its spatial-temporal existential conditions by dynamizing space-time. In particular, consciousness integrates itself into the world in order to dynamize it—that is, in order to create dynamized space and dynamized time. Hence, the world is structured and restructured according to the intentionality of consciousness. In this way, consciousness utilizes the world as a source of energy that allows a conscious being to develop into an enhanced, superior version of itself according to its own structure in the context of an organism.

Everything that we can definitively say about the physical world, and about the past of the physical world, is based on the classical worldview, which is founded on two major theoretical pillars, depending on the scale of our analysis: Newtonian mechanics and the general theory of relativity.

In fact, the general theory of relativity is a geometric theory of gravitation and of space-time, explaining the behavior of the universe on the large scale. On the other hand, the quantum world is not directly observable, and it can be used only for calculating probabilities. Hence, quantum mechanics, pioneered by Niels Bohr, Werner Heisenberg, Wolfgang Pauli, and Erwin Schrödinger, is a theory of physical probability.

The term "quantum" derives from the Latin language, and it means an amount of something. In the context of quantum mechanics, the term "quantum" means the smallest amount of energy that can be measured. The central concept of quantum physics is that of a wave, here meaning a disturbance or oscillation that travels through space-time accompanied by a transfer of energy. The basic properties of a wave are its amplitude (i.e., the distance from the center line, that is, the still position, to the top of a crest or the bottom of a trough), its frequency (i.e., the number of cycles occurring per second; specifically, it can be measured by counting the number of crests of waves that pass a fixed point in one second), and its length (i.e., the distance over which the wave's shape repeats; for instance, the distance between two adjacent crests). From the perspective of quantum mechanics, the concept of a physical system is equivalent to the concept of a state. This, in turn, is a vector in a multi-dimensional geometric space that allows length and angle to be rigorously measured.

Continuing in the context of quantum mechanics, a molecule can be thought of like a mountain range (described by a wave-function) filled with infinitely many energy steps, where each energy step, representing a quantum of energy, is a quantum state. A molecule stands on one of these quantum states, and all the other infinitely many quantum states are empty, they are virtual states. Moreover, each quantum state is characterized by a wave form. When a system stands on one of these states, the other states also exist, but potentially. This means that they cannot be observed, and they actually look empty. Those virtual states are potential modes of being, by virtue of which a molecule can jump into other quantum states. Due to Heisenberg's uncertainty principle, we know that molecules can make "quantum jumps," because they have empty states into which they can jump.

The distinction between being actually (actuality) and being potentially (potentiality), the concepts of dynamized time and of dynamized space, and the distinction between the modes of being and the modes of enhanced being provide important insights to the understanding of the fundamental difference between the general theory of relativity and quantum mechanics. The objects with which the general theory of relativity is concerned are actual beings. Therefore, Einstein's theory of gravity and of

space-time deals with actuality—with being actually, in the physical world—whereas the objects with which quantum mechanics is concerned are potential beings. This means that quantum mechanics is concerned with potentiality, with being potentially, in the physical world. Therefore, the reality of the physical world is not one, since it consists of two different realms (the realm of actuality and the realm of potentiality), but the reality of the physical world is unified in the sense that there is a structural continuity between the reality described by quantum mechanics and the reality described by the general theory of relativity, since both of them are parts of the intrinsic program of development of the physical world.

In accordance with the aforementioned inquiries into dynamized time and dynamized space, the worldview of quantum mechanics and the worldview of the general theory of relativity represent both the reality of the world and the reality of consciousness. In fact, consciousness is fundamental to the way in which "existence" is perceived in the context of modern positive science. A successful scientific theory (such as the general theory of relativity, quantum mechanics, etc.) is a mathematical framework—that is, an abstract system—from which we can derive predictions that agree with observation. Therefore, physical objects, such as time, black holes, quarks, bosons, etc., which are said to "exist" in the physical world are names that physicists give to mathematical structures that are parts of successful hypothetico-deductive systems. In physics, a hypothetico-deductive system is said to be successful if the predictions, or the generalizations, that derive from it agree with observations and logic. This is the meaning of the term "existence" in the context of the natural sciences.

Antimatter is exactly the same as matter, except that the particles that make up antimatter have exactly the opposite charge to the particles that make up matter. We could say that antimatter is the mirror image of matter, except that it has an opposite charge. For example, for every proton, there is an antiproton; for every electron, there is an antielectron, etc. Let us compare the electron with the antielectron, also called the positron. Then we find the following:

Electron (a negative one elementary electric charge): e^-.

Positron or antielectron (a positive one elementary electric charge): e^+.

The mass of the electron is the same as the mass of the positron: $m_{e^-} = m_{e^+}$.

The spin of the electron is the same as the spin of the positron: $Spin_{e^-} = Spin_{e^+}$.

The electron charge and the positron charge are exactly opposite to each other:
$q_{e^-} = -1.6 \times 10^{-19} C$ and $q_{e^+} = +1.6 \times 10^{-19} C$,
where C denotes coulomb (the standard unit of electric charge).
The discovery of antimatter is due to the British theoretical physicist Paul Dirac. In 1928, trying to bridge quantum theory with Albert Einstein's theory of relativity, he proposed the Dirac equation—namely, a relativistic equation of motion for the wave function of the electron, thus describing the behavior of an electron when it is moving at relativistic speeds. At this point, let us recall from mathematics that the equation $x^2 = 1$ has two solutions—namely: $+1$ and -1. Similarly, Dirac's equation has two solutions: according to the first solution, the energy of the electron is a positive number, but, according to the second solution, the energy of the electron is a negative number. However, this mathematical result cannot hold in nature, because, according to classical physics, energy is always a positive number. For this reason, Dirac interpreted the results of his famous equation as follows: the particle that appears to carry the positive energy is an electron with its negative charge, whereas the particle that, according to the solutions of the Dirac equation, appears to carry a negative amount of energy is the electron with the opposite charge—namely, the antielectron. Therefore, mathematics suggested the existence of antimatter. Just as the equation $x^2 = 1$ has two solutions (namely, $+1$ and -1), so the Dirac equation suggests the existence of a particle and an antiparticle. The observation of antimatter—that is, the practical confirmation of its existence—was achieved in 1932 by the American physicist Carl Anderson, who, for this reason, shared the 1936 Nobel Prize in Physics with Victor Hess. In particular, using a cloud chamber (that is, a particle detector used for visualizing the passage of ionizing radiation), Anderson observed and photographed a positron (specifically, he recorded a particle coming from the bottom and curving to the left, indicating a positive charge, and this particle had the mass of an electron, meaning that it was a new kind of particle, a positive electron, or a positron). What happens when matter comes into contact with antimatter? When a particle (e.g., an electron) comes into contact, "collides," with its respective antiparticle (e.g., a positron), they annihilate each other (that is, they cancel each other out), leaving behind an amount of energy (according to Albert Einstein's formula $m = \frac{E}{C^2}$, where m denotes the mass of the corresponding particle-antiparticle pair, E denotes energy, and c^2 denotes the speed of light squared). Hence, the creation of the universe was made possible due to a fundamental asymmetry between matter and antimatter—that is, the amount of matter exceeded the amount

of antimatter, and, thus, the amount of matter that survived the interaction with antimatter continued to create the universe.
V.M.: What is the second objective of our work?
S.O.: To mold and, indeed, create history.
V.M.: What is history?
S.O.: History is a series of acts through which consciousness controls and manages regions of the world. In other words, history is a series of interventions of consciousness in the space-time continuum, which underpin the distinction between the notion of before and the notion of after. The fact that the integration of beings into the world underpins their organization, which, in turn, underpins the completion and the integration of their presence, does not imply that the presence of beings is passive or that the study of beings as agents is of secondary significance vis-à-vis the study of beings as organisms of the world. Being as agency is best understood as dynamic intentionality. The reality of a being as an agent is manifested through the discarding of those possibilities which do not comply with the given being's nature or with its program of development, as well as though the identification and the embracing of those elements which are akin to the given being's nature and reinforce it. In other words, being as agency is manifested through self-affirmation. In the context of their integration into the world, beings are structurally interrelated and interdependent; they affirm themselves within each other, but the nature of each being is not altered by this interaction. The constitutive elements of being, or the categories of being, interact with the constitutive elements of the reality of the world. Therefore, apart from being modes of being, they become modes of enhanced being. The intentionality of the structure of a being, or the character of a being's intentionality, is manifested through the dynamization of the existential conditions of that being by that being. Therefore, consciousness restructures both the world and itself in a way that ontologically upgrades the corresponding being into an enhanced, superior version of itself, so that its organic integration does not bring about its substantive alteration.

Lecture II

V.M.: Before inquiring into conscious life and into the functioning of philosophizing and scientific consciousness, we must have a clear understanding of life and of the major philosophical and biological perceptions of life, which is one of the most important manifestations of existence. The term "life" refers to a set of phenomena (such as reproduction, development, and homeostasis or maintenance) that

characterize organisms. The term "organism" refers to any being that embodies the properties of life, and it is contrasted to those objects which, lacking an organic constitution, are characterized by inertia and apparent stability. It goes without saying that life contains inorganic matter. However, life restructures inorganic matter in an organic way, thus differentiating it from inorganic matter. In other words, life is entwined with inanimate matter and consciousness, and it underpins the structural continuity between them. For this reason, there are both differences and similarities between organic matter and inorganic matter, which I would like you to explain in relation to the issue of the nature and the substance of life.

J.O.: The structure and the properties of atoms, molecules and, in general, of all ordinary matter are due to primarily electrical interactions between electrically charged particles. The fundamental building blocks of ordinary matter are the negatively charged "electron," the positively charged "proton," and the uncharged "neutron." In a neutral atom, the number of electrons equals the number of protons that exist in the nucleus, and the net electrical charge is zero. If one or more electrons are removed (resp. added), then the remaining positively (resp. negatively) charged structure is called a "positive ion" (resp. a "negative ion").

From the perspective of quantum mechanics, particles are discrete packets, "quanta," of energy with wave-like properties. In other words, according to quantum mechanics, energy is not continuous, but it is always parceled up into some tiny discrete "lump" (which is what "quantum" literally means: a discrete thing). In essence, an electron is a circular standing wave.

In simple terms, to construct an atom, one needs some protons and neutrons for the construction of the nucleus, and then one has to put some electrons around the nucleus until the whole system is electrically neutral (in fact, once you have a positively charged nucleus, it attracts electrons, which automatically form shells around the nucleus). However, it should be mentioned that the construction of an atomic nucleus is a complex process, because protons, being positively charged, repel each other. As a result, they have to come very close to each other in order for the nuclear force to start operating and, thus, keep them together, given that there exist sufficiently many neutrons. This process requires extremely high temperatures (hundreds of millions of degrees Kelvin). Such high temperatures existed briefly after the Big Bang.

According to the "Bing-Bang" cosmological model, gravity underpinned and, actually, determined the transition from the "Bing-Bang" cosmological "soup" to the galactic structure that we observe today:

gravity started from the initial conditions of the Big Bang and made the universe much more complex because, even though the density of the universe was almost uniform, there were density quantum-mechanical fluctuations. Put slightly differently, there were small differences in the density of the universe from one region to another. Thus, a region of the universe with density slightly greater than the mean density of the universe acted upon itself by its own gravity and, gradually, it made itself denser. Consequently, instead of expanding with the rest of the universe, it drew matter into the given region. Ultimately, this region collapsed upon itself and did not participate in the universal expansion. In this way, a physical object was made out of such a region. Gradually, the universe was filled with small density inhomogeneities resulting from inflation due to quantum-mechanical fluctuations, which ultimately merged into the structures of the universe that we observe today.

Even though there is a structural continuity between inorganic matter and organic matter, life—by transforming inorganic matter into organic matter—implies an important differentiation in matter. Thus, the differences between inorganic matter and organic matter can be summarized as follows:

Firstly, inorganic matter is governed by inertia, which is the resistance of any inorganic body to any change in its velocity. On the other hand, organically structured living beings sense things, react to external stimuli, and move on their own.

Secondly, inorganic matter reacts according to Newton's third law of motion: for every action in nature, there is an equal and opposite reaction. In other words, the reaction of an inorganic body is quantitatively determined by external mechanical forces (specifically, by tensile force, compressive force, and shear force) that are applied to it. On the other hand, the reactions of organically structured living beings manifest peculiar qualitative features that are not strictly analogous to the stimuli that cause reaction, and they depend on organic relations that govern each living being according to its structural program.

Thirdly, according to the Standard Model of particle physics, the minimal constituent matter elements of inorganic bodies are uniform—that is, subatomic particles are identical (so that no exchange of two identical particles, such as electrons, can lead to a new microscopic state). Thus, all the atoms of which any inorganic body is composed are identical to each other. By contrast, the minimal constituent matter elements of organic matter (such as DNA) are subject to differentiations, which underpin the actualization and the manifestation of the structural program of an organic being. In fact, due to their differentiation, the cells of an organic being

underpin its organic constitution, which determines the corresponding organic being's unity and cohesion (namely, the attraction of molecules for other molecules of the same kind). Furthermore, it is important to mention that eukaryotes (that is, organisms whose cells have a nucleus enclosed within a nuclear envelope), such as the human being, have two types of DNA: the DNA of the cells (namely, the agent of the genetic information of the cells) and the mitochondrial DNA (namely, the DNA located in mitochondria, which are double membrane-bound organelles supplying cellular energy and controlling the cell cycle and the cell growth; mitochondrial proteins—that is, proteins transcribed from mitochondrial DNA—vary depending on the tissue and the species).

Fourthly, inorganic bodies are connected with each other under specific conditions in order to form chemical compounds, which are always characterized by the same quantitative data, described and explained by the following laws: Antoine Laurent Lavoisier's "law of conservation of matter," according to which matter is neither lost nor gained during a chemical reaction; Joseph Louis Proust's "law of constant composition," according to which the constituent elements in a compound are always present in a definite proportion by weight; and John Dalton's "law of multiple proportions," according to which, in the formation of two or more compounds from the same elements, the weights of one element that combine with a fixed weight of a second element are in a ratio of small whole numbers (that is, integers), such as 2 to 1, 3 to 1, 3 to 2, or 4 to 3. On the other hand, organically structured living beings exchange some of their constituent elements with some of their environment's constituent elements in the context of a dynamic process that is called assimilation. In biology, assimilation is the absorption and digestion of food or nutrients by an organism.

Fifthly, inorganic bodies exist in definite and fixed quantities according to Lavoisier's "law of conservation of matter," Proust's "law of constant composition," and Dalton's "law of multiple proportions." On the other hand, organically structured living beings (specifically, "parents") create new living beings (specifically, "offspring") similar to them in the context of the reproductive process.

Sixthly, with few exceptions (such as radioactive nuclides, or nuclear species which are unstable structures that decay to form other nuclides by emitting particles and electromagnetic radiation), inorganic bodies are incapable of self-transformation. On the other hand, organically structured living beings follow life cycles (namely, developmental stages that occur during an organism's lifetime).

Furthermore, consciousness is a state in which a being can understand, process, and modify one's internal and external environment. Therefore, it can be described as a complex system of concepts. Consciousness is manifested by the creation of multiple feedback loops whereby a conscious being can create models in order to pursue certain goals. Animals can understand their position in space, and many of them can also understand their relationships with other beings, but only humans can understand the future and restructure their spatial-temporal existential conditions according to their intentionality, thus creating history.

The continuity of living organisms is ensured by the succession of generations. On the one hand, each living organism is organically self-contained; on the other hand, the succession of generations ensures the continuity of the corresponding species. Intimately related to the study of the continuity of living organisms is the neo-Darwinian concept of a mutation: an abrupt jump in the continuity of living organisms or, more specifically, an alteration in the nucleotide of the genome of an organism.

However, the aforementioned scientific approaches to the properties of life cannot sufficiently address the issue of the nature and the substance of life, because "scientific explanation" is founded on experience. In particular, "scientific explanation" is founded on both rational, abstract thinking and empiricism. Whereas rationalism can underpin the investigation of the most general questions about reality, empiricism orients scientific consciousness to local results and *a posteriori* methods of inquiry. Therefore, ontology is necessary in order to inquire into the nature and the substance of life.

It goes without saying that materialist ontological theories have formulated over-statements and over-simplifications by arguing that the properties of life are reducible to chemical reactions, while spiritualist ontological theories have formulated over-statements and over-simplifications by articulating interpretations that are founded on mere intellectual speculation and ignore empirical data. Nevertheless, the careful study of the history of philosophy with regard to the issue of the nature and the substance of life can provide us with useful information and ideas.

V.M.: Explain the argument that there is a type of continuity between life and spirit.

J.O.: According to ancient philosophy, there is a type of continuity between life and spirit. Inspired by pre-Socratic philosophy, Epicurus formulated a theory of hylozoism that is founded on the concept of a primal breath animating matter. This hylozoist perspective is similar to the biblical Jews' and the Kabbalists' teachings about God's "ruach" (breath and spirit), since Greek philosophy exerted a significant influence on

ancient Judaism and, particularly, on the Jewish Kabbalah (which is a synthesis of Pythagoreanism, Neoplatonism, and biblical mysticism).

In particular, the Jewish way of worshiping God was influenced by Greek philosophy, since it included not only chants and readings from the Bible but also interpretations of the Jewish religious texts—that is, a philosophical task. Greek culture influenced the Bible itself: the expressions "God the Most High" and "I am that I am" are indications of Platonic influences on the Bible. Moreover, the epithets "Lord of the Powers" and "Almighty" (in Greek: "Pantocrator"), which were adopted by Hellenized Jews, were originally applied to Hermes. Moreover, the Greek translation of the books of the Old Testament that was produced in 271 B.C.E. by seventy Jewish interpreters by orders of Ptolemy II Philadelphus, King of Ptolemaic Egypt, is characterized by a linguistic attempt to imitate Greek anthropomorphism. As a result of their contact with and influence by Greek philosophy, the Jews' old tradition of prayer to a silent God, which underpins pure mysticism, was gradually supplemented with philosophy—specifically with a discourse about God, which underpins the development of theology.

The primary Kabbalistic treatises are the *Wisdom of Solomon*, the *Zohar* (or *Book of Light*), and the *Sefer Yetsira* (or *Book of the Creation*). The treatise *Wisdom of Solomon* is said to have been written in Hellenistic Alexandria, and it is attributed to Philo of Alexandria, a Hellenistic Jewish philosopher, whose purpose was to harmonize Greek philosophy with Jewish spirituality. Philo's allegorical and symbolic treatises, as well as his concept of the Logos as God's creative principle, were important for several Christian Church fathers, influencing early Christology. The *Zohar* was written by Simeon bar Yochai and first printed in Mantua in 1558. The author of the *Sefer Yetsira* is unknown, and scholars place its origin at sometime between 100 B.C.E. and 800 C.E., but it was originally published in Provence in the thirteenth century C.E. by Rabbi Isaac ben Abraham.

In addition, Gematria is congruent to the Kabbalah. Gematria is the practice of assigning a numerical value to a name, word, or phrase according to an alphanumerical cipher. The traditional Kabbalists maintain that any words that enumerate to the same value have something in common, so that words can be replaced with other words of the same numerical value, thus revealing hidden ways of interpreting sacred texts.

The most important Kabbalistic symbol is the Tree of Life, which consists of ten "Sefirot" (singular, "Sefira")—that is, emanations or levels of reality—through which the "Ein Sof" (namely, the Infinite and Unknowable One) reveals Himself and continuously creates and sustains

both the physical realm and the chain of higher intelligible realms. The ten Sefirot of the Kabbalistic Tree of Life are connected together by twenty-two paths corresponding to the Hebrew letters and to the Major Arcana of the Tarot cards.

According to Rabbi Isaac (ben Solomon) Luria (1534–72), known also as "Ha-Ari Hakadosh" ("The Holy Lion"), who was one of the most influential Jewish mystics in the Galilee region of Ottoman Syria, and developed the so-called Lurianic Kabbalah (a school of Kabbalah named after him), the holy "Sefirot"—or the attributes/emanations through which the "Ein Sof" (the unmanifested Infinite Unity) reveals Itself and creates both the physical realm and the chain of higher spiritual realms—are surrounded by the profane "Qliphoth." The Hebrew term "Qliphoth" literally means "Peels," "Shells," or "Husks" (the singular form of "Qliphoth" is "Qlippah," and the singular form of "Sefirot" is "Sefira"). According to Isaac Luria's biblical homilies and Kabbalistic system, "in essence, the Qlippah comes before holiness in every matter . . . The parable of this is that the Qlippah of the stem is exceedingly beautiful but its entire end is Ha-Qlippah as it was said by Rashbi [*Zohar*, 1:19]: 'This is Qlippah to that and this is nucleus to that'" (Ha-Ari Hakadosh, *Sefer Liquitim*, Parshat Bereshit). Therefore, the Lurianic Kabbalah differentiates the holy nucleus of the cosmos and of the beings and things that exist in the cosmos from their impure, profane peel. In this way, it reveals important esoteric knowledge regarding Eden and the narrative of the Book of Genesis.

The ten Sefirot of the Kabbalistic Tree of Life are the following, listed in order from the Beginning to the End *(the J.O. shows the Kabbalistic Tree of Life as he/she explains the Sefirot)*:

1) Kether, meaning Crown; its major esoteric emblems are the following: the crown, the point, and the swastika. It symbolizes participation in the good-in-itself and union with the deity.
2) Chokhmah, meaning Wisdom; its major esoteric emblems are the following: the uplifted rod of power, the straight line, and the tower. It firstly symbolizes the world that is created by the intentionality of human consciousness. Secondly, it symbolizes the attribute of being creative and spiritual.
3) Binah, meaning Understanding; its major esoteric emblem is the chalice. It firstly symbolizes the external world. Secondly, it symbolizes the attribute of being logical, analytical, and organized.
4) Chesed, meaning Mercy; its major esoteric emblems are the following: the pyramid, the tetrahedron, the wand, and the scepter. It firstly symbolizes historical time, which is a creation of human

consciousness. Secondly, it symbolizes the attribute of being sociable, generous, and optimistic.

5) Geburah, meaning Power; its major esoteric emblems are the following: the pentagon, the five petaled rose, the sword, the spear, and the chain. It firstly symbolizes natural time. Secondly, it symbolizes the attribute of being calculated, cautious, and serious.
6) Tiphareth, meaning Beauty; its major esoteric emblems are the following: the cube and the rosy cross. It firstly symbolizes the dynamization of space and time by human consciousness—that is, the restructuring of reality by the intentionality of human consciousness. Secondly, it symbolizes the attribute of being balanced and empathic.
7) Netzach, meaning Victory; its major esoteric emblems are the following: the lamp, the rose, and the seven veils. It firstly symbolizes emotions; secondly, it symbolizes space as a creation of human consciousness; thirdly, the attribute of being persistent, forceful, and daring.
8) Hod, meaning Splendor; its major esoteric emblem is the apron. It symbolizes, firstly, rational thought; secondly, natural space; and, thirdly, the attribute of being thankful, accommodating, and devoted.
9) Yesod, meaning Foundation; its major esoteric emblems are the perfumes and the sandals. Firstly, it symbolizes moral consciousness; and, secondly, the awareness of the true will.
10) Malkuth, meaning Kingdom; its major esoteric emblems are the following: the Equal-Armed Cross, the double cubed altar, the magic circle, and the triangle. It firstly symbolizes materialization; secondly, it symbolizes the brain; thirdly, discourse and effective communication; and, fourthly, leadership.

Kether, Chokhmah, and Binah constitute the supernal triad of the Kabbalistic Tree of Life. In terms of ontology, Kether is pure Being, Chokhmah is Energy, and Binah is Form. Tiphareth is the Law of Harmony and the channel of God's Love in the world. Tiphareth corresponds to the Son through Whom one can know the Father. Tiphareth, Chesed, and Geburah constitute the second triad of the Kabbalistic Tree of Life, while Yesod, Netzach, and Hod constitute the third one; these two triads combined constitute the Hexad, or Hexalpha, also known as the Seal of Solomon. Moreover, the second triad (namely, Tiphareth, Chesed, and Geburah) is often mentioned as the subjective principle. From this perspective, it refers to the realm of culture, while the

third triad (namely, Yesod, Netzach, and Hod) is often mentioned as the objective principle, referring to the realm of historical objectification.

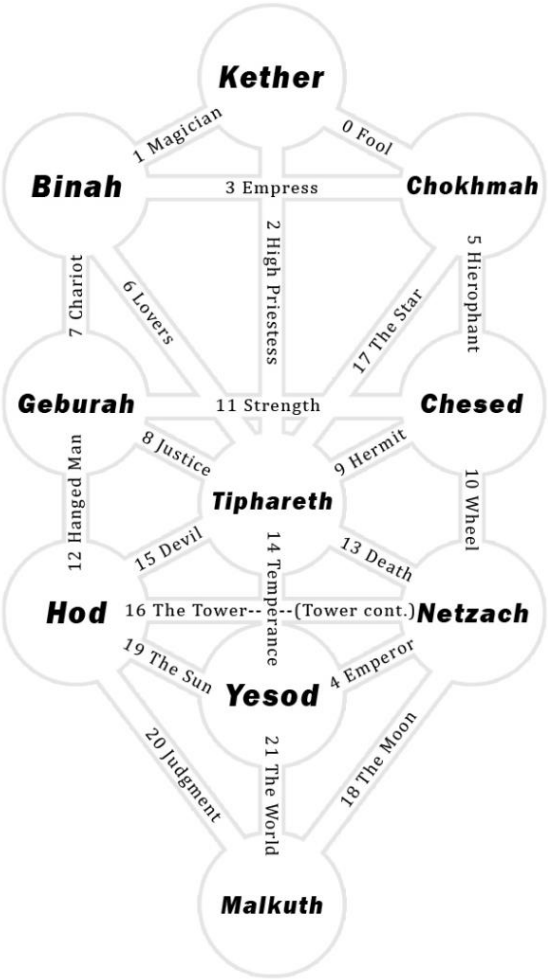
In the history of esotericism, the Kabbalistic Tree of Life has been used by non-Jewish mystics as an ecumenical esoteric system of correspondences and symbols that can be adjusted to the aims of various mystics. Thus, for many esotericists, the Kabbalah has come to mean an attempt to formalize, or even mathematicize occultism in accordance with the legacy of the Pythagorean "school." For instance, the Renaissance saw the birth of Christian Kabbalah, which consists in a Christian reinterpretation of Kabbalistic doctrines and symbols. Ramon Llull, a philosopher, logician, Franciscan tertiary, and writer from the Kingdom of Majorca, was the first Christian scholar to acknowledge and appreciate the Kabbalah as a tool of conversion. However, among the first and most important systematic propagators of Kabbalistic studies beyond exclusively Jewish circles were the Italian philosopher Giovanni Pico della Mirandola, the Venetian Franciscan friar Francesco Giorgi, the German scholar Johann Reuchlin, and Paolo Riccio, a German Jewish convert to Christianity who became a professor of Philosophy at the University of Pavia and subsequently a physician to Emperor Maximilian I.

For a systematic study of the Kabbalah in the context of Western occultism, one should consult the following books:

Johann Reuchlin, *On the Art of the Kabbalah (De Arte Cabalistica)*, translated by Martin and Sarah Goodman, University of Nebraska Press, 1993 (originally published in 1517); and

Arthur Edward Waite, *The Holy Kabbalah*, Dover Occult, 2003 (originally published in 1929).

Figure 2-4: The Kabbalistic Tree of Life. Source: Wikimedia commons, Author: Alan SyncBook Press: https://commons.wikimedia.org/wiki/File:Tree_of_Life_Tarot.jpg



The Hellenization of the Jews originally manifested in the field of language. The Jewish population which was most rapidly Hellenized was that of Egypt. Many Jews of Egypt used to take Greek names, such as: Apollonius, Artemidorus, Diodotus, Demetrius, Dionysus, Diophantus, Heracleia, Heracledes, Hermeias, Theodotus, Theodorus, Dositheus,

Jason, etc. One can even find Greek names among the Maccabees—for instance, two generals of the Maccabees' army had Greek names: Dositheus and Sosipatrus. Moreover, in the fourth book of the Maccabees, the classical Greek term "polity" is used by the Jews in order to refer collectively to those who are faithful to the Torah.

The Jews of Egypt developed a highly Hellenized literature. The historiographer Demetrius, a Jewish courtier of Ptolemy IV, wrote an exposition of the Jewish religion following a philosophical style, which indicates the Greek spiritual influence on the Jews of Egypt. Artapanus of Alexandria, another famous Hellenized Jew, wrote an allegorical novel in which Moses is presented as the founder of the Orphic Mysteries. The Orphic cult was one of the most ancient Greek monotheistic religious systems. According to the initiates of the Orphic Mysteries, there is only one divine King-Lord. He is self-caused, everything that has been created has been created by Him (the One), and all the different gods of the ancient Greek pantheon are different manifestations of the omnipotent One. A Jew named Ezekiel composed a Greek tragedy on the theme of the Exodus. Philo the Elder wrote the epic *On Jerusalem* in Homeric hexameters. The Jewish epic poet Theodotus wrote an epic *On Shechem*, connecting the name of Shechem, an Israelite city of the tribe of Manasseh, with Sikimios, the son of the Greek god Hermes. The Jewish philosopher Aristobulus of Paneas coined the theory that Pythagoras and Plato knew the Bible.

Furthermore, there was a peculiar interrelationship between Hellenism and Judaism. On the one hand, seeking an experiential knowledge of absolute values, Hellenism made an inner "pilgrimage" to the East. As we read in Josephus's *Antiquities of the Jews*, many Greeks turned their minds towards Jewish monotheism because they were fascinated by its mysticism and radical transcendentalism, which provided many people with consolatory and reassuring myths and mystical teachings in an era of large-scale historical change, turmoil, and existential insecurity. In particular, due to Alexander the Great, his epigones, and the Roman elites, the ancient Greek model of a city-state was replaced by an imperial model known as the "ecumene." The Greek term "ecumene" means that a group of people participates not only in the same geographical or geopolitical entity, but also in a common spiritual horizon. The creation of the ecumene was intimately related to the levelling of different peoples—that is, to the intermixture of the civilizations of different nations and the development of religious syncretism. The levelling of the different peoples of the ecumene was followed by the intensification of the individuation of the human being *per se*. The individual human being was not any more a

member of a homogeneous cultural community or a citizen of a city-state. In the context of the vast ecumene, the individual was feeling that one was too small to make a significant impact on social-historical becoming. Therefore, the individual started to live for one's own sake. In other words, the individual's primary goal was the survival and the intensification of one's individuality. Thus, naturally, the Greek soul turned towards Eastern religious doctrines of salvation and towards Eastern ascetic traditions, and tried to imitate them, combining the legacy of classical philosophy and Eastern religious teachings and myths about inner life. In particular, as we read in Arrian's book *The Discourses of Epictetus* (III, 22), the philosopher Epictetus described himself as a man "without a city, without a house, without possessions, without a slave" and as a man who has "no wife, no children, no praetorium, but only the earth and heavens, and one poor cloak." On the other hand, Judaism, seeking its further integration into the historical world and a more creative way of historical self-actualization, exited its narrow geographical and ethnic space and turned to Hellenism. The consequences of the previous duplex interaction, underpinning a synthesis between Greek philosophy and mystical Jewish monotheism, became epoch-making when Christianity was born and Rome fell.

The Stoics' hylozoism is founded on the concept of a divine fire animating matter, and this hylozoist perspective is similar to several biblical passages where the divine spirit is symbolized by fire (Acts 2, for instance). Furthermore, the conception of a principle that animates the body underpins both Plato's and Aristotle's philosophy. In Plato's philosophy, the soul is placed between spirit—the energies of which are ideas—and matter. In Aristotle's philosophy, the ontological differentiation of the principle that animates matter underpins Aristotle's argument that this principle is the organizing form of the body, and governs the entire body. In line with Plato's arguments regarding the placing of the soul between spirit and matter, Neoplatonism has articulated its own ontological hierarchies. However, the aforementioned arguments regarding the existence of an external principle that animates matter are inconclusive because they are based on a sense of logical necessity, and, therefore, they deal with the ontological component of the issue of life more in terms of logical reduction than in terms of ontology itself.

V.M.: Explain the controversy between mechanism, also known as mechanical philosophy, and dynamism, also known as dynamic philosophy, in the context of modernity.

J.O.: In the seventeenth century, and in relation to important advances in the scientific discipline of medicine, the philosophical inquiry into the

nature and the substance of life was systematized, and it started considering clearly scientific data. Thus, in the context of modern philosophy, the philosophical inquiries into the problem of life can be distinguished into two general categories: mechanism (also known as mechanical philosophy) and dynamism (also known as dynamic philosophy). According to mechanism, which is largely inspired by ancient atomism, the constitution of reality, including life, is a result of random physical-chemical phenomena. However, Descartes's philosophy replaced the previous materialist variety of mechanism with a spiritualist variety of mechanism, according to which, in contrast to animals, the human being is governed by spirit, therefore making the human being a cognizing organism.

Descartes's attempt to explain the life of animals by means of a monistic philosophy, and the life of the human being by means of a dualistic philosophy, is characterized by important flaws. In the twentieth century, several distinguished representatives of mechanism, such as Daniel Auger, Jacques Loeb, and John Searle, while endorsing an anti-materialist, "anti-physicalist," perspective, argue that there is a type of continuity between matter and life, including consciousness as an outgrowth of life. In addition, such careful and thorough proponents of mechanism reject the argument that life is a transcendent principle by maintaining that—in spite of the continuity between organic matter and inorganic matter, and in spite of the fact that both organic matter and inorganic matter are subject to the same natural laws—life comprises the set of the differences between organic matter and inorganic matter. However, the scientific corroboration of the aforementioned arguments of mechanism does not imply their definitive confirmation, either in the context of philosophy or in the context of science itself. Even though mechanical philosophy can provide epistemologically satisfactory propositions, it cannot properly address the fact that there exists a substantive difference between life and matter. Moreover, mechanical philosophy analyzes the data of life in a way that cannot give rise to a synthetic study of the principle of life and of structural questions.

The brain (the central nervous system of humans), like that of animals, contains nerve cells that are highly specialized in transmitting messages. Each nerve cell, called a neuron, consists of the central body of the cell, the neuraxon, and the dendrites. At the end of the neural tube, there is a special structure called a synapse, through which the neurons communicate with each other. When a message created in one neuron is about to be transmitted to the next, the first neuron releases specialized chemicals called neurotransmitters. The released neurotransmitters are

taken up by specially shaped regions, called receptors, on the cell membrane of the next neuron involved in the particular synapse.

As regards the naive materialists' approach to the mind–body problem, it should be mentioned that modern biology implies that the functioning of a biological organism can be compared with a bio-chemical factory, but it does not legitimize the argument that consciousness is caused by the brain. It goes without saying that modern biologists have discovered many correlations between neural activity and conscious experiences, but a "cause-and-effect relationship" between neural activity and conscious experiences cannot be deduced solely on the basis of an observed association, or "correlation," between them. According to an old statistical adage, "correlation does not imply causation." Hence, in the case of anxiety disorders for instance, symptomatic pharmacotherapy (which is based on the administration of neuroleptic or antipsychotic drugs, narcotic-hypnotic drugs, tranquilizers, and antidepressants) proves that there exist important neural correlates of consciousness, but the explanation of the phenomenon of consciousness cannot be exhausted in or entirely reduced to the explanation of its biological substrate. For this reason, for instance, the treatment of anxiety disorders cannot be constrained to symptomatic pharmacotherapy—it calls for other types of treatment, too, such as psychoanalytic therapy. Indeed, if consciousness were only a consequence of bio-chemical processes in the brain, or the central nervous system, then the academic discipline of psychology should be totally abolished and totally replaced by bio-chemistry. Furthermore, apart from the "down-top" pathways that feed forward data from the sense organs up to the brain, there are also "top-down" response pathways, which are intimately related to the intentionality of consciousness. In several cases, their effects (positive or negative) are more important than the effects of the "down-top" pathways.

In view of the foregoing, regarding the mind–body problem, one can reasonably argue that modern biology and, in general, modern natural science, far from confirming I materialist arguments, corroborate Aristotle's hylomorphism and Thomas Aquinas's Aristotelianism. A very good approximation of hylomorphism in relation to the explanation of the relationship between consciousness and the brain has been formulated by the American philosopher James Porter Moreland: a CD (namely, a compact disc) does not actually contain music, but it contains only pits, recessed areas on a CD where data are stored. Moreover, a CD does not "create" music. However, if the configurations on a CD are placed into the adequate retrieval system, then music can be played. If the CD is damaged, then the CD player cannot properly read the configurations, and

it, therefore, cannot play the music. In much the same way, consciousness can read pathways in the brain, and thus access and process stored information. If these pathways are changed or damaged, then the underlying information (received and stored by the brain) will not be available or could be read in an altered way. Consciousness reads or interprets the configuration of neurons, which store information that the brain receives from the environment.

The argument of classical mechanism, according to which any living organism is merely a set of physical-chemical phenomena, contradicts the second law of thermodynamics, according to which the state of entropy of the entire universe, as an isolated system, will always increase over time. In particular, according to the second law of thermodynamics, no process can have as its sole result the transfer of heat from a cooler to a hotter body. Intimately related to the second law of thermodynamics is the concept of entropy, which provides a quantitative measure of disorder. Entropy counts the number of different microscopic configurations that have the same macroscopic appearance (or, in other words, how much information one could stuff into a macroscopic object if one kept track of the microscopic details). The second law of thermodynamics is equivalent to the "maximum entropy principle" and the "minimum energy principle": the maximum entropy principle states that, for a closed system with fixed internal energy (an isolated system), the entropy is maximized at stable equilibrium. The minimum energy principle states that, for a closed system with fixed entropy, the total energy is minimized at stable equilibrium. Hence, the second law of thermodynamics states that the quality of the energy of the entire universe, as an isolated system, is degraded irreversibly. In fact, physical, chemical, and electrical energy can be changed into heat, but the reverse (e.g., the transformation of heat into physical energy) cannot be fully accomplished without outside help or without an inevitable loss of energy in the form of irretrievable heat. This does not mean that the energy is destroyed, but it means that it becomes unavailable for producing work. This very degradation of energy is entropy. William James Sidis, in his book *The Animate and the Inanimate* (published in 1920), argues that life implies a reversal of the second law of thermodynamics.

The attempt to reconcile the aforementioned argument of classical mechanism, according to which any living organism is merely a mechanistic physical-chemical system, with the second law of thermodynamics by arguing that the world is in a state of maximum entropy, consisting of beings that are imperfect and weak manifestations

of life, is also unsuccessful, because it contradicts both the continuity and the dynamism of life itself.

In contrast to mechanism (mechanical philosophy), dynamism (dynamic philosophy) highlights the differences between matter and life. The major representative of the first historical phase of dynamism was Leibniz. Leibniz founded his variety of dynamism on the Stoics' hylozoism, thus departing from Descartes's mechanism. The major representative of the second historical phase of dynamism was the French physician, physiologist, and encyclopedist Paul Joseph Barthez. Barthez employed the expression "vital principle" as a convenient term for the cause of the phenomena of life, distinguishing it from both the principle of matter and the principle of spirit. Thus, he refused to commit himself to either spiritualism or materialism. The major representative of the third historical phase of dynamism was the French anatomist and pathologist Marie François Xavier Bichat, the acknowledged father of modern histology. In his famous physiological research works, Bichat, rejecting reductionism, recognized three essential "vital systems": animal life, sensible organic life, and insensible organic life. He located the primary seat of animal life in the brain, of sensible organic life in the heart, and of insensible organic life in the lungs, and he argued that various physical-chemical factors tend to destroy organic life. The aforementioned varieties of dynamism converge to the argument that there is a discontinuity between physical-chemical phenomena and life. However, dynamism started following an alternative intellectual path at the end of the nineteenth century: physical-chemical phenomena constitute the basis of life, but, apart from them, life has also a final cause (or purpose) which consists in the preservation of the unity of each and every organism through which life is manifested. The aforementioned teleological approach to life has been called "neofinalism" (in French, "néo-finalisme") by the French philosopher Raymond Ruyer.

Every rigorous inquiry into the phenomenon of life and every rigorous attempt to understand the significance of a being are necessarily dependent on the study of the structure of a being—especially, on the study of the intrinsic program of ontological actualization of a being, which underpins the transition from being potentially to being actually. The term "development" refers to a smooth growth and expansion of an organically structured living being according to the given being's structure, whereas the term "evolution" refers to a sequence of smooth and rather slow transformations according to a procedural logic; in its pure form, the concept of evolution is associated with the passive role that British empiricism assigns to consciousness. Thus, the term "development"

implies a deliberately organized process of amelioration, which can be studied in terms of a model of constrained optimization.

Intimately related to the study of life is the study of consciousness. Consciousness proceeds from life, but it is not ontologically posterior to life, because consciousness exists potentially within the tendency of a being to exist. It is also intrinsic to instinct, which is a condensed form of logic. Moreover, consciousness underpins the adaptation of the organically structured living beings to their environment. Finally, as Bergson has correctly pointed out, consciousness is inextricably linked to action. When the human being ascends to the highest levels of consciousness, which correspond to reason and morality, it spiritualizes matter. The four most important means by which human consciousness spiritualizes matter are art, science, technology, and institutions, which imply the integration of ideas into matter and the restructuring of matter according to the intentionality of consciousness.

In fact, the spiritualization of matter through the integration of ideas is the most important underpinning of economic development. For instance, it is worth pointing out that the American economist Robert Solow (who was awarded the Nobel Prize in Economics in 1987) has found that, in the United States, during the period from 1909 to 1949, about one-eighth of the increment in labor productivity could be attributed to increased capital per man hour, and the remaining seven-eighths to a factor that is called "Solow residual" and consists of technological progress and other essentially spiritual factors that improve efficiency. The American economist Edward F. Denison has studied the contribution of different elements to growth in real Gross National Product in the United States during the period from 1929 to 1982, and he has shown that advances in knowledge, education, and other cultural and institutional factors play the most important role in economic growth. Moreover, the Russian-Soviet philosopher and scientist Alexander Bogdanov, one of the acknowledged founders of the science of planning and organizational theory, has argued that socialism is meaningless without a "universal organizational science," which would combine and coordinate different scholarly disciplines.

V.M.: What do the terms "spirit" and "spirituality" mean?

J.O.: The nature of the human being is bipartite in the sense that the human being is comprised of two elements: the body and the soul. The soul is the personal way of carrying and manifesting the impersonal life-force. Spirit is not a separate, third substance of the human being, but it is a power of the human soul. In particular, "spirit" is a term by which we refer to everything that transcends the logic of natural necessity. Thus, if a person—specifically, a soul—thinks, behaves, and acts in ways that

transcend the logic of natural necessity (alternatively put, if one is elevated above a mode of being that is determined merely by biological and physical constraints and commands), then such a person is spiritual. Moreover, at the level of the individual human being, spirit manifests itself as a radical way of representing and combining things in order to create new forms. In other words, a spiritual person does not merely recall, represent, or combine things, but such a person manages to create new forms. For instance, a car or a violin is an ontologically new form, since it is something that does not exist in nature.

By the term "spirituality," we mean one's personal quest for, or encounter with, the sacred and with ultimate existential questions. It may or may not lead to, or arise from, one's participation in a religious community. However, spirituality does not only refer to an experience of transcendence, it also emphasizes a personal bond between the individual and the whole, whose nature can be transcendent. Therefore, the concept of spirituality is inextricably linked to the concepts of meaning and purpose. Spirituality has at least two components: the first component is a relationship with a reality that surpasses the realm of the ego, and, with regard to this component, spirituality is intimately related to a deeply erotic attitude, since "eros," in its pure sense, implies self-transcendence and sharing, or communion. The second component is a peculiar type of knowledge that combines intuition, thought, experience, affect, and motivation.

In its pure, or primitive, form, religion derives from the natural, instinctive tendency of the individual to protect oneself, survive, and control one's environment, whereas spirituality derives from free will, because spirituality is a conscious choice of a mode of being that transcends the logic of natural necessity. Therefore, religion gives rise to a society founded on needs and necessities, whereas spirituality gives rise to a society founded on freedom and truth.

Lecture III

V.M.: Explain the religious conception of humanity.

Orator: At the time when people were hunter-gatherers, the predominant form of religion that people created was animism. One of the main features of animism is that spirits are local entities (a particular tree, rock, or storm) rather than universal deities acting according to a hierarchy of rules. As a result of the Agricultural Revolution, also known as the Neolithic Revolution, people became tied to a particular location due to sedentary patterns of agriculture, as they were farmers and herders.

Therefore, a new form of religion was needed to correspond to the new socio-economic conditions. In these conditions, polytheism was instituted and prevailed. In particular, deities were invented to whom people promised absolute devotion in exchange for absolute control of animals and plants. The role of the deities is that of mediating the relationship between humans on the one hand and animals and plants on the other. Polytheism is a vision of life that aims to transcend the uncontrollability of human and natural events. The following millennia were spent building empires, supported by the invention of money and writing. Polytheism is no longer appropriate. Man began to feel the need for a single, all-powerful and unifying deity to rule the world. What was needed now was a single deity who embodied the concept of destiny, but was fully involved in the historical events of the people who believed in it. Such a deity is a God who defends and sustains his people. Thus, monotheism was born, the most important pioneers of which were the Jews, who found in the idea of Jehova the archetype of the unique national God. Christian monotheism, derived from Jewish monotheism, offered an ecumenical, universalizing form of monotheism. Thus, Jewish monotheism expressed the ideals and needs of a national kingdom and a nomadic population, whereas Christian monotheism expressed the ideals and needs of a globalizing model, humanism, and an imperial structure.

According to the religious conception of humanity, the human being is a creation of God. The starting point of all monotheistic religions is the creation of the world, and especially of humanity. Akhenaten, the Egyptian pharaoh-creator of monotheism, and subsequently Moses, Jesus, and Muhammad have played key roles in the formation of the principles that underpin all the theological doctrines that have attributed to humanity a clear relationship with God. According to these religious traditions, humanity is God's creation, owes everything to humanity's creator, and humans must obey divine commandments in order to claim rewards for their earthly life. Even those religions which are not structured as fully developed, coherent theological doctrines believe in a creation myth and emphasize humanity's obedience to God's will.

Furthermore, it is worth pointing out that the Darwinian theory of evolution holds that nature favors those factors that help the species survive, and, indeed, religion is one of the most important factors that have helped the survival of the human species in terms of both the management of social life and the tackling of psychological issues. Hence, the scientific conception of humanity admits that religion has offered significant evolutionary advantages to the human species. The most important evolutionary advantages that religion has provided for the species *Homo*

have to do with the rationalization of reality, the management of anxiety, and the exercise of social control.

V.M.: Explain the scientific conception of humanity.

Orator: According to the theory of evolution formulated by Charles Darwin in his seminal book *On the Origin of Species* (published in 1859), the biological definition of humanity is that the human being is a "thinking monkey," a monkey that has invented science, logic and mathematics, morality, law and religion. Since Darwin formulated his theory, the scientific discipline of biology has clarified the origin of life and has established that all living organisms, whether bacteria, fungi, plants, or animals, including the human being, originate from the primordial cell. All individual life forms are structured with the same elementary components that are assembled by the same biosynthetic processes. Although, of course, there are some differences which depend on the nature of the substances used or the source of energy used and the type of work involved in different physical processes, the basic processes are always the same, and all living organisms use the same genetic language.

The aforementioned scientific knowledge allows us to study the evolutionary history of humanity. In the twentieth century, biologists came to the following conclusions regarding the evolutionary history of humanity: (i) the evolutionary history of humanitybegan some sixty-five million years ago, when dinosaurs dominated the plains of Europe and North America; (ii) at that time, humanity's ancestors, who could barely be identified as primates, were jumping from tree to tree and other large plants; (iii) later, these primitive mammals began to acquire differences and evolve until they became the ancestors of anthropomorphic apes; (iv) still later, about seven million years ago, a subset of their descendants began to slowly but steadily develop new characteristics, and its evolutionary lineage separated from that of the chimpanzee.

At first, these evolutionary innovations involved simple, basic traits, such as the ability to walk on two legs. Over time, however, humans began to manifest truly new and more complex features, such as an ever-larger brain, the use of tools, and the development of language and culture. Eventually, this evolutionary line gave way to modern humanity.

During the aforementioned evolutionary process, which started from the common ancestor between humans and chimpanzees, there are no intermediate links between humans and chimpanzees, but we find lateral branches that separated from the main branch and then disappeared. The most recent of these side branches is represented by *Homo neanderthalensis.* Twentieth-century scientists concluded that *Homo neanderthalensis*

disappeared about 30,000 years ago, but for a relatively short period of time lived alongside *Homo sapiens*.

In the twentieth century, biologists and paleontologists came to the following conclusions regarding the timeline of human evolution: about 200,000 years ago, *Homo sapiens* began to spread across the Earth at a very rapid rate. It is possible that this genus, as early as about 150,000 years ago, may have dominated Africa and, by about 70,000 years ago, may have travelled to the Middle East. From there, these hunter-gatherers arrived in South Asia and then, via sea routes, travelled to Asia and Australia about 60,000 years ago. About 40,000 years ago, they returned to Europe. About 15,000 years ago, they reached the Bering Strait between Asia and North America via land bridges that appeared on the surface of the two continents. About 12,000 years ago, they crossed the continental mass of the Americas and, in particular, colonized the Amazonian forests and the pampas of Patagonia. The aforementioned spread of *Homo sapiens* across the planet took place at great speed, since DNA analysis of a large variety of modern human beings shows that all of the approximately eight billion people living on Earth in the beginning of the twenty-first century are descended from a limited group of five thousand females and a similar number of males who lived some 150,000 to 200,000 years ago. This ancestral group of humanity probably originated in Africa, because Africa alone contains the largest number of modern DNA variants in the world.

During the period of *Homo sapiens'* spread across the globe, two very important events took place that gave human evolution a strong forward momentum: language and religion.

During the evolution of *Homo sapiens*, the emergence and development of language is the most important event that separates us from all other animal species, including the anthropomorphic apes which are the closest relatives of humans. At this point, it is worth pointing out that standard languages, whether we are dealing with the natural language of humans, mathematics, or computer programming languages, include (1) alphabets (collections of initial symbols), (2) rules for constructing sentences with these symbols, (3) rules of deduction, or transformations of sentences, (4) the definition of axiom systems, and (5) theorems formulated according to the rules of deduction. Any form of communicative competence developed by other biological species is not even remotely comparable to human language. Human language has enabled the birth of scientific knowledge, from which comes technology, the achievements of which are changing the existential state of humanity.

Why, I wonder, do only humans have the ability of language? The answer can be given on many levels. Firstly, at the level of anatomy, an important

difference that still exists between humans and anthropomorphic apes (the chimpanzee being the closest to the human species) is that the latter lack the vocal apparatus necessary for speech production. For example, in the chimpanzee, the larynx is located very high behind the base of the tongue. In the human being, it is located lower down in the throat. For this reason, the human larynx has the advantage of widening the resonance chamber in the throat and mouth, allowing the emission of a huge range of sounds (pressure waves). These sounds are then articulated into words and phrases which follow the rules of grammar and syntax.

At the level of paleontology, the study of the differences between humans and higher primates is based on the comparison of fossil finds in order to investigate corresponding anatomical correlations of speech. An important correlation between anatomy and speech ability refers to the dimension of the opening of the sublingual canal located at the base of the skull through which the nerve that goes to the tongue passes. Indeed, fossil analysis shows that humans have a sublingual neural canal that is much larger than that of all anthropomorphic apes.

At the level of the respiratory system, we find another important difference between humans and anthropomorphic apes, as a certain kind of breathing is necessary to enable the emission of sounds and words. In particular, *Homines sapientes* have developed a relative enlargement of the spinal canal in the region of the thoracic vertebrae, especially in the upper part of the thoracic cavity, whereas all other anthropomorphic apes have developed in a different way. The thoracic nerves control the thoracic muscles and diaphragm and play an important role in calibrating the breathing necessary for the emission of words. In the fossils of anthropoid apes, the thoracic nerves do not show such enlargement and, for this reason, cannot perform the sounds that humans are able to produce. Analysis of the thoracic vertebrae of *Homo sapiens* shows that the enlargement of the spinal canal occurs simultaneously with the enlargement of the submandibular canal. In the twentieth century, after comparing the fossil findings of *Homo sapiens* and hominid apes, scientists concluded that we inherited our features from archaic *Homo sapiens* about 500,000 years ago. This is the most likely time period in which language evolved. Of course, words and language are not abilities that suddenly appeared, but evolved separately, each in its own way. It is also valid to claim that language may have passed from a phonetic (musical) phase to a subsequent linguistic (verbal) phase.

Coming back to the above remarks on language, I would like to emphasize the social role that language may have had on people. Robin Dunbar, a distinguished cognitive anthropologist from the University of Oxford,

believes that, at this point, it is possible to formulate the hypothesis of the "social brain." Basing his conclusions on research on the neocortex of the brain, in his seminal volume *The Human Story: A New History of Mankind's Evolution* (London: Faber and Faber, 2004), Dunbar argues that there is a significant correlation between social group size and neocortex size in primates: the group size predicted by this correlation analysis for humans is about 150, meaning that the number of individuals with whom a normal human adult has reciprocated relationships of trust, obligation, and reciprocity is approximately 150. Beyond the 150 layer, there are at least two further layers: the layer of acquaintances, and the number of faces one can put names to. Furthermore, according to Dunbar (ibid), the main mechanism by means of which primates link their groups is "social grooming," and the amount of time monkeys spend grooming other group members is directly related to their typical group size (so that "the larger the group, the more time the animals spend grooming each other").

In light of Dunbar's claims, we can be led to the conclusion that, in humans, language may have replaced grooming in anthropomorphic monkeys, so that human societies could be held together in increasingly larger groups (see: R. Dunbar, *Grooming, Gossip and the Evolution of Language*, London: Faber and Faber, 1996). Beware, then, of any attempt by modern marketing, modern propaganda, modern communications, modern entertainment industry, and modern lifestyle to de-humanize you by privileging the principle of grooming over the principle of language, or the principles of looking and feeling over the principles of thinking and discourse. Intellectual development and intellectual work essentially differentiate the human being from the mere ape. Humanity's greatest existential and identity dilemma as a thinking ape is whether humans will choose to exist more as "thinking subjects" or more as "apes" and how much of each.

In studying the overall evolutionary and historical trajectory of *Homo sapiens*, we find that the development of *Homo sapiens'* consciousness and intentionality, or purposefulness, was based on thinking and discourse—that is, on the fact that *Homo sapiens* decided and was able to work intellectually with great effort to transcend the stages of grooming and sensation. In the twentieth century, biologists, archaeologists, and historians arrived at the conclusion that, 70,000 years ago, *Homines sapientes* initiated a Cultural Revolution.

In addition, the language used up to that time to describe the external world and to communicate between humans gradually acquired another important feature: the ability to convey information about things that do

not exist in physical reality. In this way, myths, legends, and religions were born. Now, *Homo* has all the basic tools to become a creative producer of history.

V.M.: Explain the development of consciousness.

Orator: The evolutionary history of humanity is defined by the increase in brain size, as the latter made possible the rise of consciousness, both in its primitive form and in its higher order. Higher order consciousness, in turn, made possible the birth of language and intentionality or purposefulness.

Consciousness is essentially linked to intentionality, through which human beings can access external reality and enter into relationships with each other. Undoubtedly, there are conscious states that are not intentional, and there are intentional states that are not part of our consciousness. Nevertheless, the connection between consciousness and intentionality plays a crucial role in understanding human beings and history.

The subjective states of consciousness include beliefs, desires, perceptions, and emotions. An important philosophical problem consists in how to determine the relations between the subjective states of consciousness and external, extra-conscious reality. Intentionality is always directed towards something, and we need to understand what that something might be. Starting our investigation from simple cases, intentionality may be directed at primitive biological instincts such as, for example, thirst and hunger. Hunger is the desire or mental impulse to eat. Eating is the event that fulfills the feeling of hunger. We can interpret thirst, and all other biological forms of intentionality, in a similar way. Moreover, intentionality can be directed towards sensory impressions, such as sight, smell, touch, and hearing. In other cases, it can be directed towards objects that express emotional, empathic, and logical mental states.

In general, by "intentionality," we mean the property of mental states (e.g., thoughts, beliefs, desires, hopes, and so on) to be directed towards an object or towards some state of affairs. Intentionality gives an external direction to states of consciousness. In this way, it can operate according to a hierarchy of relations ranging from a minimum to a maximum. The levels of this hierarchy of relations are called "orders of intentionality," in the terminology of cognitive anthropologist Robin Dunbar. Specifically, bacteria and certain insects have zeroth-order intentionality, while brain-equipped organisms are conscious of their mental states. For example, brain-equipped organisms know when they are in danger or hungry. Therefore, brain-equipped organisms have first-order intentionality. First-order intentionality means that a being is self-aware, consciously referring to itself. However, there are also types of higher-order intentionality.

Intentionality can be directed towards the beliefs of other people—we say that it is second-order intentionality.

In other words, in the terminology of cognitive anthropologist Robin Dunbar, we can distinguish the orders of intentionality as follows: most vertebrates can recall their mental states, at least in an elementary way, that is, by knowing that they know. Organisms that know that they know have first-order intentionality. Organisms that, moreover, know that someone else knows something have second-order intentionality. Organisms that, in addition, know that someone else knows that someone else knows something have third-order intentionality. As the number of subjects in the intentionality sequence increases, so does the number of hierarchical orders. This sequence can reflexively be extended indefinitely, but, in the context of their everyday life, most people rarely reach intentionality of an order higher than fourth, and they can very hardly rise to the fifth order—that is, to the following type of reasoning: "Theodore knows that Christina believes that George thinks that Nicolas supposes that Natasha intends to do something."

Fourth-order intentionality is required, at a minimum, for the development of literature that goes beyond mere narrative because, for example, an author wants his/her readers to believe that literary hero A thinks that literary hero B intends to do something. The same level of minimum skills are required for the development of science, since doing a scientific task requires asking whether the world could exist otherwise and going beyond the level of sensory experience, and then asking someone else to do the same.

In general, the emergence of intentionality is an epiphenomenon of the relationship between the cognitive capacities of primates (including humans) and the dimensions of the cerebral neocortex. The neocortex is a layer about six millimeters thick that surrounds the entire reptilian brain, and is present in all mammals. In mammals, this envelope ranges from ten to forty percent of the total brain mass. In humans, it can reach up to eighty percent of the brain mass. The neocortex is the thinking part of the brain— therefore, the thicker it is, the greater the cognitive abilities.

During the evolution of biological species, the enlargement of the primate brain occurred mainly at the front of the skull, where the frontal lobe is located. The other regions of the brain, the lateral and posterior regions, are responsible for the functions of vision, sensory perception, and memory. According to numerous clinical studies, the dimensions of the human frontal lobe are correlated with superior intelligence, which is the most important element of superiority over anthropomorphic apes. In humans, the surface area of the frontal lobe is more than four times the

volume of the chimpanzee frontal lobe, and the rate of increase is exponential. Also, the development and function of neurons in the human brain determine the development of *Homo's* ability to grasp higher orders of intentionality or purposefulness.

Of all the known complex systems that exist in the physical universe, the most complex is the human brain. If we were to build a computing machine that modelled the human brain, then that computing machine would have to be the size of a city block, would have to be cooled by an entire river, and would need a nuclear power plant to power it (while the human brain runs on only about 20 Watts).

In light of the above, to determine when *Homo* developed the ability to grasp higher orders of intentionality, we must first deal with the comparison between the dimensions of the frontal lobe and the dimensions of the hominid brain. There is a strong degree of correlation between frontal lobe volume and the order of intentionality: first-order intentionality for non-humanoid monkeys, second-order intentionality for humanoid monkeys, and fifth-order intentionality for humans. Again, by resorting to paleontology, we can ascertain in which hominid fossil the corresponding brain size may have appeared and, thus, understand the critical points—specifically, the moments of acquisition of the various orders of intentionality, and the consequent cognitive abilities of hominids. The scientific research carried out during the twentieth century on this issue has shown that third-order intentionality appeared in *Homo erectus* about two million years ago, fourth-order intentionality manifested in archaic *Homo sapiens* about 500,000 years ago, and fifth-order intentionality appeared about 200,000 years ago in modern *Homo sapiens*, from which modern humans are descended.

The main conclusion that can be drawn from the above analysis is that anthropomorphic apes and humans differ from each other in one fundamental aspect: the ability with which humans can separate themselves from the world of their sensory experiences. The ability to transcend the sensory realm allows us to reflect on our experiences and hypothesize about their nature, and it provides us with concepts of our past, future, and "social self." Unlike human beings, anthropomorphic monkeys have only a direct experience of the world, are limited to immediate experience, and live in the present as they perceive and recall it.

V.M.: Explain the transition of humanity from nature to culture.

Orator: Culture manifests itself as science, arts, ethics, institutions, religion, and ideology. In all times and in all cultures, religion represents the highest and most universal expression of the creativity of human consciousness.

The universality of religion derives from the fact that, during the evolution of the species *Homo*, all human communities known to date have always believed in an existential "other." More specifically, they have all believed in the existence of a spiritual world parallel to and different from that of the senses. Humanity has performed rituals and prayers to propitiate and to gain the favor of spiritual and invisible entities believed to inhabit this world, in order to induce them to turn a merciful gaze on human beings afflicted by various sufferings.

The level of consciousness to which a being must rise to develop religious consciousness is very high. In fact, the ability to conceive religion is an exclusive privilege of the human species. No other biological species living on Earth can formulate anything even remotely resembling religion. Since humans are a product of evolution, we must carefully investigate the factors that may have favored the emergence of our religious impulse.

In order to explain religion as a social activity and as a social institution, we need at least fourth-order (perhaps even fifth-order) intentionality, so that we can handle syllogisms of the following type: "John supposes (1) that Mary believes (2) that John believes (3) that there is a divine being intending (4) to influence people's future (because this divine being understands people's desires (5))." Until people can interact and form a community on the basis of fourth-order (or even fifth-order) intentionality, we cannot yet speak of a fully developed religion, but only of religious beliefs. The existence of a common belief—that is, the fact that there are things that mean the same to everyone—is the keystone of religion. Hence, a true communion of words, a sharing of words as a basic characteristic of any genuine dialogue, is a major underpinning of religion.

Given the above observations about the level of intentionality that religion requires, and given that only the human species can pursue and achieve fourth-order intentionality, we can explain why only humans, among all the biological species living on Earth, have religious systems. Also, some humans can pursue and achieve fifth-order and sixth-order intentionality, and this fact may explain why, among humans, only a relatively small number of individuals are successful religious leaders. It goes without saying that the truly successful religious leaders and the truly successful novelists constitute a very small and very high elite. Hence, few people are founders of religions, cultural currents, and legal systems.

Undoubtedly, the activities of the religious or mythmaking and, in general, the institutionalizing consciousness presuppose the existence of a well-formed language. Without language, consciousness cannot rise to fifth-order intentionality, and cannot form social groups of high organization.

Based on the point that, in order to *understand* religion, one needs a well-formed language and at least fourth-order intentionality (while the *creation* of a religion requires at least fifth-order intentionality), we can determine when religion made its first appearance in the evolutionary history of hominids. Specifically, in view of the foregoing, we can argue that the first appearance of religion in the evolutionary history of hominids coincided with the time of the first appearance of language. Fifth-order intentionality, associated with *Homo sapiens*, manifested itself much later, when fifth-order intentionality in conjunction with a well-formed language equipped with advanced grammar and advanced syntax expressed religion as both a social institution and a metaphysical system.

Darwin's theory of evolution favors everything that can help the species to survive. The evolutionary advantages that the human species has derived from religion are social cohesion, social control, and creative management of existential anxiety. To achieve these goals, religion uses powerful means, such as belief in immortality, metaphysics, and mysticism.

In the light of what I have mentioned so far, the human being seems to be a "thinking ape" who invented religion, art, morality, science, a plethora of technologies, ideology, and a plethora of institutions. Human life can have meaning if, and only if, humanity is happy. In his *Metaphysical Principles of Virtue*, Immanuel Kant concludes that happiness is a continuous state of well-being, enjoyment of life, and complete satisfaction with the state one is in. Some of the most enduring and important channels through which humanity seeks happiness are religions, ideologies, and esoteric societies.

Religions are religious anthropologies, while ideologies are non-religious anthropologies. If we compare modern ideologies with monotheistic religions, we shall find profound analogies. There is, however, a fundamental difference. Religion is based on a metaphysical doctrine that is independent of historicity, whereas ideology is based on a philosophical and social doctrine that is dialectically related to historicity and expresses an attempt to universally interpret and manage history through ideas, institutions, and material possibilities. Hence, "religion" is not the same as "ideology."

A doctrine of immortality, a metaphysical thought, and a ritual system are the three basic components of religion. However, in order to have a more complete understanding of religion, we must also refer to mysticism. According to conventional, traditional religiosity, believers will be able to see the deity and have direct contact with the deity after death as a reward for living an honorable and respectful life. On the other hand, according to mysticism, certain selected people can achieve a personal contact with the deity already in this life: this is the mystical union with the deity. Mystics

believe that they can initiate the journey that will lead them into the presence of the deity while still alive in this world. In general, mysticism advocates immediacy and temporality as well as a form of autonomous radical individualism based on interiority and psychological introspection. The idea of mystical union with the deity has been a dimension and version of religion for thousands of years. For example, various forms of mysticism can be found in Judaism, Christianity, Islam, and Hinduism.

Regardless of the individual differences between mystical systems, the mystical experience, as narrated by mystics, has the following common characteristics: the impression that the mind is separated from the body, transcending the limitations of space, the spread of a powerful light to which the mind is drawn, and the unbridled joy and ecstasy achieved at the moment one experiences "divine union." Neuroscientists and anthropologists have investigated the conditions under which and the ways in which people can reach this mystical ecstasy.

Robin Dunbar has established that the hypothalamus, a region of the brain that controls the action of opioids, if properly stimulated, will release endorphins. Therefore, we can assume that the state of trance achieved at the end of meditation is the process of maximizing the production of opioids. From this point of view, the mystics themselves initiate a technique that causes the release of opioids in the hypothalamus. This induces ecstasy, which then brings about the vision of a dazzling light, the exit of the mind from the body, and the experience of mystical union with the deity. Mysticism is also associated with the experience of near-death, which is usually associated with the depletion of the oxygen supplying the brain. The techniques that mystics use to reach a state of trance are related to a way of reducing the flow of oxygen to the brain or a way of forcing the hypothalamus to emit opioids in order to alter the spatial position of the mind.

The study of religion in the context of the theory of evolution, the study of our brain functions and cognitive abilities as products of natural selection, and the analysis of the biological background of mysticism lead us to examine the relationship between culture and its biological basis.

Many nineteenth-century and twentieth-century philosophers and social scientists have radically differentiated culture from genes, arguing that culture is a separate thing in itself. In other words, that culture has acquired a life of its own that manifests itself in a way that is autonomous from genetics, even if the latter is the biological background of culture. According to these positions, civilization comes only from civilization.

On the other hand, widely respected evolutionary biologists and paleontologists such as Gerald Edelman, Richard Dawkins, and Edward

Osborne Wilson conclude (albeit with different shades of opinion) that, on the basis of the most recent developments in genetics and molecular biology, there is a significant relationship between genes and culture. Thus, the problem is not whether or not this relationship exists, but how binding it is. The main difference between the various scholars lies in how long and how complex the causal chain linking genes to culture is. As the speed of cultural evolution increases and, in general, as civilization develops and becomes more complex, the link between genes and civilization weakens, even if it is never completely broken.

Therefore, we must reject both extreme idealism and extreme physicalism. Extreme idealism, as Vladimir Lenin has aptly pointed out in his book *Materialism and Empirio-criticism* (published in 1909), is based on two principles: (1) the complete detachment of the intellect from matter, and (2) the relativity of knowledge. Thus, in the context of extreme idealism, the intellect is in danger of falling into fantasy or even irrationality, and the fact is overlooked that, as Lenin has rightly pointed out in his aforementioned book, the sum total of relative truths underpins the knowledge of the absolute truth in a subject area. Moreover, in every scientific truth, despite its relativity, one can find an element of absolute truth. In addition, Lenin, in his *Philosophical Notebooks*, rightly rejects "vulgar materialism," too, pointing out that the difference between the ideal and the material is not absolute, and that the thought of transforming the ideal into the real is very important for history. For instance, the process of mathematization of scientific knowledge shows that mathematical abstractions are linked to the actual development of all aspects of the material life of society (e.g., industry, technology, financial policy, and production planning), forming a vast and ever-expanding field of applications of mathematics. The generalized experience of these applications leads to the philosophical notion that mathematical representations of the properties of things exert an active influence on the real world—the ideal is transformed into the real.

The distinguished Italian philosopher and Freemason Giuliano Di Bernardo, in his volume *The Future of Homo Sapiens* (2021, p. 127), has made the following wise remarks:

> The theory of evolution, . . . examined in its historical development, brings us to an extraordinary conclusion: nature has reached its terminus. Scientists have discovered the steps by which nature created and developed the species Homo and have thus essentially replaced nature. This gives way to an important conclusion: that the decision of what the "new" man of the future shall be no longer belongs to nature but to man himself.

As culture develops and humanity matures, the new humanity of the future will be increasingly determined by humanity itself rather than by nature. It was nature that favored religion as a survival mechanism in modelling humanity, specifically as an underpinning of social control and social cohesion. However, the more culture develops and the more humanity matures, the more humanity extricates itself from the existential shackles of nature and, indeed, rejects nature in favor of the intentionality of human consciousness. Thus, traditional religion, or formalized religion "as we know it"—in contrast to ethics (moral philosophy) and its capacity to unite humanity around a project for the future—becomes obsolete and, indeed, a phenomenon of atavism.

V.M.: Explain the production of history by humanity.

Orator: According to the Darwinian theory of "natural selection," species survive by changing or adapting their genes to their environment. For example, bears survived in the Arctic regions. Living there, their fur became increasingly white so that they could not be distinguished from the snow. For many generations, the light-colored bears survived and reproduced, so that today all polar bears have white fur. In this way, bears have adapted their genes to their white snowy environment. Darwin called this process evolution by "natural selection."

This process of adapting genes to the environment has been going on since life appeared on planet Earth. This explains, according to Darwin's theory, the emergence of many types of plant and animal life over millions of years. Those individuals that were best adapted to their environment managed to survive, multiply, and become the dominant variety. In other words, from the beginning of life, it was the genes that were always adapted to the environment.

Then came the revolutionary human species. It was a revolutionary species because it reversed the process of "natural selection"; it was the first species to adapt the environment to its genes rather than its genes to the environment. It succeeded because it had a superior mind and used it, making the environment more comfortable and more conducive to the development of life.

Homo sapiens played a key role in this revolutionary achievement by using three main ways to change the environment that other animals, having less developed brains, could not imitate: (1) reason, which includes logic and language, (2) tool making and use, and (3) the use of fire. This combination of reason and the use of tools and fire gave humans powerful new technology that no other animal possessed. Technology affected all aspects of human life and enabled humans to become the masters of their environment rather than its servants. While the animals' preeminent mode

of existence is adaptation to reality, the preeminent mode of existence of the human being is the reconstruction of reality according to the intentionality of consciousness and, therefore, the production of history, which is the most authentic expression of the creativity of the human being. Hence, the human being, among all biological species living on planet Earth, has the exclusive privilege of living not in physical space-time but in historical space-time. This awareness underpins our Order's conception of "occultism": the manipulation of reality according to the intentionality of consciousness. This is the quintessence of our Order's occult philosophy.

V.M.: God is the name that consciousness gives to the positive void that consciousness conceives and recognizes as the ultimate source of the meaning of the beings and things that exist in the world. In particular, Immanuel Kant, in his book entitled *A New Elucidation of the First Principles of Metaphysical Cognition*, wisely argued that, by the term "God," we should refer to the ground of all real possibility: "nothing can be conceived as possible unless whatever is real in every possible concept exists and indeed exists absolutely necessarily"; "for if this be denied, nothing at all would be possible; in other words, there would be nothing but the impossible" (*Kants gesammelte Schriften*, 1:395). Within the mythological core of every spiritualized, elevated religious system, one can discern and understand the power and structural stability of moral consciousness. Please, demonstrate the manner in which Grand Masters of spirituality have attempted to spiritualize religion and elevate it to a tool for the deepening and expansion of moral consciousness.

Orator: Confucius, the great lawgiver of ancient China, taught the principles of "solidarity" and "reciprocity." He said to the Chinese that the most important benefit that they should desire for their country is justice, and that humanity should look for the great law of duty. According to Confucius, justice is equity to render to everyone that to which one is entitled. In addition, Confucius taught that humans can overcome economic, social, and moral crises through the practice of virtues and the exercise of study.

Zoroaster, whose words became law to the ancient Persians, said that the best servant of God is a person whose heart is upright, who loves freedom, has due regard for what is just to everyone, who does not run after material riches, and whose heart wishes well to every living being. According to Zoroaster, those who are charitable and merciful in their judgments alone are just. According to the central premise of Zoroastrianism, which is exposed in the sacred volume called *Avesta* written in the Zend language, Ahura Mazda or Ohrmazd is the highest

spirit of worship. Ahura literally means light, and Mazda literally means wisdom. Moreover, according to the *Avesta*, Angra Mainyu or Ahriman, being the destructive spirit, contends with Ahura Mazda upon the earth for the souls of men; man may in this life fall under the evil sway of Ahriman, but Ahura Mazda will be supreme at the end.

The historical Gautama Buddha (also known as Siddhartha Gautama in Sanskrit), although born a prince, chose the life of a Hindu monk. He reformed certain rigid and ritualistic aspects of Hinduism, and he taught humans a path that leads to the elimination of suffering and passions. According to Buddha's teachings, this path consists of heartfelt concern for others, love, and compassion, the highest expression of which is the altruistic intention to become enlightened in order to do good to all beings. It is said that Buddha was once asked, "is it better to walk alone or in company?" He answered: "it is better to walk alone than to walk with a fool."

Moses, the greatest leader and lawgiver of the ancient Israelites, delivered the Israelites out of bondage and led them to the Promised Land. On Mount Sinai, Moses told the Israelites that God gave him the Ten Commandments that constitute the foundation of the Jewish moral system (according to Deuteronomy 5:1–22): (i) "I am the Lord your God who brought you out of the land of Egypt, out of the house of bondage; You shall have no other gods before Me." (ii) "You shall not make for yourself a carved image." (iii) "You shall not take the name of the Lord your God in vain." (iv) "Observe the Sabbath day, to keep it holy." (v) "Honor your father and your mother." (vi) "You shall not murder." (vii) "You shall not commit adultery." (viii) "You shall not steal." (ix) "You shall not bear false witness against your neighbor." (x) "You shall not covet your neighbor's wife; and you shall not desire . . . anything that is your neighbor's." In its entirety, the Mosaic or Sinaitic Law comprises 613 commandments, and it is the most important form of law adapted to the needs of nomadic peoples. It helped the Israelites to maintain their existential otherness in general, and their monotheistic doctrine in particular, among different peoples and great civilizations.

Hermes Trismegistus, the mythical Greco-Egyptian Patriarch of Wisdom, taught that the first step of the soul seeking reunion with God is to recognize its own ignorance, and that it is God's wish to be known by humanity, since humanity is God's most glorious creation. According to Hermes Trismegistus, knowing God requires the second birth of the spirit, the unveiling of the "essential" human within, meaning that everyone who seeks to know God must acquire wisdom, practice virtue, do ill to no human, and be detached from worldly things. In other words, according to

Hermes Trismegistus, a human being can become divine by reflecting the divine virtues that are equivalent to the essential self, which is the image of God.

Socrates, a great sage of classical Athens, taught that "there is only one good, that is, knowledge, and one evil, that is, ignorance" (as we read in Diogenes Laertius's *Lives of Eminent Philosophers,* Chapter 5, par. 31). Moreover, Socrates has argued that the "feeling of wonder shows that you are a philosopher, since wonder is the only beginning of philosophy" (as we read in Plato's *Theaetetus*, 155d).

Plato, the most important student of Socrates, taught humans self-knowledge, and revealed to them the world of pure ideas and eternal truths. From the perspective of Plato's philosophy, he who lives in harmony with himself, with his fellow-humans, and with the good-in-itself alone is just; the good-in-itself is the ontological underpinning of the cosmic order.

Thus spoke Jesus of Nazareth, who sacrificed his life for the salvation of humanity: "Do not think that I have come to abolish the Law or the Prophets; I have not come to abolish them but to fulfill them" (Matthew 5:17). "God is spirit, and his worshipers must worship in the Spirit and in truth" (John 4:24). "Love the Lord your God with all your heart and with all your soul and with all your mind and with all your strength . . . Love your neighbor as yourself" (Mark 12:30–31). "You are gods!" (John 10:34). Moreover, according to the Gnostic Gospel of Thomas (verses 112–113), when Jesus was asked "When will the kingdom come?", he answered as follows: "It will not come by watching for it. It will not be said, 'Look, here!' or 'Look, there!' Rather, the Father's kingdom is spread out upon the earth, and people don't see it."

Muhammad ibn Abdullah, the prophet of Islam, taught that God is One, and that in Him should we repose our hopes and to Him should we pray. According to Muhammad, when dealing with others, we should remember that God is watching over us, and that He will deal with us as we have dealt with others. Therefore, we must avoid evils such as cruelty, dishonesty, pride, antagonism, jealousy, selfishness, and callousness. Muhammad said: "The Muslim is the one from whose tongue and hand the people are safe, and the believer is the one from whom the people's lives and wealth are safe" (Sunan an-Nassa'I 4995). According to Islam, Allah did not send down one *Sharia*—that is, divine law—for all humankind. In Surat Al-Ma'idah (5:48), in the Quran, we read the following: "To each of you we have ordained a law and a way," meaning that, even though God created humanity from a single Father (Adam) and a single Mother (Eve), He did not create us all as a single people; He ordained that humanity

should emerge as different nations and different tribes. Thus, there is diversity among humankind as there is diversity among flowers, and this diversity is meant to enhance the splendor of Allah's creation and to function as a means by which humans recognize each other. The most spiritually exalted branches of Islam are Sufism, whose interpretation of Islam is strongly influenced and inspired by the Hermetic "school" of Mesopotamia, and Iranian Shiite mysticism. Furthermore, it is worth stressing that the Quran proclaims that the status among men and women is classified on the basis of faith and conduct rather than birth, dismissing any arrogant claim to a birthright of superiority as false (Sura Al-Hujurat).

Finally, as indicated by the Christ Mystery, the Great Secret of Human Enlightenment is humanity's potential autonomy and, indeed, divinity. Pontius Pilate—who served as the fifth prefect of the Roman province of Judaea under Emperor Tiberius and is best known from the biblical account of the trial and crucifixion of Jesus (Matthew 27:2, 19; John 18:31–37, 19:12–13; Mark 15:5, 15)—represents natural law, or the logic of natural necessity. The Jews who shouted against Jesus, "Crucify! Crucify!" (John 19:6), and that "he must die, because he claimed to be the Son of God" (John 19:7), represent written law, or the logic of social necessity. Those who have not been raised above both natural law and written law (or those who have not been raised above the logic of necessities through that process of enlightenment which makes one a Master of one's destiny and genuinely creative) cannot accept the truth that transcends an established natural or social order, and they crucify the true, creative, and liberating Word, assuming that its content is a "stumbling block" or a "foolishness" (1 Corinthians 1:23).

On the one hand, due to doctrinal theological differences, religions cannot construct a unitary project. On the other hand, spiritually sensitive and cultivated people, independently of their religion, can undertake a common project based on ethics. Religions have failed in the objective of finding common solutions for humanity, but humanity can look for them in terms of ethics in the hope that the road on which to travel, once weakened on theological bases, may be easier to cross and the project easier to achieve. By instituting a global moral system, the leaders of the diverse religions can reach a consensus for the purpose of promoting a unitary project to bring about the spiritual, ethical, and material healing of humanity. Such a global moral system should be based on the basic rule that "what you do not wish done to yourself, do not do to others," and it should include the following four fundamental imperatives: "do not kill," "do not steal," "do not lie," and "do not commit impure acts." The aforementioned global moral system is in agreement with all major

religions without, however, being the product of a unitary world religion and without implying a syncretism of all religions (deism).

V.M.: Brother/Sister Orator, explain the ancient Egyptian Mysteries, from which Freemasonry, Judaism, and Christianity have taken inspiration, especially regarding rituality and symbology.

Orator: The major Egyptian Mysteries are the Mysteries of Isis and Osiris, and the Mysteries of Serapis.

In Egyptian mythology, Ra was the major Egyptian solar deity, which corresponds to the sixth Sefira of the Kabbalistic Tree of Life, that of Tiphareth, and he sprang from the Cosmic Egg. In particular, in the *Egyptian Book of the Dead*, Ra is shown beaming in the Cosmic Egg. Furthermore, according to Egyptian mythology, Ptah was the Egyptian god of Memphis and a divine defender of craftsmen and artists. He was credited with the invention of arts, and he was a metalworker and architect. From the perspective of the Kabbalistic Tree of Life, Ptah corresponds to Kether. Another important ancient Egyptian god of knowledge and illumination was Thoth. He was the divine Egyptian champion of science, music, literature, wisdom, and invention as well as the keeper of the divine Records. In Hermopolis, Thoth was believed to have hatched the Cosmic Egg: when Thoth first sang, the sound from his voice created four gods and four goddesses, who continued the world's creation by word and song. Moreover, he helped Isis to restore Osiris, sustained the Child Horus, and became the judge of Horus and Seth. From the perspective of the Kabbalistic Tree of Life, Thoth corresponds to Chokhmah.

Isis was the first daughter of Geb (the Egyptian god of the Earth) and Nut (the Egyptian goddess of the Sky), and she was chosen by her oldest brother and husband, Osiris, to share the throne. From the perspective of the Kabbalistic Tree of Life, Isis corresponds to Binah. Osiris represented the spirit of vegetation; from the perspective of the Kabbalistic Tree of Life, he corresponds to Malkuth. Isis created marriage, taught women to grow corn and weave clothes, and taught men to cure disease. Their brother Seth killed Osiris and, in several versions of the story, dismembered Osiris' body. With the assistance of other deities, including Thoth and Anubis, Isis searched for the pieces of Osiris's body and reassembled it. Her efforts were the prototype for mummification. According to ancient Egyptian mythology, the god that invented mummification was Anubis, who was presiding over funerals and was directing the dead to the afterworld. From a Kabbalistic perspective, Anubis corresponds to the eighth Sefira of the Kabbalistic Tree of Life, that of Hod. The ninth Sefira of the Kabbalistic Tree of Life, that of

Yesod, corresponds to Shu, one of the primordial Egyptian gods who was a personification of dry air. According to the Heliopolitan cosmology, Shu separated Nut from Geb as they were in the act of love. Thus, he created duality in the manifest world. Ancient Greeks associated Shu with Atlas, the primordial Titan who held up the celestial spheres (in fact, "Shu" means "he who holds up").

Isis's love and grief for her husband Osiris and her powerful magical spells restored breath and life to Osiris's body. Isis copulated with Osiris and conceived their son Horus. However, from this point on, Osiris lives on only in the Duat (the underworld). Horus is a solar god and is often identified with the ancient Greek god Apollo. From the perspective of the Kabbalistic Tree of Life, Horus as Lord of Force corresponds to Geburah, while Amoun, an ancient Egyptian god of expedition and discovery, corresponds to Chesed. In ancient Egyptian mythology, it is often mentioned that the wife of Horus was Hathor-Sekhmet, a female deity who can be identified with both the ancient Greek goddess of love, Aphrodite, and the Anatolian mother goddess Cybele, who ruled life and death. From the perspective of the Kabbalistic Tree of Life, Hathor-Sekhmet corresponds to Netzach.

The myth of Osiris symbolizes the human soul's drama. Osiris himself symbolizes the soul proper: what in ancient Greek is called "nous," or what the Kabbalists call "neshamah"; this refers to the repository of the uncreated spirit. Within the profane men, Osiris is dead through the lower nature, symbolized by Seth. Seth symbolizes the animal soul, in particular selfish sentiments and passions. Thus, Osiris is buried in the shrine of the "mummy" of the materialistic world and thrown into the Nile, which symbolizes materialistic life. But Isis, who symbolizes the realm of the "sacred" and the love for the divine—that is, for the real truth—seeks, with the help of Anubis (who symbolizes divine wisdom) and Thoth (who symbolizes divine grace), the parts of Osiris's fragmented corpse (or being) in order to reassemble them, thus giving rise to an integrated and spiritually revived harmonious soul, symbolized by Horus.

The cult of Serapis was introduced during the third century B.C.E. on the orders of the Greek Pharaoh Ptolemy I "the Savior" of the Ptolemaic Kingdom, a Hellenistic Kingdom based in ancient Egypt, as a means to unify the Greeks and the Egyptians in his realm. Serapis, depicted as Greek in appearance but with Egyptian trappings, was a syncretistic deity derived from the worship of the following gods: the Egyptian god Osiris; the Egyptian god Apis, who was an intermediary between humans and Osiris; the Greek god Hades, who was the god of the dead and the king of the underworld; the Greek goddess Demeter, who was the goddess of

grain, agriculture, harvest, growth, and nourishment; and the Greek god Dionysus, who was the god of the grape-harvest, wine, fertility, ritual madness, religious ecstasy, sacrifice, and theater.

Figure 2-5: The holy ankh, or key of life, is an ancient Egyptian hieroglyphic symbol representing the word "life." Coptic Christians adapted it into the "crux ansata," a shape with a circular rather than oval loop.

Given that the symbol of a rose plays an important role in esotericism, it is worth mentioning that, in ancient Egypt, the priests of Memphis used to consecrate a Rose bush to Isis, who was called the Queen of Roses. The name of the sacred site of Memphis where the symbolic rose was found was called Kab.

V.M.: Brother/Sister Orator, explain the significance of monotheism.

Orator: First of all, it is useful to mention that monotheism was not originally developed by the Hebrews, and that the Hebrews were not the only monotheistic nation in the ancient world. At the zenith of ancient Egyptian civilization, Pharaoh Amenhotep IV (who reigned for approximately 16 years: 1352–1336/1334 B.C.E.), also known as Akhenaten (meaning Effective Spirit of Aten), attempted to change the polytheistic religion of Egypt into a monotheistic one by promulgating the monotheistic worship of Aten, the Sun-God. Aten means bright disk and, thus, the falcon-headed picture of Ra-Harakhti (namely, the previous Egyptian religion's solar deity) was replaced by the symbol of a solar disk whose rays ended in human hands, some of which were holding the holy ankh, the symbol of life. In contrast to the old Egyptian religion's solar deity, which was representing the physical Sun, Aten was a symbol of the trans-cosmic One, the One life-giving source, which creates and preserves the cosmos through its light.

Moses, the most important prophet in Judaism, was initiated into the mysteries of the Egyptian monotheism, and, when the Egyptians decided to abandon Akhenaten's monotheistic religion and return to polytheism, Moses preserved and expanded the monotheistic tradition among the

Hebrews. Thus, Judaism became the strictest and most effective custodian of monotheism in the ancient world. Moreover, in the Hebrew Bible, the manifestations of God's glory are often described as light or fire, and one of the Hebrew names for God is Adonai, which is derived from the root "ada" (which means radiant).

In ancient Greece, in the context of the Orphic Mysteries, there was also a monotheistic religious dogma; in contrast to Homer's popular polytheistic beliefs, the Orphic hymns celebrated the One and Universal God. According to Iamblichus and Porphyry, even though Pythagoras was making references to several gods, he was teaching that there is only one God and that the various different gods mentioned in the Greeks' popular polytheistic religion are powers or energies of the One God. In the same spirit, Plutarch, one of the most influential Platonist philosophers, believed in one, unitary, transtemporal God, with different names for the deity's different aspects.

Why is monotheism so important for the spiritual development of humanity? Monotheism leads to the ontological liberation of God from the cosmos, and, therefore, in the context of monotheism, the divine Word (in Greek, the Logos) is free from any cosmic laws and necessities. In the context of monotheism, God's mode of being is freedom, and this conception of the deity has important anthropological consequences. The essential autonomy of God from the natural cosmos implies that God relates to the natural cosmos only according to God's own free will (in Greek, "thēlema"). Therefore, the laws that govern the natural cosmos (that is, the Creation) do not restrain the divine Mind. Hence, since God is the most important existential mirror in which humans can see and evaluate themselves, and since the mode of being and the nature of the God with whom one identifies oneself determine one's way of thinking about oneself and about one's existential perfection, the God of monotheism is the perfect archetype of human emancipation and spiritual perfection. If one is mirrored in a God who is essentially and totally free from the physical universe, then one develops a sense of autonomy vis-à-vis natural and logical necessities.

V.M.: How do religion and spirituality relate to science?

Orator: The primitive form of religion is mainly based on instinct and selfishness, whereas science is mainly based on reason and observation. From this perspective, there is a fundamental contradiction between religion and science. Over many centuries, this fundamental contradiction has often been manifested through cultural conflicts and hostility, and often a crude religious mentality has led to the establishment of tyrannical regimes. However, a new situation arises when religion is spiritualized:

when humanity moves away from primitive religiosity to higher levels of spiritualized religion, thus understanding spiritualized religion as a tool for the deepening and expansion of moral consciousness. Spirituality does not contradict science, since spirituality refers to the liberation of humanity from necessities, and science consists in a corrective and liberating action of human consciousness.

Spirituality provides a liberating type of knowledge that is based on a causal logic, since it seeks to disclose and interpret the efficient and final causes of the beings and things that exist in the world. Science provides a liberating type of knowledge that is based on a functional logic, since it seeks to explain and/or change the functions of physical and social systems. Spirituality and science in unison can provide a holistic type of knowledge, which is the major demand of classical philosophy. Hence, the spiritualization of religion, or the substitution of religious mentality with spiritual mentality, is a necessary condition for building a better person in a better society.

Finally, it is worth pointing out that the first Apologists and Fathers of the Church of Christ purposely avoided calling their faith a religion. Instead, they characterized their spiritual community as the "Ecclesia of Christ." The Greek term "ecclesia" literally means "gathering of those summoned," and ancient Greeks used the term "ecclesia of the dēmos," meaning the assembly of citizens in a city-state. In Latin, the old Greek term "ecclesia" was preserved, hence the Latin Church Fathers' use the term "ecclesia Christi." However, in the English language, the term "ecclesia" has been largely replaced by the term "church," which derives from the German term "Kirche" and the Dutch term "kerk." These, in turn, derive from the Greek term "kyriakōn (dōma)," meaning "Lord's house," since, in Greek, "kyrios" means "master" or "lord." Whereas the terms "church," "Kirche," and "kerk" may be latently linked to the feudal mindset and may give rise to various forms of spiritual despotism, the term "ecclesia" implies egalitarianism, freedom, and the experience of participating in a specific spiritual system.

The Illuminati Grand Master, philosopher, and academician Giuliano Di Bernardo has brilliantly summarized the history of religion within the broader context of the history of civilization as follows:

> Knowing that we are the product of evolution, we should ask ourselves what selective pressures favored the emergence of the religious impulse. If religion exists and is universal, then what is the evolutionary advantage derived from it? What purpose does religious behavior serve? Why do men and women, in all ages, kneel, flagellate, kill and allow themselves to be killed in the name of god?

The answer to these questions is not simple. However, using the method of historical narration, we can formulate some explanatory hypotheses.

The first fundamental question we can ask ourselves is: "when, in human history, did religion first appear?" To attempt an answer, we can use the comparative analysis between levels of intentionality, on the one hand, and the size of the brain in the fossil record, on the other. What order of intentionality must a religious belief possess in order to manifest itself?

In its most basic forms, religion requires the existence of a world other than the one we live in that we perceive with our senses. For this to be possible, at least second-order intentionality is required. As Dunbar pointed out in *The Thinking Monkey*, in order to participate in religious activities, I must believe that a parallel world exists, populated by beings endowed with intentions that can be modified by my prayers. I, in other words, believe [1] that there are deities, who intend [2] to determine my future. If I am unable to influence the intentions of these beings in any way, religion has no role to play; such entities are little different from the raging floods or erupting volcanoes that engulf us without warning. A religion, if it is to have any real value, must be able to influence the future that is destined for us. But second-order intentionality is not really sufficient to sustain belief in a metaphysical reality. If religion is to have any practical utility, then the deities must be able to understand what I want. It seems therefore that religion must involve a third-order intentionality: I believe [1] that there are deities whom I can persuade to understand [2] what I really want [3] and who, having done so, will intervene to help me. This seems to me sufficient to explain the evolution of a religious feeling. It is not yet sufficient, however, to explain the communal character of religion, the larger-scale phenomenon of rituals and all forms of public devotion, which are so important in the religions we practice. Religion in its human form is nothing if it is not a social activity: we come together in common and shared rituals and beliefs to form a community. To achieve this, we need at least a fourth (and perhaps a fifth) order of intentionality: I suppose [1] that you think [2] that I believe [3] that there are deities who intend [4] to intervene to alter our future because they understand our desires [5]. If, and until, we cannot come together like this, we have no religion but only personal beliefs. It is the shared beliefs that make religion what it is.

It is clear that all this presupposes the existence of a well-articulated language: without language, consciousness cannot rise to the fifth order of intentionality, nor can it form highly organized social groups.

The consequence of what has just been said is that only human beings, unlike all other animal species, being able to aspire to a fourth-order intentionality, can have religion. This also explains why, within religions, few humans, capable of rising to the fifth or sixth order, are the founders, such as Moses, Jesus or Mohammed.

Since all our brain functions and cognitive abilities are products of natural selection, it follows that human knowledge is limited by its neural basis. The problem that arises is to examine the relationship between culture and its biological basis. The existence of this relationship is supported by some scholars while it is denied by others.

Philosophers and social scientists of the last century have repeatedly argued that culture has definitively separated from genes to the point of becoming a thing in itself. It has acquired a life of its own that manifests itself autonomously and independently of the genetic mechanisms that gave rise to it. In conclusion, culture derives from culture.

My way of thinking, in this regard, shares the conviction of authoritative evolutionary biologists and palaeontologists, such as G. Edelman, R. Dawkins and E. O. Wilson, who, with different nuances, re-propose, in the light of the most recent developments in genetics and molecular biology, the gene-culture relationship. The question, therefore, is not whether such a relationship exists or not, but how binding it is. The main difference between the various scholars is precisely how long the thread that binds genes to culture is. The faster cultural evolution moves, the weaker the link between genes and culture becomes, although it is never completely broken.

At the end of this examination, we can define man as a "thinking ape," who invented science, logic and mathematics, art, morality and religion. His creations also include agriculture, kingdoms, empires, churches, money, writing, and great civilizations. A monkey that has sent nature into the attic and is replacing god.

This is the man whose prehistoric and historical path has been laced with wars that have brought mankind endless anguish.

War must be examined not only on the basis of the common foundation, but also with reference to the different epochs in prehistory and human history. Prehistory, which we can date from the origins of hominids to the agricultural revolution, given the numerical smallness of human groups (Dunbar believes they were at most made up of 150 individuals), is of no particular relevance. Instead, the differentiation between wars becomes fundamental from when human history begins, around 12,000 years ago, with the birth of agriculture, which definitively marks the end of an economy based on hunting and berry picking. Until that time, we can assume that wars were mainly determined, among hunter-gatherers, by reasons that concerned hunting territory and relations within groups.

The hunter-gatherers were profound connoisseurs of the secrets of plants and animals, which constituted the main source of their food. They formed small groups that moved constantly in search of game to hunt. When one territory no longer had any animals, they moved to another not far away. They were not sedentary. These living conditions were comfortable and had certain advantages. Firstly, the food was balanced and nutritious. Movement for hunting also kept their bodies in perfect physical shape. Finally, diseases were rare. I think that when we talk about the

golden age of mankind, we are referring precisely to the living conditions of hunter-gatherers.

The agricultural revolution radically changed this way of life. Contrary to what one might think, it was not brought about by kings, shamans or merchants, but by certain plants such as wheat, rice and the potato. It is precisely wheat that drives the birth of agriculture. The *sapiens* began to devote more and more effort to the cultivation of wheat. After several millennia, wheat is the main crop in many regions of planet Earth. It required, however, a collective effort from dawn to dusk. The availability of food increased enormously, but the grain-based diet is poor in proteins, minerals and vitamins. The first diseases began due to the feeding and domestication of animals such as sheep, chickens, donkeys, goats and pigs, which transmitted diseases such as smallpox, measles and tuberculosis to humans. Nevertheless, with the surplus of food, *sapiens* were able to multiply exponentially. The *surplus of* food supplies meant that more and more individuals lived together first in villages and then in cities linked by trade networks. Religion, which had taken the form of *animism* in the hunter-gatherers, was transformed into *polytheism* and the first groups were formed governed by religious authorities, who, in the name of an invented god, exercising social cohesion and control, claimed power-sharing with the state.

Homo sapiens is now the one and only hominid inhabiting planet Earth, of which he is slowly but surely gaining total control. The conditions are already in place for the establishment of kingdoms and empires, the birth of writing, and the minting of the first coins.

Kingdoms are presented as territories, more or less large, in which the king, assisted by a group of loyalists, exercises all forms of authority: political, economic, social, military, cultural. He defends his subjects from whom he demands part of their possessions in the form of taxes. The fundamental characteristic of the kingdom is that the subjects have elements in common, such as language, religion, and culture. Kingdoms, however, inevitably turn into *empires* when kings conquer regions inhabited by peoples with different languages, customs, and religions. The empire, therefore, is a political order in which the emperor governs a diversity of peoples, each with its own cultural identity and separate territory, without altering its fundamental structure and state identity. The boundaries of the empire are flexible due to the conquest of new territories and peoples.

If we take a look at human history, we find that the empire has been the most successful form of political organization. Also because of its stability (empires last for centuries), most of humanity has spent its life within an empire. Subjugated peoples, on the whole, have shown that they have enjoyed their subordination to the authority of the emperor. The damage caused by bloody wars was counterbalanced by stability and the development of science, the arts and philosophy, the activities of which were financed with the proceeds of conquests. It may seem paradoxical,

but humanity's greatest cultural achievements are a consequence of the exploitation of conquered and subjugated populations.

The empire is an expression of a particular conception of life, based on the unity of the world, on a set of universal principles to govern humanity, on the quasi-divine role of the emperor. This vision characterized the Assyrian and Babylonian empires. It was taken up by the Persians and Alexander the Great. It continued with the Roman emperors, the Muslim caliphs, the emperors of Europe, China and South America, and on to the present day.

Kings and emperors expressed the secular power of the state, which was echoed by the religious power represented by the most important churches, from Christian to Orthodox to Islamic, which exercised it absolutely and with greater social control.

From the agricultural revolution to the 18th century, all regions of the earth were ruled by absolute and dictatorial governments. In the state/individual relationship, it was always the state that prevailed. The freedom of the individual was decided by the state, which set the limits and conditions. Religious power was stronger than secular power, since it was exercised in the name of god and considered men humble creatures who owed everything to their creator.

For over 12,000 years, mankind lived in a state of absolute subservience to the authority of kings, emperors, and popes, in all possible forms. In the Renaissance, however, with the mists of the Middle Ages behind us, the wailing of a new world is felt that places man and not god at the center of the universe. The vindication of the scientist's right to research freely, together with criticism of the Holy Scriptures, gave rise to an attitude of sharp contrast with the authorities of the Catholic Church, which was forced to repress it. The condemnation of Galileo is exemplary. The signals in favor of the individual became stronger when the American Revolution of 1776 and the French Revolution of 1789 issued the *Bills of Rights* and the *Declaration of the Rights of Man and of the Citizen* respectively. To have concrete consequences, however, we will have to wait for the arrival of liberal ideology, which will transfer individualism from politics to economics. It is liberalism knocking at the doors of history. It is the beginning of a process that will last about two centuries, at the end of which the individual will become the protagonist of human affairs.

Liberalism, underpinned by the Industrial Revolution, triumphed in Europe and the United States of America. The radical changes taking place in society, made possible by the technological applications of science, give rise to a euphoric sense of optimism towards the future that is increasingly imagined as a source of well-being and happiness.

Economic theorists invent *capitalism* as a corollary of liberalism. The pair <liberalism-capitalism> becomes a conception of man and life, a philosophical anthropology: "The highest human values, such as justice, freedom and happiness depend on economic growth. If there is growth,

humanity lives in prosperity. If growth stops, then humanity lives in suffering." Subsequently, *democracy* entered the <liberalism-capitalism> binomial and the <liberalism-capitalism-democracy> trinomial was formed, which still dominates the Western world today.

In the historical reality of 19th century Europe, however, liberal ideology not only fails to deliver on its promises but also deepens the gap between rich and poor while the poor live in a state of ever increasing destitution. It is in this context that Marx proposes communist ideology, with the task of reversing the individual/society relationship in favor of society. His vision finds practical implementation with the revolution that leads to Stalin's dictatorship.

Liberal anthropology is challenged not only by communism but also by internal contradictions, such as the onset of economic crises. To illustrate this, it suffices to mention two crises that arise in the international economy: the crisis of 1929 and the crisis of 2008.

The first aspect to consider is the *politics/economy relationship* in which politics once again plays a primary role over economics. Individuals or groups can enrich themselves but within the limits imposed by the state. Exemplary in this respect is the view of the Chinese government, which, following the moral rules of Confucius, favors the enrichment of the individual because if all individuals live well, the whole society lives well and is happy. For this reason, China has taken capitalism from liberalism and used it as a tool to improve society, proving that capitalism is not the exclusive preserve of democratic regimes. The enormous advantage of enshrining the supremacy of politics over economics is also in avoiding devastating economic crises.

The second aspect concerns the *family*. The exaltation of the individual has resulted in the destruction of the family. Wives rebelled against husbands, children against fathers. Society no longer had the support of its fundamental pillar and went into crisis. A crisis that manifested itself on all levels: from education to the relationship between the sexes, from culture to the organization of institutions. Quicksand was created that little by little sucked everything down. The sense of responsibility has dissolved like mist in the sun and a state of omertà has been created that affects everyone.

This state of crisis pervading society has been further aggravated by the foundation on which it rests: *democracy*. With the false belief that we are all equal, everyone has wanted to exercise the right to count like everyone else. The hierarchical relationship that has always underpinned society has been broken, opening the door to creeping anarchy. Differences that expressed competencies on specific aspects were annulled, resulting in the growth of an indistinct, extremely dangerous whole. Roles lose their contours and the superior becomes similar to the inferior. It is a Babel that grips social reality.

The anthropology of totalitarianism wants to restore meaning and consistency to the authority of the state in managing the economy,

strengthening the family and eliminating the disastrous consequences of democracy. Is this a return to the past? In a sense yes, but in a new perspective projected towards the future of humanity.

Thus, the Illuminati Grand Master Giuliano Di Bernardo has articulated a vision of a historical One-God, arguing as follows:

> The god of the (mainly monotheistic) religions was a powerful but invented entity. With his attributes, he did not intervene in human history. The One-God, on the other hand, is a concrete being, operating in history, deciding the fate of men. Not all men will be gods, but only a small circle. Some of them, chosen by the One-God, will form the Council of Sages, who will support the One-God in governing the world. The Council of Sages has similarities with the government of philosophers outlined by Confucius and Plato in *The Republic*. If Plato's philosophers were the wisest of their time, in the One Society of the future, the wisest will be men who have acquired the most knowledge in science, technology, economics, politics, ethics, and aesthetics. The One and the Council of the Wise will hold absolute power. Consequently, they will not need any other powers to govern the world, such as those of the past that involved religions.
>
> What will be the One-God's vision of man and life? By what principles will he rule planet Earth? After having eliminated religions as irrelevant, he should be inspired by the rules of a universal ethic, which could have an esoteric and initiatory foundation (see, in this regard, *Freemasonry: Splendour and Decadence*, Amazon, 2022). Faced with the distinction between good and evil, the One-God should choose the good and act according to the principles and rules of universal ethics. He, however, is a man, and man is a combination of good and evil. It is possible, therefore, that The One-God decides to act according to evil.
>
> If he is inspired by the good, then he will give mankind prosperity and happiness by eliminating incurable diseases, improving material conditions (housing and food), cleaning the atmosphere of pollution and the oceans of plastic.
>
> If, on the other hand, he is inspired by evil, then the absolute power at his disposal could make him make terrible choices, such as the elimination of that part of humanity that is not functional to the maintenance of the global society.
>
> I hope that the One-God is an Enlightened One because that would give the certainty that he will act according to the good. But that is only my hope. To give more meaning to this hope, it would be necessary to place the Order of the Illuminati at the center of the globalization process, to give it authority and power in the formation of the inner circle of god-men who will rule the world.

V.M.: Explain the significance of Rosicrucianism.
Orator: During the Renaissance, Rosicrucianism played a very important role in the attempt to spiritualize religion, or substitute religious mentality with spiritual mentality. In essence, Rosicrucianism is not a particular Order, but a spiritual and cultural movement. The legendary history of the Rosicrucians goes back to medieval Germany. The earliest public notice of the Fratres of the Rose and Cross appeared in 1614 in a pamphlet printed at Kassel in Germany, entitled *Fama Fraternitatis Rosae Crucis* (meaning *The Fame of the Fraternity of the Rose and Cross*). In 1615, a new edition of the previous pamphlet appeared, to which was added *Confessio Fraternitatis* (meaning *The Confession of the Fraternity*), giving great promises about future revelations. The following year saw the publication of a new Rosicrucian manifesto entitled the *Chymical Wedding of Christian Rosenkreutz*, whose authorship is attributed to Johann Valentin Andreae, a Lutheran theologian from Tübingen, and it describes the path to salvation through ecstasy and enlightenment. Andreae's work immediately attracted the attention of alchemists, theosophists, doctors, and astrologers, who founded Rosicrucian fraternities not only in Germany but also in France, Austria, England, and the Netherlands. In 1652, the Welsh mystical philosopher and alchemist Thomas Vaughan, who was writing under the pseudonym of Eugenius Philalethes, translated the aforementioned Rosicrucian pamphlets into English.

The *Fama Fraternitatis* starts as follows:

> Seeing the only Wise and Merciful God in these latter days hath poured out so richly his mercy and goodness to Mankind, whereby we do attain more and more to the perfect knowledge of his Son Jesus Christ and Nature, that justly we may boast of the happy time, wherein there is not only discovered unto us the half part of the World, which was heretofore unknown and hidden, but he hath also made manifest unto us many wonderful, and never heretofore seen, Works and Creatures of Nature, and moreover hath raised men endued with great Wisdom, which might partly renew and reduce all Arts (in this our Age spotted and imperfect) to perfection; so that finally Man might thereby understand his own Nobleness and Worth, and why he is called *Microcosmus*, and how far his knowledge extendeth in Nature.

The *Fama Fraternitatis* announced the existence of a discreet fraternity founded in the late Middle Ages by a German nobleman, the "pious, spiritual, and highly-illuminated Father" Fr. C.R. (Christian Rosenkreutz). According to the *Fama*, dissatisfied with monastic life, the young C.R. travelled with a Father P.A.L. to the East in search of knowledge, and he reached Cyprus, where his friend died in 1393. Brother C.R. then went on

to Damascus, where he conducted further studies, and subsequently went to Egypt, where he remained for a long time. He journeyed along the Mediterranean Sea and visited Fez, where he studied the accumulated scientific and philosophical knowledge as well as the magic—that is, the alchemical treatises—of the Arabs.

In the study of alchemy, practice and experiment are necessary, thus paving the road to modern natural science. These need to be preceded by theoretical knowledge, which constitutes the philosophical or spiritual aspect of alchemy. In general, alchemy has two aspects: the material and the spiritual. The argument that alchemy was merely a primitive form of chemistry is untenable by anyone who is familiar with works written by its chief adepts. Additionally, the argument that alchemy is only a set of philosophical and theological teachings and that the alchemists' chemical references are only allegories is equally untenable by anyone who is familiar with the history of alchemy, since many of alchemy's most prominent adepts have made significant contributions to chemistry, and they have not been notable as teachers either of philosophy or of theology. In Antoine-Joseph Pernety's *Dictionnaire Mytho-Hermétique* (Paris: Delalain, 1758), one can find a very important explanation of alchemical terms upon the material plane.

In fact, by the late Middle Ages, the Arabs and the Persians had already created big libraries, and had collected and translated many ancient Greek philosophical and scientific books. In the eighth century, during the reign of the Abbasid caliphs al-Mansur and Harun al-Rashid, many works of ancient Greek philosophers and scientists, such as Aristotle, Hippocrates, Galen of Pergamon, Ptolemy, and Euclid, were translated into Arabic by such prominent translators and scholars as Ibn Bakhtishu, Theodore Abu Qurrah, al-Bitriq, and his son Yahya. In the ninth century, the Abbasid caliph al-Mamun, son of al-Mahdi, supported the study of ancient Greek philosophy and science and was an advocate of philosophical freethinking. Moreover, in Baghdad, caliph Harun al-Rashid founded an intellectual center called the "House of Wisdom" (in Arabic, Bayt al-Hikma), which was generously sponsored and developed further by caliph al-Mamun.

Finally, Brother C.R. crossed over into Spain, where he learned the Jewish Kabbalah and the philosophy of the Moors. It is worth pointing out that the Moors were the first great improvers of natural sciences in medieval Western Europe, since they had acquired their sciences from the Greeks through the Byzantine Empire, and they created renowned universities in Spain, such as those of Seville, Cordova, and Granada. During his journey in search of wisdom, Brother C.R. observed how the

Eastern sages shared their knowledge without jealousy, hypocrisy, or egoism.

In 1402, Brother C.R. returned to Germany, and he settled down to codify the vast amount of knowledge that he had collected. However, in Germany, he found only hostility and indifference. Thus, in a German place now unknown, Brother C.R. established a fraternity of sympathetic brethren: the Fratres R.C., who met annually in secret in the "House of the Holy Spirit," as they called their assembly, with the aims of advancing science and an esoteric approach to religion, and of furthering a deep reformation of knowledge.

Six ordinances were laid down to govern the original members of the Rosicrucian Fraternity in the conduct of their lives:

1. None of them should profess any great powers, knowledge, or authority to the outer world, but they should do good and heal the poor freely.
2. No peculiar habit should be worn when out in the world to make them conspicuous or liable to persecution.
3. On at least one day in every year all Fratres should assemble to record their work and communicate to each other their gains of knowledge.
4. Every Frater should seek one or more suitable persons to succeed him.
5. "R.C." or "C.R." should be their seal, mark, and character.
6. The Fraternity should remain a secret or private one, at least for a hundred years, if not longer.

Around 1450, the eldest Fratres or Magistri designed and executed a funeral Chamber in which Brother C.R. should be buried when his end came. Brother C.R. died in 1484, and he was entombed. On the door of the vault, the inscription "Post centum viginti annos patebo" was engraved on a brazen plate, meaning that the vault should be opened after 120 years. This long period passed by, and, even though Roman Catholicism had been fiercely attacked by the Religious Reformation, the work of the Rosicrucians had continued in peace and secrecy. In 1604, the Fratres then forming the central group of the Rosicrucian Fraternity opened the door of the secret chamber and entered the vault, where the Founder's body was lying in perfect condition, clothed in the symbolic robe and the insignia of his office of Magus (Head) of the Fraternity. Also stored here were the original books and achievements of the earliest members.

Through the aforementioned symbols and allegories, the original Rosicrucian manifestos (the *Fama* and the *Confessio*) are concerned with the following goal: the union of religion and science in the context of a

spiritual quest for the real truth of both the visible and the invisible things. This is the context in which Rosicrucians study systems of Eastern mysticism and delve into the natural sciences. In other words, the Rosicrucians promote the development of science in general while simultaneously cultivating an esoteric approach to religion that ultimately seeks to transform religion into spirituality. The unifying principle that pervades the entire spectrum of the Rosicrucians' endeavors is their longing for communion with the one, ultimate source of the meaning of all beings and things, visible and invisible.

The Rosicrucian ethos underpins the "Invisible College," a fraternity of scientists and freethinkers that was formed in England during Oliver Cromwell's dictatorship.

In the 1650s, Oliver Cromwell—an English Parliamentarian, military leader, and Puritan who was elected Member of Parliament for Huntingdon in 1628 and for Cambridge in the Short (1640) and Long (1640–49) Parliaments—revolted against the British Monarch King Charles I of the House of Stuart, and he established the most radical dictatorship in Europe. Cromwell was one of the signatories of King Charles I's death warrant in 1649. He was a fervent Puritan and an advocate of a model of liberal oligarchy, with which he tried to replace the Stuarts' model of centralized monarchy.

Cromwell's attempt to liberalize the British regime, reinforce the power of the British Parliament vis-à-vis the British Monarchy, and combat the extravagance and the corruption of the British nobility was, in principle, a just and humanistic project. However, Cromwell failed to understand the deep contradictions of the British society, and to bring about radical changes (especially regarding the material needs of the population). Thus, the movement that supported the Parliament of England was ultimately dominated by wealthy merchants and sympathetic landowners who wanted a compromise with the monarchy. More specifically, it was essentially a dictatorship of an oligarchy of the landowners and the rising bourgeoisie.

Not only did Cromwell impose a fierce dictatorial regime on the British Isles, he also attempted to substitute his Puritan doctrine for the Anglican Church and to control the British intellectual life. Thus, in 1650, Cromwell proclaimed and appointed himself Chancellor of the University of Oxford. However, in 1648 at Oxford, Dr. John Wilkins, who was an Anglican priest, the author of *Mathematical Magic*, a student of alchemy, and the Warden of Wadham College, assembled a group of prominent scientific researchers, such as chemist Robert Boyle, physicist and astronomer Robert Hooke, anatomist William Petty, and young future

architect Christopher Wren. Cromwell and his Puritan regime were not sympathetic towards esoteric studies, astronomy, and mathematics. Thus, Wilkins's group became a secret scholarly society known as the "Invisible College," which was inspired by Rosicrucianism and modelled after the fraternity of the Rosicrucians that is described in the aforementioned Rosicrucian manifestos.

After Cromwell's death in 1658, the monarchy was gradually restored. King Charles II, the eldest surviving son of Charles I, ascended to the British throne on 29 May 1660. He was a keen supporter of the new generation of English scientists. In 1659, Christopher Wren was appointed Professor of Astronomy at Gresham College in Bishopsgate, and his rooms became the principal meeting place of the Invisible College. In 1660, several members of the Invisible College, including Wilkins, Wren, Boyle, and Ashmole, participated in the formation of the Royal Society for the advancement of science. In 1662, King Charles II granted a royal charter to the Royal Society.

V.M.: Explain the Hermetic principle "as above, so below; as below, so above."

Orator: A century after Alexander the Great conquered Egypt and founded his city, Alexandria, in 331 B.C.E., Greek settlers in Alexandria had begun to apply the epithet "megistos kai megistos theos megas" (meaning "greatest and greatest the great god") to Hermes. This dignity derives from the epithet "two times great," which Egyptians had applied to Hermes's Egyptian equivalent, the god Thoth. The Greco-Egyptian Thoth-Hermes was the spirit of inventiveness. Sometime between the first century B.C.E. and the end of the first century C.E., a new figure appeared: "Hermes Trismegistus" (meaning "Thrice Greatest Hermes"), a name with which Greek settlers in Egypt unified the Greek god Hermes and the Egyptian god Thoth, since both were associated with magical knowledge, the dead, and healing. According to the Hermeticists' legends, the *Hermetica* was a collection of forty-two books of Greco-Egyptian magical wisdom that were written by Hermes Trismegistus, who was believed to be an ancient patriarch of civilization.

The Hermetic principle "as above, so below; as below, so above" refers to the interplay between spirit and matter, as well as to the interplay between thought and form. In the language of alchemy, this principle is related to and represented by "distillation." Distillation is the process of separating the components or substances from a liquid mixture by using selective boiling and condensation (for instance, this is the method by which brandy and other "spirits" are produced). The alchemists refer to the gas that is let off during the phase of vaporization as the "spirit" (representing our

thoughts and emotions), and they refer to the fixed matter that is produced during the phase of condensation as the "body." By analogy, nature has its own distillery: the heat of the Sun evaporates the water of the Earth, the water (moisture) goes up into the clouds, and then it rains. From the perspective of alchemy, distillation and the Hermetic principle "as above, so below; as below, so above" mean that we constantly create and manifest our world, and so magic is the power of consciously controlling what we send out.

From the perspective of the Hermetic Art, or Alchemy—and, specifically, according to the *Libellus II* of the *Hermetica*—we can interpret the Trinity formula Father–Son–Spirit in terms of the Trinity formula Space–Place–Motion.

V.M.: The most ancient known alchemist is the Greek alchemist Zosimos of Panopolis, who flourished in the Greco-Roman Egypt in the late third century C.E. and in the early fourth century C.E. The tenth-century C.E. Byzantine encyclopedic dictionary *Souda* writes that "chemeia," from which alchemy (in Greek, "alchemeia") is derived, is the art of making silver and gold (the two so-called perfect metals), thus emphasizing the practical aspect of alchemy. Byzantine scholars maintain that alchemy has two aspects: the material and the spiritual. For instance, the Byzantine rhetorician Aineias Gazaios (fifth/sixth century C.E.), in his philosophical dialogue entitled *Theophrastus*, refers to the art of transmutation, by which it was sought to produce silver and gold from other, less precious metals. With regard to spiritual alchemy, Aineias Gazaios delineates transmutation as an art that leads to the spiritualization of matter and the salvation of the psychosomatic nexus of the human being. In the Middle Ages and in the Renaissance, both material and spiritual alchemy expanded throughout the Islamic world and Western Europe, and various alchemical"schools" were developed by Arab, Persian, and Western European alchemists, suchas: Al-Farabi (born in Damascus, in the ninth century C.E.), Ibn Umayl (a tenth-century C.E. Arab alchemist who amalgamated Greek alchemy with Islamic spirituality), Al-Tughra'I (a Persian alchemist born in Isfahan, in the eleventh century C.E.), Roger Bacon (a thirteenth-century English philosopher and Franciscan friar), the famous Swiss physician Paracelsus (born Theophrastus von Hohenheim, 1493–1541), to name a few. One of the major alchemical principles is "Solve et Coagula," meaning "volatilize and fix," or, more specifically, dissolve and coagulate. Explain the stages of the alchemical work.

Orator: The stages of the alchemical work are the following:

1) *Calcination:* chemically, it involves heating a substance over an open flame or in a crucible until it is reduced to ashes.

Psychologically, it signifies the breaking down of old imperfect forms, the overcoming of egotism, and the destruction of one's attachments to material possessions. The celestial body that corresponds to Calcination is Saturn. The element that corresponds to Calcination is Fire. The color that corresponds to Calcination is magenta.

2) *Dissolution:* chemically, it consists in dissolving the ashes from Calcination in water. Psychologically, it signifies a further breaking down of artificial psychological structures by total immersion in the unconscious. The celestial body that corresponds to Dissolution is Jupiter. The element that corresponds to Dissolution is water. The color that corresponds to Dissolution is light blue, the characteristic color of Symbolic Masonry.
3) *Separation:* chemically, it refers to the isolation of the components of Dissolution by filtration and, subsequently, to the discarding of any ungenuine or unworthy material. Psychologically, it signifies the elimination of inner impurities, the "rediscovery" of our essence, and the reclaiming of visionary "gold." The celestial body that corresponds to Separation is Mars. The element that corresponds to Separation is air. The color that corresponds to Separation is orange-red.
4) *Conjunction:* chemically, it consists of the recombination of the saved elements from Separation into a new substance. Psychologically, it signifies the union between the feminine psychological qualities with the masculine ones, as well as the recombination of the purified powers and aspects of our psyche into a more harmonious and adequately organized whole. The celestial body that corresponds to Conjunction is Venus. The element that corresponds to Conjunction is earth. The color that corresponds to Conjunction is green.
5) *Fermentation:* chemically, it refers to the growth of a ferment (bacteria) in organic solutions (for instance, the process of fermentation in winemaking turns grape juice into an alcoholic beverage, and, through fermentation, one can produce yogurt and cheese). At the symbolic level, Fermentation is preceded by the Putrefaction of the hermaphroditic "child" of Conjunction, resulting in its death and resurrection to a new level of being; and the stage of Fermentation signifies the empowerment of the product of Conjunction in order to ensure its survival. Psychologically, the process of Fermentation signifies the concentration of the mind to a high goal, intense prayer, desire for mystical union with the good-

in-itself, and the rousing of psychosomatic energy. The celestial body that corresponds to Fermentation is Mercury. The substance that corresponds to Fermentation is sulfur. The color that corresponds to Fermentation is turquoise.

6) *Distillation:* chemically, it consists in the boiling and condensation of the fermented solution to increase its purity (as it is the case for example in distilling wines into brandy). Psychologically, it signifies further purification of the self through introspection in order to free oneself from irrational passions and sentiments, and to ensure that no impurities derived from the "ego" or the "id" inhibit the completion of the alchemical process. The celestial body that corresponds to Distillation is the Moon. The substance that corresponds to Distillation is mercury. The color that corresponds to Distillation is dark blue.
7) *Coagulation:* chemically, it refers to the precipitation or sublimation of the purified Ferment from Distillation. Psychologically, it signifies a psychological state characterized by beauty, integrity, and incorruptibility. The celestial body that corresponds to Coagulation is the Sun. The substance that corresponds to Coagulation is salt. The color that corresponds to Coagulation is purple.

V.M.: Brother/Sister Orator, given that one of the major goals and principles of our Order is the development and promotion of science, please explain the difference between science and pseudoscience.

Orator: The main purpose of positive science is to explain observations. This definition helps one to tell science and pseudoscience apart. Scientific work comprises observations and explanation.

In science, by "explanation," we mean that we have a model, which is a simplified description of the object of scientific research—put simply, a segment of the real world—and this model allows scientists to make observation statements that agree with measurements. In other words, a scientific model is simpler than the collection of all the available data, but it underpins the formulation of statements about all the available data, and these statements agree with measurements. This is due to the fact that a scientific model captures certain patterns in the data, and patterns are simplifications.

By the term "pseudoscience," we mean theories that clearly contradict evidence, but their advocates believe in them, either by denying the available evidence or by denying the scientific method. However, what we call pseudoscience today may once have been considered scientific truth. For instance, a superstitious variety of astrology was once a respectable

scientific discipline. Moreover, theories accused of being pseudoscientific sometimes turn out to be scientifically valid. For instance, the argument that the continents that exist today on planet Earth broke apart from one large tectonic plate was considered to be pseudoscientific until evidence confirmed it.

In hindsight, pseudoscience is a byproduct of (specifically, the waste produced by) scientific work, but some people cling to pseudoscientific arguments for longer than is reasonable. However, in the history of science, pseudoscientific arguments have often played a positive role, because they have forced scientists to improve their methods in order to debunk pseudoscientific arguments. For instance, in the eighteenth century, single-blind trials were invented in order to debunk the practice of Mesmerism. At that time, scientists had already begun to study and apply electromagnetism, but many people were understandably mystified by the first batteries and electrically powered devices. Exploiting their confusion, Franz Mesmer, a German physician, claimed that he had discovered a very thin fluid that penetrated the entire universe, including the human body. When this fluid is blocked from flowing, Mesmer argued, the result is that people fall ill. According to Mesmer, it is possible to control the flow of this fluid and thus cure people, and he proposed a specific method, which has been named after him as Mesmerism.

Mesmerism is based on the hypothesis that the aforementioned cosmic fluid is magnetic and enters the body through poles. The North Pole was supposed to be on the head, and, from this point, the cosmic magnetic fluid, originally deriving from the stars, was supposed to have entered the body. The South Pole was supposed to be at the feet, and it was supposedly connected with the magnetic field of the Earth. According to Mesmer, the flow of this cosmic magnetic fluid could be unblocked by "magnetizing" people. In the late 1770s in Paris, as the Enlightenment was on the wane, Mesmer started having so many patients that he could not treat them individually. For this reason, he established a collective treatment known as the "baquet," which has been described by the historian Claude-Ann Lopez as follows:

> Thirty or more persons could be magnetized simultaneously around a covered tub, a case made of oak, about one foot high, filled with a layer of powdered glass and iron fillings, then with a number of "mesmerized" water-bottles, symmetrically arranged. The lid was pierced with holes through which passed jointed iron branches, to be held by the patients. In subdued light, absolutely silent, they sat in concentric circles, bound to one another by a cord. Then Mesmer, wearing a coat of lilac silk and carrying a long iron wand, walked up and down the crowd, touching the diseased

parts of the patients' bodies (Claude-Anne Lopez, "Franklin and Mesmer: An Encounter," *Yale Journal of Biology and Medicine*, vol. 66, 1993, p. 326).

After being "magnetized" by Mesmer, patients frequently reported feeling significantly better. Scientists of the time, including Benjamin Franklin and Antoine Lavoisier, set out to debunk Mesmer's claims. In particular, with the assistance of his associates, Franklin conducted the following experiment: he blindfolded a group of patients. Some of them were told that they would get a treatment, but they did not actually receive any treatment, whereas others were given a treatment without their knowledge. Finally, Franklin and his associates found that the supposed effects of Mesmerism were not related to the actual treatment, but to the belief of whether one received a treatment. In other words, the positive effects of Mesmerism were of a psychological rather than physical, nature.

In the case of Franklin's research methodology, the patients did not know whether they received an actual treatment, but those conducting the experiment did. Such experiments can be improved by randomly assigning people to one of the two groups so that neither the people leading the experiment nor those participating in it know who received an actual treatment. This method is known as a "double blind trial." In fact, this method was invented in order to debunk pseudoscientific arguments of a system of alternative medicine called homeopathy. Homeopathy was invented by a German physician called Samuel Hahnemann in the nineteenth century, and it is based on the hypothesis that diluting a substance makes it more effective in treating illness. In 1835, Friedrich Wilhelm von Hoven, a public health official in Nuremberg, got into a public dispute with Johann Jacob Reuter, who was a distinguished advocate of homeopathy. Reuter claimed that dissolving a single grain of salt in one hundred drops of water, and then diluting it thirty times by a factor of one hundred would produce "extraordinary sensations" if one drank it. With the assistance of his associates, Friedrich Wilhelm von Hoven conducted the following experiment: he prepared fifty samples of homeopathic salt-water following Reuter's recipe, and fifty samples of plain water. Today, we would call the plain-water samples a "placebo." The aforementioned one hundred samples were numbered and randomly assigned to trial participants by repeated shuffling. The assignments of bottles to patients were kept secret on a list in a sealed envelope, so that neither von Hoven nor the patients knew who got what. Fifty people participated in the aforementioned experiment. For three weeks, von Hoven collected reports from the study participants before finally opening the sealed envelope to see who had received what. It turned out that only

eight participants had experienced anything extraordinary—five of whom had received the homeopathic dilution, and three had received plain water. Hence, the effect was not statistically significant. It is worth mentioning that von Hoven was a member of the "Society of Truth-Loving Men," a society of rationalists and skeptics that was founded in the nineteenth century in order to counter quackery and fraud.

Lecture IV

V.M.: Brother/Sister Orator, give us further information about the structure and the role of the Grand Lodge of our Order.

Orator: The interests of the Order are managed by a body that is styled the Grand Lodge of the Order, and is hereinafter referred to as "The Grand Lodge." The Grand Lodge possesses the supreme superintending authority, and it has the exclusive and inherent power of enacting laws and regulations for the government of the Order, as well as altering, repealing, and abrogating them, always taking care that the Ritual and the fundamental ethos of the Order be preserved. In addition, the Grand Lodge has the exclusive and inherent power of investigating, regulating, and deciding all matters relative to the Order, particular Lodges, or individual Brothers and Sisters, and it may exercise this power either of itself or by a delegated authority that it may appoint at its discretion.

The Grand Master is accorded the prefix title "Most Venerable and Most Puissant Brother/Sister." The person, male or female, filling this office is elected every nine years by the Supreme Electoral College according to its by-laws. The Grand Master has the sole power of filling the Offices of the Grand Lodge through appointment, and has the power of removing any Grand Officer if he/she deems it right for the good of the Order. The regalia of the Grand Master (comprising an apron, a cap, a pair of gauntlets, a gold chain collar, a gold chain collar jewel, a breast jewel, and a scepter) are distinctly characterized by their purple color (Fig. 2-12). Purple is symbolically associated with imperial rule and dignity, because it is a combination of power and love, represented by the colors red and blue respectively. The Grand Master uses a scepter instead of a gavel. Said scepter is a purple rod measuring a total length of twelve inches, and has a gold Cross of Salem with crosslets attached to its top. The triple lobes at the end of each arm of the Great Cross symbolize the three alchemical elements of sulfur (representing the active principle), mercury (representing the passive principle), and salt (representing the synthesis between the two previous principles). The four major arms of the Great

Cross symbolize the four elements or states of matter: air/gas, fire/plasma, earth/solid, and water/liquid.

The regalia of the other Grand Lodge Officers comprise an apron, a cap, a pair of gauntlets, a collar, a gold collar jewel, and the Order's breast jewel, and they are distinctly characterized by dark blue, symbolizing high levels of honor, fidelity, friendship, and love. The gauntlets and the cap are dark blue, and they are gilt embroidered with the corresponding emblem of Office within two branches of gilt embroidered laurel. The apron is of white fabric and has a triangular flap, all bordered with a four-inch richly gilt embroidered ribbon of dark blue. It is fringed with gold, and the emblem of office is displayed in the center of the apron within two branches of gilt embroidered laurel. Moreover, there are two tassels of gold suspended by dark blue ribbons from beneath the flap. Finally, the apron carries three Taus or Levels of gold, placed as in the case of Venerable Masters. A four-inch, matching, richly gilt embroidered, dark blue collar is also worn, and from this hangs the jewel of office. The Deputy Grand Master and the Assistant Grand Master have the distinction of wearing their distinctive gold jewels of office suspended by gold chain collars, like the Grand Master.

The Grand Master's gold chain collar jewel comprises an extended Pair of Compasses over a Quadrant, with a Rayed Eye inscribed in an Equilateral Triangle at the center. It is surmounted by the inscription "Grand Master," and is enclosed within a wreath of laurel. The Deputy Grand Master's gold chain collar jewel is similar to that of the Grand Master, with the difference that it is surmounted by the inscription "Deputy Gr. Master." The Assistant Grand Master's gold chain collar jewel is also similar to that of the Grand Master, with the difference being that it is surmounted by the inscription "Assistant Gr. Master." The gold collar jewels of the other Grand Lodge Officers are similar to those of the corresponding Lodge Officers, with the difference that they are enclosed within a wreath of laurel.

The active members of the Grand Lodge are called Grand Officers, and they rank in the following order:

1. The Grand Master.
2. The Deputy Grand Master is accorded the prefix title "Very Venerable and Very Puissant Brother/Sister." In the absence of the Grand Master, the Deputy Grand Master will possess all the powers and privileges of the Grand Master. Should any vacancy occur in the Office of the Grand Master, the Deputy Grand Master will have power to perform the duties of the Office until the successor to that Office takes possession of the same. In case the

Office of the Grand Master falls vacant, the President of the Council of the Conservators will summon the Supreme Electoral College to elect the new Grand Master according to the Order's Constitution and the Supreme Electoral College's by-laws.

3. The Assistant Grand Master is accorded the prefix title "Very Venerable and Very Puissant Brother/Sister." In the absence of the Grand Master and the Deputy Grand Master, the Assistant Grand Master will possess all the powers and privileges of the Grand Master.
4. The Senior Grand Overseer is accorded the prefix title "Venerable and Puissant Brother/Sister."
5. The Junior Grand Overseer is accorded the prefix title "Venerable and Puissant Brother/Sister."
6. The Grand Orator is accorded the prefix title "Venerable and Puissant Brother/Sister."
7. The Grand Director of Ceremonies is accorded the prefix title "Venerable and Puissant Brother/Sister."
8. The Grand Secretary is accorded the prefix title "Venerable and Puissant Brother/Sister." The Grand Secretary will issue summonses for meetings of the Grand Lodge and its Special Committees, and attend and take minutes of their proceedings, receive the returns of the subordinate Lodges and organizations and enter them in the books of the Grand Lodge, receive the fees and contributions payable by Lodges or individual members to the Grand Lodge's Funds, and act according to the instructions of the Grand Master and the by-laws of the Grand Lodge. Moreover, he/she will transmit to the proper quarters the reports of the proceedings of the Communications of the Grand Lodge as well as all other documents that may be ordered to be transmitted, and lay them before the proper authority. The Grand Secretary may appoint clerks in order to assist him/her in the performance of his/her duties. The clerks on the Grand Secretary's staff must be Master Masons of at least three years' standing.
9. The Grand Treasurer is accorded the prefix title "Venerable and Puissant Brother/Sister." The Grand Treasurer is the Order's chief financial officer, and he/she acts according to the instructions of the Grand Master and the by-laws of the Grand Lodge. The Grand Treasurer may appoint clerks in order to assist him/her in the performance of his/her duties. The clerks on the Grand Treasurer's staff must be Master Masons of at least three years' standing.

10. The Grand Expert is accorded the prefix title “Venerable and Puissant Brother/Sister.”
11. The Grand Director of Music is accorded the prefix title “Venerable and Puissant Brother/Sister.”
12. The Assistant Grand Secretary is accorded the prefix title “Venerable and Puissant Brother/Sister.”
13. The Assistant Grand Director of Ceremonies is accorded the prefix title “Venerable and Puissant Brother/Sister.”
14. The Grand Inner Guard is accorded the prefix title “Venerable and Puissant Brother/Sister.”
15. The Grand Outer Guard is accorded the prefix title “Venerable and Puissant Brother/Sister.”

The Grand Lodge has the right to cancel or suspend the Charter of any subordinate Lodge or organization, and has the right to suspend or dissolve the membership of any person whenever the Grand Lodge deems such action consistent with the protection of the Order’s interest. Such decisions are made by the rule of majority, and by open ballot held among the members of the Supreme Disciplinary Board. The members of the Supreme Disciplinary Board are the Senior Grand Overseer (serving as the President of the Supreme Disciplinary Board), the Junior Grand Overseer (serving as the Vice President of the Supreme Disciplinary Board), and the Grand Orator (serving as the Secretary of the Supreme Disciplinary Board). In any case of dispute to be adjudicated by the Supreme Disciplinary Board, the Grand Inner Guard serves as the Prosecutor, and the Grand Outer Guard serves as the Interrogator. The Prosecutor and the Interrogator draw up the indictment, and they send it to the Secretary of the Supreme Disciplinary Board, which conducts the corresponding trial. The official acts of the Supreme Disciplinary Board are void and of no effect when disapproved of by the Grand Master.

The active members of the Grand Lodge have the right and the duty to provide by-laws for the Order and its subordinate organizations. Such decisions are made in plenary sessions of the active members of the Grand Lodge, by the rule of majority and by open ballot held among the active members of the Grand Lodge, and they are enforced only after approval by the Grand Master.

Every subordinate Lodge of the Order has the right to conduct its own affairs according to its own wishes and by-laws, provided that its acts and by-laws are not contrary to the letter or the spirit of the Ritual and the Constitution of the Order. However no by-laws will become operative until approved by the Grand Master.

V.M.: Brother/Sister Orator, give us further information about the structure and the role of the Council of the Regional Directorates of our Order.

Orator: In our Order, the term "Regional Directorate" refers to the governing body of a group of Lodges in a given area, usually a city, state, or country. In particular, each Regional Directorate of our Order consists of three Regional Grand Officers, specifically: the Regional Grand Master, who is the head of the corresponding Regional Directorate and therefore supervises the Venerable Masters of all the Lodges that belong to the corresponding Regional Directorate; the Deputy Regional Grand Master, who is the chief supervisor of the Senior and the Junior Overseers of all the Lodges that belong to the corresponding Regional Directorate; and the Assistant Regional Grand Master, who is the chief supervisor of the Secretaries of all the Lodges that belong to the corresponding Regional Directorate, and acts as the general secretary of the corresponding Regional Directorate. The Regional Grand Master, the Deputy Regional Grand Master, and the Assistant Regional Grand Master of each and every Regional Directorate are directly appointed by the Order's Grand Master; they serve at his/her discretion, they provide the Grand Master with information and advice, and they prepare reports to the Grand Lodge on the situation, problems, and development of the corresponding Regional Directorate. The plenary sessions of the Council of the Regional Directorates of our Order are chaired by the Order's Deputy Grand Master whenever the Grand Master is not present. Every active Regional Grand Officer of every Regional Directorate of our Order participates in every plenary session of the Council of the Regional Directorates of our Order.

All the Regional Grand Masters of the Order's Regional Directorates are accorded the prefix title "Very Illustrious Brother/Sister."All the Deputy Regional Grand Masters of the Order's Regional Directorates are accorded the prefix title "Illustrious Brother/Sister." All the Assistant Regional Grand Masters of the Order's Regional Directorates are accorded the prefix title "Illustrious Brother/Sister."

The regalia of the Regional Grand Officers comprise the following items: an apron, a sash, a pair of gauntlets, a cap, a collarette, a collarette jewel, and the Order's breast jewel. The apron is of white fabric and has a triangular flap, all bordered with a four-inch ribbon of dark blue. Moreover, there are two tassels of gold suspended by dark blue ribbons from beneath the flap, and the apron carries three Taus or Levels of gold placed as in the case of Venerable Masters. The corresponding emblem of office is displayed in the center of the apron as follows: the emblem of a Regional Grand Master is a dark blue Pentagram (Pentalpha) within the

dark blue circular inscription "Regional Gr. Master of . . .," and then follows the name of the corresponding Regional Directorate; the emblem of a Deputy Regional Grand Master is a dark blue Pentagram (Pentalpha) within the dark blue circular inscription "Deputy Regional Gr. Master of . . .," and then follows the name of the corresponding Regional Directorate; and the emblem of an Assistant Regional Grand Master is a dark blue Pentagram (Pentalpha) within the dark blue circular inscription "Assistant Regional Gr. Master of . . .," and then follows the name of the corresponding Regional Directorate. The cap, pair of gauntlets, sash, collarette, and collarette jewel of a Regional Grand Officer are all similar to those of a Venerable Master of a Lodge of our Order, with the difference being that a Regional Grand Officer's gauntlets, sash, collarette, and cap are dark blue, the gold triple tau at the front of the Regional Grand Officers' caps is inscribed in a gold circle, and the Regional Grand Officers' gauntlets are adorned with the emblems of their respective Offices embroidered in gold.

V.M.: Brother/Sister Orator, give us further information about the structure and the role of the Council of the Conservators of our Order.

Orator: The Council of the Conservators of our Order is composed of thirty-three members: a President appointed annually by the Order's Grand Master, twelve Past Grand Officers of the Order as specified by the Council of the Conservators' by-laws, and twenty Past Maters of the Order's Lodges as specified by the Council of the Conservators' by-laws. The Council of the Conservators is the custodian of the Order's founding documents; it is responsible for the historical continuity of the Order through the observance of the Order's Constitution and Rituals as well as through the preservation of the Order's founding principles and ideology, and it fulfills its role as the Order's intelligence agency and as a check on the Order's Grand Lodge. All the members of the Council of the Conservators are accorded the prefix title "Very Enlightened Brother/Sister," and they wear special regalia that are distinctive of this special Masonic Body. In particular, the regalia of the members of the Council of the Conservators comprise the following items: an apron, a sash, a pair of gauntlets, a cap, a collarette, a collarette jewel, and the Order's breast jewel. The apron is of white fabric, has a triangular orange-red flap, and is bordered with a four-inch ribbon of dark blue. The flap is fringed with gold, and in the center of the flap is a gold equilateral triangle having a gold triple tau therein. Moreover, there are two tassels of gold suspended by orange-red ribbons from beneath the flap. The cap, pair of gauntlets, sash, collarette, and collarette jewel of a Conservator are similar to those of a Venerable Master of a Lodge of our Order, with the

difference that a Conservator's gauntlets, sash, collarette, and cap are dark blue, the gold triple tau at the front of the Conservators' caps is inscribed in a gold equilateral triangle, and the Conservators' gauntlets are adorned with a gold equilateral triangle having a gold triple tau therein.

The President of the Council of the Conservators and ten more members of the Council of the Conservators constitute the eleven-member Supreme Electoral College, which is constituted and operates according to its own by-laws. The meetings of the Supreme Electoral College are very infrequent and normally only held for the election of the Grand Master or in order to fill a vacancy that may arise in this Body.

The President of the Council of the Conservators, who is also the *ex-officio* President of the Supreme Electoral College, in addition to the regalia indicating that he/she is a member of the Council of the Conservators, wears a garter round his/her left arm and a gold breast star on his/her left breast. The garter of the President of the Council of the Conservators is purple, edged with gold, and bears the gilt embroidered words "Finis Coronat Opus," meaning "The End Crowns the Work." On the left breast, he/she wears a seven-pointed star featuring the coat of arms of the Order in the center. Moreover, the President of the Council of the Conservators carries a gold-mounted, flat-ended ceremonial baton whose shaft is covered in purple velvet ornamented with eleven gold five-pointed stars. The other ten members of the Supreme Electoral College, who are also members of the Council of the Conservators, in addition to the regalia indicating that they are members of the Council of the Conservators, wear a breast star, identical to that of the President of the Council of the Conservators, on the left breast, and a garter, similar to that of the President of the Council of the Conservators, round their left arm, with the difference that their garter is dark blue instead of purple.

Our Order can be likened to a Pyramid with a Double Capstone: the Council of the Conservators of our Order is the Inner Capstone that stands protected within the Outer Capstone, which is the Order's Grand Lodge. If the Grand Master should fail to perform his/her duties rationally in accordance with the Order's Constitution and Ritual and render himself/herself unworthy of the obedience of the Lodges, he/she shall be subjected to some new regulation, which will be formulated by the Council of the Conservators, according to its own by-laws. It will be dictated by the occasion, because the Order, being a private exclusive fraternity, has no reason to provide for an event that it deems improbable, if not inconceivable. Moreover, the Council of the Conservators manages our Order's research laboratories, think-tank, and inner special committees. Finally, I would like to inform you that, in our Order, there

are mysteries within mysteries, in the sense that there are an exoteric initiation and an esoteric one. Therefore, we perform formal labors consisting in publishable symbolic and didactic ceremonies as well as deeper philosophical, scientific, political, and occult labors sealed with deep secrecy. The Path can only be found in silence and secrecy.

V.M.: Brother/Sister Orator, given that our Order has a peculiar chivalric ethos, explain the origins of chivalry.

Orator: The word chivalry is developed from the image of a warrior on the horseback. The literal meaning is horsemanship, from Latin *caballus* (*cheval* in French). Chivalry, referring to the medieval institution of Knighthood, can be considered from three points of view: the military, the social, and the religious.

Historical evidence corroborates the argument that (Christian) chivalry began with the Roman Emperor Constantine I the Great in the fourth century. The Emperor faced a battle with Maxentius. On the 28th of October 312, just before the battle, he stated that he had a vision of a flaming cross in the heavens with an inscription *In Hoc Signo Vinces*, IHSV ("In this sign you will conquer"). Subsequently, he had a new imperial standard made in this design, and it was carried into the battle and guarded by fifty elite soldiers called the *Praepositi Laberorum*. In this way, he managed to win the fervent support of his Christian subjects, who were increasing in number, and this gave him a considerable advantage against his enemies. Emperor Constantine I was victorious under the aforementioned sign, and the first Christian Empire of Byzantium was founded. The fifty warriors were recognized into a sacred Knightly Guard, called *Torquati* (Golden Knights) and *Perfectissimi* (Most Perfect Knights)—thus was formed the first Christian Chivalric Order.

Flavius Valerius Aurelius Constantinus (ca.280–337 C.E.), commonly known as Constantine I and Constantine the Great among Roman Catholics, or Saint Constantine among Eastern Orthodox Christians, was a Roman emperor, proclaimed Augustus by his troops in 306, who ruled an ever-growing portion of the Roman Empire until his death. He is famous for being the first Christian Roman Emperor and for his Edict of Milan, which put an end to institutionalized persecution of Christians in the Roman Empire.

The Latin term *jus honorum* refers to the prerogative and the right of a sovereign and/or a spiritual authority (for instance, a Bishop, an Abbot of a Monastery, a Patriarch, etc.), authorized by the law and/or the custom to create and to grant the nobility, noble titles, the knighthood, coat of arms, and honorary decorations and distinctions of merit. The Latin term *fons honorum* (source, or fountain, of honors) refers specifically to the person,

assembly, and/or elected body entitled to the *potestas* (the coercive power of a magistrate), and/or to the public powers materialized by the political or spiritual sovereignty.

In 407 C.E., at the request of the North African Christian Church, the Western Roman Emperor Honorius instituted a special type of forensic practitioner known as the *defensor ecclesiae*: a permanently Church-mandated professional advocate, representing the Church in legal cases and capable of protecting the population of the abuses of the powerful. Honorius's edict instructed the *defensores ecclesiae* ("defenders of the Church") to inform high-ranking officials of the Roman provinces when the rights of the local Church were being violated. After 407, the privilege granted to the North African bishops was extended to other local Churches. For instance, Pope Innocent I (who was the Bishop of Rome from 401 to his death in 417) used a group of *defensores ecclesiae* to expel a group of "Photinian" heretics from the city of Rome.

By the end of the fourth century, in the Roman Empire, the Church was using the secular apparatus of the *imperium* (empire) to bring Pagans and Christian sectarians into the formally instituted Church. Augustine, Bishop of Hippo (the ancient name of the modern city of Annaba, in Algeria), played a major role in the articulation of the Christian political thought. In his early fifth century book, *The City of God*, Augustine merged the Roman legal scholarship in which he had received his initial training and Christian theology, to which he had been converted, thus providing the definitive justification of the Christian Republic.

In order to understand the Christian political thought, we must bear in mind the meaning of the Latin terms *sacerdotium* and *imperium* or *regnum*. The term *sacerdotium* means both the institution of the Church and its particular, higher (or teleological) moral responsibility. The term *imperium* or *regnum* (according to whether one is referring to an empire or a kingdom) means both the institution of the civil authority and its particular functions, which primarily included the maintenance of order in society and the protection of society from external enemies. The synthesis between these two powers conveys the sense of how Christian theorists conceived the nature of good government. In particular, their conception of good government was sometimes referred to by use of the metaphor of the Two Swords expounded in 494 by Pope Gelasius I.

In the early ninth century, Charlemagne (who was King of the Franks from 768, King of the Lombards from 774, and was proclaimed Holy Roman Emperor in 800, even though the Emperor of Byzantium, based in Constantinople, continued to retain the title of Roman Emperor) challenged the authority of the *sacerdotium*. In particular, he claimed the

status of direct appointment by God, without need of endorsement by the Pope of Rome. On the basis of this claim, he appointed bishops by his, rather than papal, authority, and he required these bishops to take on the sorts of duties that were demanded of secular vassals. Charlemagne's innovation confronted churchmen with the dilemma of having to choose whether to primarily support the secular ruler, within whose territory they were located geographically, or the authority of the Pontiff, who claimed their spiritual allegiance.

Pope Gregory VII, who was the head of the Roman Catholic Church and the ruler of the Papal States from 1073 to his death in 1085, tried with determination to reverse the aforementioned trend. In particular, he explicitly prohibited the lay investiture of bishops. His ruling was immediately challenged by the German King Henry IV, who bore the title of the Holy Roman Emperor. Emperor Henry IV attempted to depose Pope Gregory VII, who, in turn, responded by excommunicating Emperor Henry IV and absolving the emperor's vassals from their feudal oaths of loyalty.

However, from the eleventh century, a slow trend developed in Europe towards the strengthening of the feudal states, and the development of autonomous elites within them. The members of those elites were trained in both civil and canon law, and they drew confidence from their classical and humanistic studies to challenge the Pope's authority to pronounce in the sphere of temporal jurisdiction (the members of those elites who were most susceptible of criticism of papal authority belonged to the lower nobility and the bourgeoisie). The development and empowerment of the bourgeoisie contributed to the strengthening of the aforementioned trend.

Beyond the above-mentioned power rivalries at the elite level, the coexistence and interaction between the powers of the *regnum* and the powers of the *sacerdotium* in any European society remained unchanged for the *popolo* (the ordinary people). It was only through slow steps that Christian subjects lost their sense of a dual allegiance, and, when that happened, civil rulers realized that they could keep power without having to defer to papal authority. For instance, King Henry VIII of England established a state Church by his Act of Supremacy of 1534, independent of the Pope of Rome. In doing so, he gave a formal expression to a changed balance of power between civil and ecclesiastical authority that already existed in practice, and he emerged as one of the pioneers of British imperialism against the universalism of the Church of Rome.

The Protestant Reformation marks the period in which the doctrine of the Two Swords was replaced by the doctrine of the Sovereign State. Martin Luther (1483–1546), the leader of the Protestant Reformation, who once

said "peace if possible, truth at all costs," acted not only as a theological reformer of ecclesiastical life, but also as a highly cultured representative of German nationalism against the universalism of the Church of Rome. Hence the watchwords of the Protestant Reformation: salvation "by faith alone," and "Christian liberty" (the title of one of Luther's most famous pamphlets, 1520). According to Lutheranism, the only indispensable "outward" prerequisite of true religion is the preaching of the Word of God, which anyone must judge for oneself. The importance of any other "external" agencies, arrangements, and matters should be examined and judged only in so far as they impede or promote the Word.

The first military-monastic Orders were Crusading Orders. Jonathan Riley-Smith, in his book entitled *The Crusades: A History* (third edition, London: Bloomsbury, 2014), explains that "to crusade" meant to engage in a "holy war," since it was believed to be waged on God's behalf, as well as in a "penitential war," since those taking part believed that they were performing an act of penance. In particular, the first military-monastic Orders of knighthood were created in the Middle East by private initiatives as Christian religious societies of knights (subsequently officially recognized by the Holy See), in order to aid Christians on their pilgrimages to the Holy Land and/or to establish and manage hospitals, and they were the following:

- **The Order of the Holy Sepulchre** (Militi Sancti Sepulchri). It was founded in ca. 1099–1103 by Godfrey of Bouillon, a French nobleman and preeminent leader of the First Crusade. In 1103, it was recognized by King Baldwin I of Jerusalem, and, in 1113, it was recognized by Pope Paschal II. Since 1931, it has been known as the Equestrian Order of the Holy Sepulchre of Jerusalem. The characteristic symbol of this Order is a red Jerusalem Cross, which is made up of five Greek Crosses said to symbolize the five wounds of Christ and/or the four Gospels and the four corners of the earth (the four smaller crosses), as well as Christ Himself (the large cross). The Latin Patriarch of Jerusalem is the Grand Prior of the Equestrian Order of the Holy Sepulchre. The headquarters of this Order is located at Palazzo Della Rovere, and its official Church is the Church of Sant'Onofrio al Gianicolo, both in Rome. The Order creates "canons" as well as "knights" with the primary mission to "support the Christian presence in the Holy Land."

Figure 2-6: The red Jerusalem Cross of the Knights of the Holy Sepulchre.

- **The Order of Knights of the Hospital of Saint John of Jerusalem** (Ordo Fratrum Hospitalis Sancti Ioannis Hierosolymitani), commonly known as the Knights Hospitaller. It was founded in ca. 1099–1113 by Gérard de Martigues, a lay brother in the Benedictine Order who was appointed as rector of the Hospice in Jerusalem at Muristan (in the Old City of Jerusalem) in 1080. In 1113, it was recognized by Pope Paschal II. Officially it still remains a Christian Order, with a Catholic successor known as the Sovereign Military Hospitaller Order of Saint John of Jerusalem, of Rhodes and of Malta (or simply as the Sovereign Military Order of Malta); and a Protestant successor, known as the Most Venerable Order of the Hospital of Saint John of Jerusalem (established in 1888 by Queen Victoria of Great Britain); and both of them recognize each other. Obviously, the Sovereign of the Sovereign Military Hospitaller Order of Saint John of Jerusalem, of Rhodes and of Malta is the Pope of Rome, whereas the Sovereign of the Most Venerable Order of the Hospital of Saint John of Jerusalem is the British Monarch. Moreover, the historic Crusading Order of Saint John (or Malta) has inspired the fabrication of several Masonic chivalric degrees and Orders. The characteristic symbol of the Order of Knights of the Hospital of Saint John of Jerusalem is a white Maltese Cross (a white eight-pointed cross on a black background). Its four arms represent the cross on which Jesus suffered, while the eight points symbolise the Beatitudes given on the Sermon on the Mount (Matthew 5:3–10).

Figure 2-7: The white Eight-Pointed ("Maltese") Cross of the Knights of Saint John.

- **The Order of the Poor Fellow-Soldiers of Christ and of the Temple of Solomon** (Pauperes Commilitones Christi Templique Salomonici), also known as the Order of Solomon's Temple, the Knights Templar, or simply the Templars. This Order was founded in ca. 1118 by Hugues de Payens and the Cistercian abbot Bernard of Clairvaux. In 1128, Bernard of Clairvaux attended the Council of Troyes, at which he traced the outlines of the Rule of the Knights Templar. In 1129, the Order of the Knights Templar was recognized by Pope Honorius II, and, in 1312, through controversial procedures, it was abolished by the papal bull *Vox in excelso* issued by Pope Clement V, under pressure from King Philip IV of France. The property of the Templars was transferred to the Knights Hospitaller, except in the Kingdoms of Castile, Aragon, and Portugal. In 1317, King Dinis I of Portugal created the Order of Christ for those Templars who survived their trials throughout Europe. Thus, he reconstituted the Order of the Knights Templar, albeit under a different name, after it was dissolved by the Pope. Moreover, the historic Crusading Order of the Templars has inspired the fabrication of several Masonic chivalric degrees and Orders. The characteristic symbol of the Templars is a red Cross Pattée (i.e., a type of cross with arms that are narrow at the center, being broader at the perimeter) on a white background; and it is an emblem of honor and dignity.

Figure 2-8: The red Cross Pattée of the Knights Templar.

- **The Order of Saint Lazarus of Jerusalem**, also known as the Leper Brothers of Jerusalem, or simply as Lazarists. It was founded in ca. 1118 at a leper hospital in the Latin Kingdom of Jerusalem, named after its patron saint, Lazarus. In 1142, it was recognized by King Fulk of Jerusalem, and was recognized by Pope Alexander IV in 1255. In 1572, the Italian branch of the Lazarists merged with the Order of Saint Maurice (a Roman Catholic Order of knighthood bestowed by the royal house of Savoy) to form the Order of Saints Maurice and Lazarus (Ordine dei Santi Maurizio e Lazzaro). In 1609, King Henry IV of France administratively linked the French branch of the Order of Saint Lazarus to the Order of Our Lady of Mount Carmel (founded in 1608 by Pope Paul V at the request of King Henry IV of France) to form the Royal Military and Hospitaller Order of Our Lady of Mount Carmel and Saint Lazarus of Jerusalem (united), which remained listed as of royal protection in the French Royal Almanac until 1830. The characteristic symbol of the Lazarists is a green Maltese (i.e., Eight-Pointed) Cross.

  Figure 2-9: The green Eight-Pointed Cross of the Knights of Saint Lazarus.

- **The Order of Brothers of the German House of Saint Mary in Jerusalem, commonly known as the Teutonic Order**. It was founded in ca. 1190 in Acre, in the Latin Kingdom of Jerusalem. In 1929, the main stern of the Teutonic Knights converted into a purely Catholic religious Order. However, during the Protestant Reformation, most of the members of the Bailiwick of Utrecht of the Teutonic Order became Protestant, cut their ties with the Teutonic Order that was based in the Holy Roman Empire, and placed themselves under the protection of the United Provinces of the Netherlands, thus forming a Protestant chivalric Order. The characteristic symbol of the Teutonic Knights is a black Cross (sometimes Cross Pattée) on a white background.

Figure 2-10: The black Cross of the Teutonic Knights.

Modelled on the chivalric Orders established in the Latin Kingdom of Jerusalem, various chivalric (specifically, military-monastic) Orders were created throughout medieval Europe. However, in the thirteenth and the fourteenth centuries, the institution of the Monarchy dominated over the institution of the Church, and, in combination with the gradual decline of the feudal system, the processes were set in motion which marked the dominance of the modern European institution of the nation-state in the nineteenth century. The monarch emerged as that institution which could unite the clerics, the landed nobility, and the bourgeoisie together and integrate them into a stable social order. Thus, the monarchs created the so-called monarchical Orders of chivalry, such as the following: the Order of Saint George, founded in 1326 by King Charles I of Hungary; the Order of the Band, founded in 1332 by Alfonso XI, King of Castile; the Order of the Garter, founded in 1348 by King Edward III of England (it is the most senior Order of Knighthood in the British honors system); the Order of the Golden Fleece, founded in 1430 by Philip the Good, Duke of Burgundy (with the absorption of the Burgundian lands into Habsburg Spain, the sovereignty of the Order passed to the Habsburg monarchs of Spain); the Order of the Star (French: Ordre de l'Étoile), founded in 1351 by King John II of France (the Order was inspired by Geoffroy de Charny, who was a theoretician of chivalry and a renowned knight who fought on the French side during the early years of the Hundred Years' War, which originated from disputed claims to the French throne between the English royal house of Plantagenet and the French royal House of Valois); and the Order of the Elephant, founded by King Christian I of Denmark during the fifteenth century (it was reformed in 1693 by King Christian V of Denmark, and it is the most senior Order of Knighthood in the Danish honors system).

V.M.: Brother/Sister Orator, explain the banner of the Grand Lodge of our Order and our Order's code of *esoteric* chivalry.

Orator: The banner of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry is a purple cloth on which is depicted a black crow standing atop a white human skull. The

human skull symbolizes the following: (i) sympathy for the sufferings of the people, (ii) death, (iii) lethal force, and (iv) victory over the enemy. The black crow symbolizes cleverness and intelligence, intellectual and physical cleansing, adaptability, teamwork, reciprocity, and transformation. In ancient Greek and Roman mythology, the crow was Apollo's sacred animal (his messenger and spy). Apollo was a god symbolizing rational awareness, discretion, truth, music, purification, and healing. He corresponds to the Kabbalistic principle of Geburah. The crow's black color symbolizes Apollo's anger. In ancient Hindu mythology (according to the *Panchatantra*), there is an ongoing fight between the Crows, which are presented as good, and the Owls, which are presented as evil, because a Crow prevented the other birds from irregularly electing an Owl as their king, since the divine eagle Garuda, who was the bird of the god Vishnu, was already their king, and to take another was a sin that would result in severe punishment by the gods. Due to its persuasive arguments, the Crow achieved the cancellation of the Owl's irregular coronation. In Hinduism, Garuda's link to Vishnu, the Hindu god who preserves the moral law and fights injustice, has made him an iconic symbol of royal duty and power.

In our Order, the Skull and Crow symbol teaches a code of *esoteric* chivalry that comprises the following virtues:

- *Strength*. Seek to excel in all your chivalric undertakings, and let shrewdness and justice guide your strength. A precondition of victory is a dynamic perception of life. For example, the ancient Greeks were a warlike nation and were distinguished for their fighting ethos, whose supreme reward is considered to be glory and loud. Thus, the Greeks were able to defeat the Persian Empire. A remarkable historical finding is that material superiority does not guarantee victory. In ancient times, the few Greeks crushed numerous barbarians, not once but many times. Therefore, the mind, ideals, and virtues of war prevail over the multitude of enemies. For this reason, by extension, we reject liberalism as an underpinning of capitalism, but we endorse a Kantian variety of liberalism, which makes clear the difference between our model of rational socialism (based on the modern Western subject) and those models of collectivism which do not signify a rational social hierarchy but are a consequence of people's immaturity and ignorance.
- *Justice*. "Justice is truth in action" (as Joseph Joubert argued in his *Pensées*, in 1842). Follow the path of right, keep yourself free of prejudice and the passion of egoism, use the sword of justice

rationally, and stand against the "banality of evil" (a term coined by political theorist Hannah Arendt). Moreover, in several religious texts, the passion for righteousness is intimately linked to the quest for God.

- *Loyalty.* Your loyalty to the A∴O∴M∴P∴R∴S∴M∴ should be uncompromised. Loyalty entails discipline, and there is an inextricable link between intelligence and discipline. A successful strategist is not necessarily smarter than the rest, but he/she is definitely more disciplined. It goes without saying that the role of leadership is very important. As it is briefly said, "a flock of sheep with a lion leader defeats a herd of lions with a sheep leader."
- *Defense.* Defend those who depend upon you for protection, never disappoint those who trust you, and resist adversary forces in a principled manner. We endorse Mahatma Gandhi's principle of "Satyagraha," according to which we should not resist in such a way that we appear to be seeking the same kind of power as that of our adversaries, and we should not imitate anything except the truth. In addition, we are aware that it is one thing to avoid resembling our adversaries, it is another thing to become fatalists and fall into depression. The right is not conquered by pleas, but by struggles. In the context of our secular esoteric chivalry, we identify ourselves as *defensores humanitatis* (defenders of humanity).
- *Courage.* Live not for living itself but for "living well." The difference between "living" and "living well" has been clarified by Plato and Aristotle: living well consists of activities caused by the rational mind in accordance with the principle of excellence. Moreover, an intelligent person, a fully developed *Homo sapiens*, is a bold risk-taker because, as Hanna Arendt has pointedly argued, "there are no dangerous thoughts; thinking itself is dangerous." Furthermore, a fully developed *Homo sapiens* is primarily a thinking being. Rationality is attributed to *Homo sapiens* in virtue of the ability to reason and act upon the consequences of deliberation. Intellectual laziness leads to immaturity. Moreover, cowardice and fear to use reason in all matters leads to immaturity. Confucius has wisely argued as follows: "faced with what is right, to leave it undone shows a lack of courage"; and "to see what is right, and not to do it, is want of courage or of principle."
- *Faith.* You must have an inner existential foundation, and you must fight nihilism and postmodernism. Faith does not consist in one's trust in something that is perceivable through the senses, but it

consists of an inner pursuit and discovery of a different world that is present in one's imagination and intuition; this encounter between the two worlds—namely, between a utopia and the actual world—takes place mysteriously within the mind of each person. The great occultist Arthur Edward Waite, in his book *The Mysteries of Magic* (second edition, London: Kegan Paul, Trench, Trübner and Co., 1897, p. 49), explained the role of faith in the context of magic as follows:

> Magic is the divinity of man achieved in union with faith; the true Magi are Men-Gods, in virtue of their intimate union with the divine principle. They are without fears and without desires, dominated by no falsehood, sharing no error, loving without illusion, suffering without impatience, reposing in the quietude of eternal thought. They lean on religion, but are not weighed down thereby; they know what it is, and also that it is necessary and eternal. For debased souls, religion is a yoke imposed through self-interest by the cowardice of fear and the follies of hope; for exalted souls, it is a force which originates from an intensified reliance on the love of humanity. A Magus cannot be ignorant, for magic implies superiority, mastership, majority, and majority signifies emancipation by knowledge.

- *Humility.* "What is the first business of philosophy? To part with self-conceit. For, it is impossible for anyone to begin to learn what he already thinks that he knows" (Epictetus, *Discourses*, 2.17, tr. Thomas W. Higginson). Additionally, humility is an important medicine against fanaticism and illusion of grandeur. Thus, there is an inextricable link between humility and intelligence.
- *Nobility.* Genuine nobility is the existential state or quality of being intellectually and morally good, and, from our Order's perspective, it is semantically equivalent to dignity. Pico della Mirandola, arguably the most prominent Italian scholar and philosopher of the Renaissance, in his 1486 *Oratio de hominis dignitate* (*Oration on the Dignity of Man*), argued that humans could ascend the "chain of being" (a hierarchical structure of all matter and life) through the exercise of their intellectual capacities, thus endorsing the dignity of human existence in this earthly life.
- *Generosity.* Genuine nobility is not manifested in what you own but in what you offer. In other words, nobility is manifested through generosity. Moreover, as the French philosopher Simone de Beauvoir has pointed out, "true generosity" signifies a shift from selfishness to giving yourself, feeling "as if it costs you nothing"—thus, it signifies a divine mode of being.

- *Frankness.* Seek to live as a genuine noble not in order to gain advantages that satisfy your selfish passions, but because you recognize that this is the healthiest and correct way of living. Furthermore, keep in mind that it is utopic to think that entrepreneurs, if left free to decide on every economic issue, will give humanity prosperity and happiness regardless of what the state does. Entrepreneurs are persons who act not only on the basis of reason but who also express their own emotions and selfish passions, and the business world is stained by several psychological illnesses. Among them are anxiety disorders, depression, bipolar disorder, paraphilias, personality disorders, psychotic disorders, to name a few. Thus, a form of enlightened leadership is necessary. Arthur Edward Waite, in his book *The Mysteries of Magic* (second edition, London: Kegan Paul, Trench, Trübner and Co., 1897, p. 68), wisely pointed out that "the man of genius differs from the dreamer and the madman in this only, that his creations are analogous to truth, while those of madmen and dreamers are lost reflections and wandering images."

Figure 2-11: The Skull and Crow symbol of the M∴P∴R∴S∴M∴

Figure 2-12: Dr. Nicolas Laos wearing his regalia as Grand Master and Sovereign Grand Commander of the Esoteric Initiatory Grand Lodge of Greece and Head of the Modern and Perfecting Rite of Symbolic Masonry.

CHAPTER 3

FELLOW CRAFT OF THE M∴P∴R∴S∴M∴

While the ceremony of acceptance into the First Degree of Freemasonry is called "Initiation," the ceremony of acceptance into the Second Degree of Freemasonry is called "Passing." Every candidate to be passed to the Second Degree of the Modern and Perfecting Rite of Symbolic Masonry must have been an Entered Apprentice for a minimum of three years (this qualification is obligatory and cannot be waived by dispensation), and other specific conditions of promotion (pertaining to the candidate's performance in the Order's inner activities and projects) are often insisted upon.

In the Second Degree, the Lodge has the same layout as in the First Degree, excepting: near the north side of the Senior Overseer's pedestal stands a big floor mirror facing West; and, on the Mosaic Pavement, near the west side of the kneeling stool stands the Symbolic Ladder (the Altar of Truth, the kneeling stool, and the Symbolic Ladder are aligned with each other; and there is a comfortable distance between the kneeling stool and the Symbolic Ladder). The Symbolic Ladder has two supports and seven steps. The first support is on the north side, and its seven ascending steps bear the following inscriptions (from bottom to top): Step 1: Linguistics and Literature, Step 2: Mathematics, Step 3: Physics and Chemistry, Step 4: Biology, Step 5: Politics and Economics, Step 6: Fine Arts, and Step 7: Philosophy. The second support is on the south side, and its seven descending steps bear the following inscriptions (from top to bottom): Step 7: Wisdom, Step 6: Bravery, Step 5: Prudence, Step 4: Justice, Step 3: Altruism, Step 2: Perseverance, and Step 1: Truthfulness.

The Secretary invites only the Fellow Crafts and the Master Masons to attend the meetings of the Lodge whenever the latter works in the Fellow Craft degree, and the Outer Guard ensures that only the members of the Second Degree and the Third Degree enter into a Lodge of Fellow Crafts (with the exception of a Candidate for Passing).

Before the Opening Ceremony, the Outer Guard waits outside the door of the Lodge.

The Candidate wears the apron of an Entered Apprentice with the triangular flap up, the sash of an Entered Apprentice, white gloves, and the Order's breast jewel. Prior to the ceremony, he/she waits in the anteroom of the Lodge.
Finally, before the Ceremony of Passing, the Lodge room should be discreetly scented by burning cedar incense sticks or essential oil.

Opening Ceremony

V.M.: Brother/Sister Senior Overseer and Brother/Sister Junior Overseer, please, confirm that all within the Lodge are Craftsmen.
S.O.: Brothers and Sisters, it is the Venerable Master's command that you prove yourselves Craftsmen.
All, except the V.M., the S.O., and the J.O., rise, take Step, give the Fellow Craft Sign and retain it. When this is done, the V.M., the S.O., and the J.O. rise, take Step, and give the Fellow Craft Sign. All cut the Fellow Craft Sign and resume their seats.
J.O.: Brother/Sister Inner Guard, act in accordance with the duties arising from your office.
The Inner Guard, holding his/her sword in his/her left hand, pointing upward, opens the door of the Lodge and instructs the Outer Guard (who was standing outside the door of the Lodge, guarding the porch of the Lodge) to take his/her seat in the Lodge. The Outer Guard, holding his/her sword in his/her right hand, pointing downward, enters the Lodge and takes his/her seat in it. The Inner Guard closes and locks the door of the Lodge, takes the key and puts it on the Junior Overseer's pedestal, and says to the Junior Overseer: "Brother/Sister Junior Overseer, we are internally and externally safe." When this is done, the Inner Guard (holding his/her sword in his/her left hand, pointing upward) takes his/her seat in the Lodge.
J.O.: The Lodge is working according to our rules of privacy and exclusivity.
S.O.: Venerable Master, the Lodge is working according to our rules of privacy and exclusivity.
At this point, the Inner Guard, who was holding his/her sword in his/her left hand, pointing upward, and the Outer Guard, who was holding his/her sword in his/her right hand, pointing downward, put their swords in their scabbards.
V.M.: Brothers and Sisters, working to the Glory of the Enlightened Humanity, I declare the Lodge duly open for the purposes of our Masonic Order in the Second Degree.

V.M. gives F.C.-gavel: five knocks given as follows: 3 short equal knocks, a short pause, and 2 long equal knocks.
S.O. gives F.C.-gavel.
J.O. gives F.C.-gavel.
The D.C. (holding his/her wand in the right hand) rearranges the Square and the Pair of Compasses that lay on the open Book of Constitutions so that one point of the Pair of Compasses is elevated above the Square, implying that we are now in the midway of Freemasonry, superior to an Entered Apprentice but inferior to the degree of a Master Mason. When this is completed, the D.C. resumes his/her seat. Thereupon, the Lodge works in accordance with its agenda.

Ceremony of Passing

V.M.: Brothers and Sisters, the purpose of this meeting is to pass Brother/Sister . . . *(name and surname of the Candidate)* to the degree of a Fellow Craft. Brother/Sister Director of Ceremonies, lead the Candidate in front of the door of the Lodge.
The I.G. takes the key from the Junior Overseer's pedestal, lets the D.C. leave the Lodge, closes and locks the door, and waits on the south side of the door. The D.C., holding his/her wand in the right hand and holding the Candidate's right hand in his/her left hand, instructs the Candidate to knock on the door of the Lodge as an Entered Apprentice (namely, with three equal distinct knocks). When this is done, the I.G. opens the door of the Lodge, and the following dialogue takes place aloud so that it can be heard in the East: The I.G. asks the D.C.: "Whom have you there?" The D.C. responds: "Brother/Sister . . . (name and surname of the Candidate), an Entered Apprentice who has made such progress in the Craft that he/she hopes will recommend him/her to be passed to the degree of a Fellow Craft." Then, the I.G. asks the D.C.: "Has his/her Master ratified his/her progress in the Craft?" The D.C. responds as follows: "We trust that his/her Master has ratified his/her progress in the Craft." Thereupon, the V.M. says: "Let Brother/Sister . . . (name and surname of the Candidate), step before the Supreme Judge." The I.G. lets the D.C. and the Candidate enter the Lodge, closes and locks the door, puts the key back on the Junior Overseer's pedestal, and resumes his/her seat. In the meantime, the D.C. leads the Candidate to stand before the big floor mirror near the north side of the Senior Overseer's pedestal. The D.C. instructs the Candidate to take Step (namely, to bring the right heel into the hollow of the left foot). In that position, the Candidate looks at

himself/herself in the mirror, and the D.C. continues to hold him/her by the right hand (they both face East).

V.M. (after a few moments of silence and as the Candidate continues to look at himself/herself in the mirror): This Masonic Order is explicitly and systematically concerned with the construction of the self, with the art of creating one's individual Lodge and the socialization of such Lodges in the context of a broader structure. By the term "structure," we mean an internal reality that is governed by its own order, which it creates and recreates by itself. In other words, a structure consists of the fundamental rules that govern the behavior and the relations of the members of a system. The structure of our system, as it is illustrated by our symbols, didactic dramas, and catechisms, and as it is represented by the Venerable Master and the Lodge Officers, is the symbolic big "other," whose role is to lead the dialectic of mirroring to its completion and perfection.

According to the great structural psychoanalyst Jacques Lacan, the child's initiation into the "mirror stage" entails a type of "libidinal dynamism," caused by the child's identification with one's image and by the creation of an "ideal ego." Through the mental process of mirroring, one recognizes himself or herself as "I" by viewing himself or herself as "other," as a person "over there." The "specular image" gives rise to the ego-as-object. The insertion of an idealized image between one and oneself gives rise to mixed feelings. On the one hand, it may give rise to a feeling of hatred, specifically when one hates that idealized version of himself or herself because he or she thinks that it is much better than him or her. On the other hand, that idealized image, and thus one's ideal self, may give rise to a feeling of love, in the sense that one may think that he or she wants to be like that image. Thus, at the "mirror stage," the subject enters an imaginary order.

In order for the subject not to be lost in the realm of misunderstanding and false recognition, the presence of an adult figure carrying the child is necessary in order to play the role of a symbolic big "other." Hence, initially, the subject jubilantly assumes his or her "ideal ego" as his or her own. However, he or she subsequently turns his or her head towards the symbolic big "other," the adult authority, as if to call on this adult authority to ratify this image.

The "mirror stage" starts during infancy. The feeling of impotence, especially between six and eighteen months of life, makes the child experience his or her body as fragmented. During this time, infants have their first experience of seeing themselves in the mirror. Lacan calls this the "specular image." When an infant sees his or her reflection for the first

time, he or she is fascinated by this "other self," because he or she sees his or her body as integrated, and projects a unified ego as something distinct from what he or she is. Through the identification with an idealized image, the infant enters a quest to achieve this idealized ego. Lacan does not consider the "mirror stage" merely as a moment in the life of an infant, but as representing a permanent structure of subjectivity. More specifically, he considers this a model of an imaginary order that is characterized by the following dialectical process: the real ego ("thesis") gives rise to its contradiction, the ideal ego ("antithesis"), and, finally, guided by the symbolic big "other," the real ego returns to itself enriched by its encounter with and contemplation of its ideal ego ("synthesis"). Through this dialectical process, one can overcome contradictions, improve oneself, and intensify the presence of oneself; and this dialectical process is part of the quintessence of our Masonic journey.

My Brother/Sister, in order to be able to work as a Craftsman, you must learn to ascend and descend the Symbolic Ladder. Therefore, you will make your first esoteric journey in order to find the Symbolic Ladder.

The Director of Music plays Richard Wagner's "Faust," Overture in D minor. The D.C., holding the Candidate by the right hand, walks through the Lodge (without stepping on the Mosaic Pavement), through the North, the East, the South, and back to the West. In this way, the D.C. leads the Candidate in front of the Symbolic Ladder in the West, and they stand on the floor of the Lodge outside the Mosaic Pavement, facing East. When the D.C. and the Candidate arrive there, the Director of Music stops playing music.

V.M.: The first esoteric journey is completed. The Symbolic Ladder, which is standing in front of you, has two supports and seven steps. It ascends from the realm of the senses and historical experience to the realm of pure thinking and abstract reasoning, and it descends from the realm of pure thinking and abstract reasoning into the realm of the senses and historical experience. The first support has seven ascending steps, which bear the following inscriptions from bottom to top: Step 1: Linguistics and Literature, Step 2: Mathematics, Step 3: Physics and Chemistry, Step 4: Biology, Step 5: Politics and Economics, Step 6: Fine Arts, and Step 7: Philosophy. The second support has seven descending steps, which bear the following inscriptions from top to bottom: Step 7: Wisdom, Step 6: Bravery, Step 5: Prudence, Step 4: Justice, Step 3: Altruism, Step 2: Perseverance, and Step 1: Truthfulness. The seven ascending steps represent the means by which we can explain and change the world. The seven descending steps represent the major virtues that should underpin

our intervention in the reality of the world, and enable us to contribute to the prosperity of the world and to work for the common good.
Brother/Sister . . . *(name and surname of the Candidate)*, make the second esoteric journey, now through the South to the East, and then return through the North to the West.
The Director of Music plays a continuation of Richard Wagner's "Faust," Overture in D minor. The D.C., holding the Candidate by the right hand, walks through the Lodge (without stepping on the Mosaic Pavement) counter-clockwise, through the South, the East, the North, and back to the West. In this way, the D.C. leads the Candidate in front of the Symbolic Ladder in the West, and they stand on the floor of the Lodge outside the Mosaic Pavement, facing East. When the D.C. and the Candidate arrive there, the Director of Music stops playing music.
V.M.: The second esoteric journey is completed. The purpose of this counter-clockwise journey is to symbolically teach that there are two general methods of philosophical and scientific inquiry: the *a priori* (meaning, "from the earlier") methods, and the *a posteriori* (meaning, "from the later") methods.
The major attribute of the *a priori* methods is that they are based on primitive hypotheses usually intuitively conceived and axiomatically accepted, which deductively give rise to series of syllogisms. These, in turn, lead to ultimate conclusions which are related to the preceding propositions in a logically rigorous way, even though it is often the case that big hypothetico-deductive systems have flaws. By being "axiomatically accepted," we mean that certain hypotheses are accepted without proof, on the basis of their intrinsic merit, or because they are regarded as self-evident**.** In summary, in a "hypothetico-deductive" (or "axiomatic") system, there are two requirements that must be met in order that we agree that a proof is correct: (i) acceptance of certain statements, called "axioms," without further justification; and (ii) agreement on how and when one statement "follows logically" from another—that is, agreement on certain rules of reasoning. Inextricably linked to the aforementioned two requirements is the requirement that every person who applies hypothetico-deductive reasoning to a particular discourse understands the meaning of the words and the symbols that are used in that discourse. The more consistent and the more complete a hypothetico-deductive system is, the more its imposition is safeguarded. By the term "consistency," we mean that the axioms of a hypothetico-deductive system neither contain nor produce contradictions. By the term "completeness," we mean that the truth value of any proposition that belongs to a hypothetico-deductive system can be determined within the given

hypothetico-deductive system (that is, according to the terms and the rules of the given hypothetico-deductive system).

During Antiquity, the first *a priori* philosophical methods were developed by the pre-Socratic philosophers, whose model focuses on the determination of a principle that was assumed to be the origin of the world and to give rise to every particular reality. Inherent in pre-Socratic philosophy is a form of dogmatic scientism, which was successfully refuted by Socrates and the sophists. The sophists (namely, such orators and professional educators as Protagoras, Gorgias, Antiphon, Hippias, Prodicus, and Thrasymachus) argued that it is reasonable to question the absolute validity of previous philosophical achievements. Socrates—through the "maieutic method," which he himself developed—sought to find a reliable method for obtaining truth. "Maieutic" is a Greek word literally meaning "of midwifery." Indeed, in the context of carefully structured conversations or dialogues, Socrates would ask probing questions that cumulatively revealed his interlocutors' unsupported assumptions and misconceptions, and so his method would "give birth" to truth by eliciting a clear and consistent formulation of a thesis that was supposedly implicitly known by all rational beings. Despite the philosophical controversies between Socrates and the sophists, the philosophies of both Socrates and the sophists indicate a major shift from philosophies focused on the world to philosophies focused on the human being.

In his early dialogues, Plato delineated Socrates's maieutic method combined with the practice of "Socratic irony," which is often condensed into the paradoxical statement "I know that I know nothing," which is attributed to Socrates, paraphrasing his statements in *Apology* and *Meno*. In particular, "Socratic irony" is a method of argumentation in which one pretends to be ignorant in order to expose the ignorance or the inconsistency of someone else through adequately posed questions. However, in his middle dialogues (for instance, in *Phaedo*, *Symposium*, *Republic*, and *Phaedro*), Plato developed his own method, which is known as Plato's "dialectic." Plato's dialectic consists of two mutually complementary, particular processes of inquiry: the "ascending" process of inquiry and the "descending" process of inquiry. According to the ascending process of inquiry, consciousness starts from sensible objects (where the source of belief is sense perception) and ascends to higher levels of conceptual knowledge, which is conversant with the ultimate realities. According to the descending process of inquiry, consciousness starts from the knowledge of the ultimate realities and descends to the different levels of application, or manifestation, of those ultimate realities

in the sensible world. In other words, through the ascending process of inquiry, the philosopher's consciousness proceeds from the phenomena to the ideas, which are participated by the phenomena and of which the phenomena are imitations, whereas, according to the descending process of inquiry, the philosopher's consciousness proceeds from the knowledge of ideas to the interpretation of phenomena.

Aristotle's methodology is similar to Plato's dialectic, and it also belongs to the category of *a priori* methods. In particular, Aristotle's philosophical methodology consists of determining a science of the "whole" being, and using this science in order to interpret every particular reality. There is a significant similarity between Aristotle's method and the geometric method, which is a style of proof that was used by Euclid in order to prove geometric theorems. In the sixteenth century, the Italian Aristotelian philosopher and logician Giacomo (or Jacopo) Zabarella described the geometric method as involving two aspects: (i) the resolutive aspect, also known as the analytic side of the geometric method, and (ii) the compositive aspect, also known as the synthetic side of the geometric method. In his *Posterior Analytics*, Aristotle combines rational primitivism and empirical primitivism. Firstly, Aristotle's rational primitivism (reflecting the mentality of the *a priori* methods) is expressed by his thesis that demonstrative understanding (namely, understanding based on the geometric method) necessarily proceeds from elements that are true, primitive, immediate, and more familiar than, prior to, and explanatory of the conclusions. Secondly, Aristotle's empirical primitivism (reflecting the mentality of the *a posteriori* methods) is expressed by his thesis that we must know the primitives (namely, axioms) by induction, since, according to Aristotle, induction is the way in which perception instills universals, and definitions are some of the most important elements of an axiomatic system that will be grasped by consciousness as a result of induction. In modern science, Gottfried Wilhelm Leibniz used the geometric method emphasizing rational primitivism, while Isaac Newton used the geometric method emphasizing empirical primitivism.

The ascending and the descending processes of inquiry that constitute Plato's dialectic were reversed by Plotinus and, generally, by Neoplatonism: when the human soul descends from the World Soul into a particular (material) body, ascent (the reversal of descent) necessitates that the descended soul generate love of the World Soul and of the higher dimensions of Nous and the One. The generation of love by the descended soul gives rise to and underpins philosophy (the love of wisdom). In this way, Plotinus and Neoplatonism, in general, articulated an apophatic

approach to a totally transcendent One, which accounts for the unity and the existence of both formal reality and the material instantiation of that reality.

Plotinus, arguably the most important Neoplatonist philosopher, utilized Plato's method of inserting two intermediate ontological terms into the initial Parmenidean perception of the antithesis between being and non-being, and he argued that we can successively contemplate both the emanation and the dialectical return of the four primary hypostases: the One, the Nous, the World Soul, and Matter. According to Plotinus, it is only the One—which is the origin of every other hypostasis—that is not susceptible to any methodic approach. However, Plotinus maintains that even the One is susceptible to knowledge, yet in an apophatic way (that is, through negating concepts that might be applied to it). In fact, Plotinus's method underpins apophatic theology.

During the Middle Ages, both the Platonic-Neoplatonic methodology and the Aristotelian methodology were used, and the Aristotelian methodology was endorsed and adjusted to the intellectual needs of medieval Christendom by Thomas Aquinas, the major representative of scholastic philosophy. Moreover, in the context of modern philosophy, Neoplatonism continues to play an important role, both due to the fact that Neoplatonism is based on a robust Platonic ontology, which can be discarded only if one is ready to totally negate the reality of the world, and due to the fact that Neoplatonism has given rise to several methods of overcoming the antitheses that characterize Platonic ontology (for instance, the antithesis between beingly beings and beingly non-beings). Thus, Neoplatonism has played an important yet implicit role in the development of modern dialectical philosophies, which, in turn, underpin the development of infinitesimal calculus by Newton and Leibniz.

In the seventeenth century, the British philosopher and statesman Francis Bacon systematized the empirical method (induction), which was originally developed by Italian scientists during the Renaissance. Bacon's method is based on a double empirical and rational standpoint. In his *Novum Organum Scientiarum*, induction implies ascending to axioms as well as descending to works so that, from axioms, new particulars are inferred—and, from these, new axioms. In fact, induction starts from sensory-sensuous data and moves through natural history (providing sensory-sensuous data as guarantees) to lower axioms or propositions, which derive from the tables of presentation or from the abstraction of notions. By the term "experience," Bacon does not refer to everyday experience, but he presupposes that his empirical method corrects and extends sensory-sensuous data into facts, which go together with his

setting up of tables (tables of presence and absence, for instance, as well as tables of comparison or of degrees, more specifically degrees of absence or presence). However, Bacon's empirical method does not end here, since Bacon assumes that, from lower axioms, more general ones can be inferred by induction. Moreover, from the more general axioms, Bacon strives to reach more fundamental laws of nature, which lead to practical deductions as new experiments or works.

Descartes understood the significance of Bacon's new scientific method, and he used it in order to criticize and overcome scholasticism (even though he was, to a large extent, intellectually molded by scholasticism). Thus, his intellectual weapons were mainly of Aristotelian origin. Descartes formulated the analytic geometric method. In his *Discourse*, Descartes showed how the arithmetic operations of addition, subtraction, multiplication, division, and the extraction of roots can be represented geometrically. In general, within the framework of analytic geometry, problems can be broken down into simpler problems involving the construction of individual straight lines, thus leading to an analytic approach to geometry. Hence, Descartes's *Geometry* is based on the use of algebra, which was called an "art of analysis."

The core of the study of structures in mathematics consists of taking numbers and putting them into equations in the form of "variables," and the rules for manipulating these equations are contained in algebra. By reducing geometric problems (problems about shapes and the manner in which they behave in spaces) to equivalent algebraic ones, Descartes made a major contribution to mathematics. Furthermore, Descartes's analytic geometry is also of great philosophical significance because, by reducing geometric problems to algebraic ones, Descartes managed to formulate a type of an *a priori* geometric philosophical method whose primary principle is not an object of the external world, but it is conscious experience itself. Intellectually, Descartes moves away from objects that are external to consciousness and turns his attention to conscious experience itself. Moreover, through the algebraic representation of geometric problems, he throws light on the structure of problem-solving in general.

However, the Dutch philosopher Baruch de Spinoza, one of the most important representatives of Cartesianism, attempted to apply the geometric method in such a way as to give rise to an extreme variety of logical formalism and a suffocating, rationalist worldview. In Spinoza's totally rationally organized universe, the only ways in which the human being can manifest humanity's freedom are murder, suicide, and madness. Descartes was much more careful than Spinoza because, in contrast to

Spinoza's formalist excesses, Descartes emphasized the importance of internal experience (intuition).

Kant's philosophy was the major underpinning of the second great philosophical shift from the world (from philosophical cosmology) to the human being (towards philosophical anthropology). In his *Prolegomena to Any Future Metaphysics that Could Come Forth as Science*, Kant famously admitted that he was influenced by the Scottish philosopher David Hume's empiricism, which was inextricably linked to skepticism (through which Hume attempted to deconstruct ordinary claims to knowledge). In general, it was formulated within a cultural milieu determined by British philosophers' elaborations of Bacon's method. Kant adopted a "critical" attitude towards an *a priori* method of philosophical research and an *a posteriori* one. Thus, his philosophy gives rise to two different philosophical methods, both of which have played important roles in modern philosophy. Firstly, Kant's philosophy gives rise to an "idealist" method which, according to the modern interpretation of the term "idealism," is founded on the principle that research can be proved only by internal experience (internal experience here meaning the empirical cognition of mental states, such as sensory perception, thinking, memory, imagination, feeling, will, and desire). In the context of modern philosophy, internal experience was originally exalted by Descartes, who did not, however, negate the objective extension of consciousness. Secondly, Kant's philosophy gives rise to a "positivist" method, according to which research can be proved only by empirical means (not argumentations), and research should be mostly deductive (in the sense that deduction is used to develop statements that can be empirically tested). From this viewpoint, knowledge should be judged by logic and ideally should be true for every segment of space-time, whereas every object that is directly related to a transcendent reality should be discarded, since Kant argues that humans cannot have theoretical knowledge of things-in-themselves. However, Kant maintains that humans can have practical knowledge of things-in-themselves. Thus, Kant's method is based on the distinction between the receptive faculty of sensibility and the active faculty of understanding: the former provides us with intuitions, whereas the latter is the source of concepts. Through the receptive faculty of sensibility, the objects are "given," whereas, through the active faculty of understanding, the objects become objects of "thought."

In the second edition of his *Critique of Pure Reason* (B, ix–x), Kant maintains that, "so far as reason is to be in these sciences," something within them must be a kind of *a priori* knowledge, and this *a priori* knowledge must be related to its object in two ways: either merely to

determine its object and its concept (which must be given from elsewhere), or also to make it actual; the former is "theoretical knowledge of reason," and the latter is "practical knowledge of reason." According to Kant, the goal of theoretical reason is to assess how things are, whereas practical reason decides how things ought to be and what persons should do. However, while practical reason decides what to do, it cannot remake reality in an arbitrary manner; instead, the successful practical agent must take account of truths about the world.

In his *Transcendental Aesthetic*, Kant refers to the followers of Newton's position as the "mathematical investigators" of nature, who contend that space and time "subsist" on their own. He refers to the followers of Leibniz's position as the "metaphysicians of nature," who think that space and time "inhere" in objects and their relations. At the ontological level, Kant's position is that space and time do not exist independently of human experience, but they are "forms of intuition" (specifically, conditions of perception imposed by human consciousness). In this way, he managed to reconcile Newton's and Leibniz's arguments: he agrees with Newton that space is an irrefutable reality for objects in experience (for the elements of the phenomenal world, which are the objects of scientific inquiry), but he also agrees with Leibniz that space is not an irrefutable reality in terms of things-in-themselves. At the epistemological level, unlike David Hume, Kant argues that the axioms of Euclidean geometry are not self-evident or true in any logically necessary way. For Kant, the axioms of Euclidean geometry are logically synthetic—that is, they may be denied without contradiction. Therefore, consistent non-Euclidean geometries are possible, as Nikolai Ivanovich Lobachevski and Bernhard Riemann actually accomplished. However, Kant argues that the axioms of Euclidean geometry are known *a priori*—specifically, they depend on our intuition of space, or space as we can imaginatively visualize it. After the publication of Kant's philosophical works, numerous attempts have been made to articulate methods of philosophical research that synthesize idealism and positivism, or that at least combine aspects of idealism and positivism with each other.

Brother/Sister . . . *(name and surname of the Candidate),* make the third esoteric journey, now through the North to the East, and then return through the South to the West, and contemplate what you have already heard during this ceremony.

The Director of Music plays a continuation of Richard Wagner's "Faust," Overture in D minor. The D.C., holding the Candidate by the right hand, walks through the Lodge (without stepping on the Mosaic Pavement) clockwise, through the North, the East, the South, and back to the West. In

this way, the D.C. leads the Candidate in front of the Symbolic Ladder in the West, and they stand on the floor of the Lodge outside the Mosaic Pavement, facing East. When the D.C. and the Candidate arrive there, the Director of Music stops playing music.

V.M.: Brother/Sister . . . *(name and surname of the Candidate),* as a proof that you possess that courage which the Fellow Craft Freemasons of our Order may be called upon to exert against our enemies, and that you hold danger and even death in contempt, we now call upon you, as a proof of your fidelity to this Order, to plunge your hand into this cauldron of mercury and pluck forth this gold coin of our Order.

The V.M. drops the gold coin into a cauldron of liquid mercury that stands on his/her pedestal. The Expert brings the cauldron to the Candidate, and the Candidate, prompted by the D.C., snatches the gold coin out of the cauldron.

V.M.: Very well! No harm awaited you, and you knew it. On one side of this gold coin is the coat of arms of our Order, and on the other side of this gold coin is the coat of arms of our Order's Grand Master. This gold coin of our Order symbolizes the obligations that its regular holder has to the Order, and the obligations that the Order has to each regular holder of such a gold coin. You will keep this gold coin as a token of fidelity, and in order to remind you that all the ceremonies of Masonry are but faithful representations of the realities of life, and that you may be ever ready to lay down your life for the triumph of the principles of the Modern and Perfecting Rite of Symbolic Masonry. As long as you are a regular and conscientious Craftsman of our Order, this gold coin will act as a seal of fraternal solidarity and relief, because we reward loyalty with loyalty; but if, alas, you lose the trust of our Order's Hierarchy, this same gold coin will become a seal of punishment and suffering.

The D.C. instructs the Candidate to keep the gold coin in his/her pocket, and the Expert places the cauldron back on the Venerable Master's pedestal and resumes his/her seat.

V.M.: Brother/Sister . . . *(name and surname of the Candidate),* make the fourth esoteric journey, walking through the North to the East, and then return through the South to the West. Contemplate what you have already heard during this ceremony.

The Director of Music plays a continuation of Richard Wagner's "Faust," Overture in D minor. The D.C., holding the Candidate by the right hand, walks through the Lodge (without stepping on the Mosaic Pavement) clockwise, through the North, the East, the South, and back to the West. In this way, the D.C. leads the Candidate in front of the Symbolic Ladder in the West, and they stand on the floor of the Lodge outside the Mosaic

Pavement, facing East. When the D.C. and the Candidate arrive there, the Director of Music stops playing music.

V.M.: The fourth esoteric journey is completed. Contemplate deeper. Make the fifth esoteric journey, walking through the North to the East, and then return through the South to the West.

The Director of Music plays a continuation of Richard Wagner's "Faust," Overture in D minor. The D.C., holding the Candidate by the right hand, walks through the Lodge (without stepping on the Mosaic Pavement) clockwise, through the North, the East, the South, and back to the West. In this way, the D.C. leads the Candidate in front of the Symbolic Ladder in the West, and they stand on the floor of the Lodge outside the Mosaic Pavement, facing East. When the D.C. and the Candidate arrive there, the Director of Music stops playing music.

V.M.: The fifth esoteric journey is completed. Brother/Sister . . . *(name and surname of the Candidate),* you will now take the Solemn Obligation of a Fellow Craft. For this reason, you will kneel on your right knee, the left foot formed in a square, you will place your right hand on the Altar of Truth, and you will lift your left arm with the elbow forming a right angle, the forearm extended vertically to the ground, the thumb extended in the form of a square, and the rest of the fingers joined together facing inward.

The V.M. gavels once and rises, holding his/her gavel in the right hand on his/her left breast. All other attendees rise, and (with the exception of the D.C.) they constantly point their outstretched right index finger at the Candidate's heart (both while the latter walks towards the kneeling stool and during the time that the Candidate remains kneeling). The D.C., holding the Candidate by the right hand, conducts him/her directly to the kneeling stool, which is placed near the west side of the Altar of Truth, and then the D.C. releases the Candidate's right hand. The D.C. assists the Candidate to kneel on his/her right knee, the left foot formed in a square, to place his/her right hand on the Altar of Truth, and to lift his/her left arm with the elbow forming a right angle, the forearm extended vertically to the ground, the thumb extended in the form of a square, and the rest of the fingers joined together facing inward. Additionally, the D.C. assists the Candidate to repeat the Oath, which is read by the Venerable Master.

V.M.: Repeat your name at length and say after me: I . . . *(name and surname of the Candidate),* in the presence of the Universal Cause, often referred to as the Grand Geometrician of the Universe, and in the presence of this Lodge of Fellow Craft Freemasons, promise and swear that I shall never directly or indirectly reveal the secrets and mysteries of the inner workings and the covert projects of the Second Degree of the Autonomous

Order of the Modern and Perfecting Rite of Symbolic Masonry, nor any of the secrets and mysteries that I have already received, except to an equal Fellow Craft Freemason who has likewise lawfully received them within this Order; that I shall act as a devoted fighter for liberation from exploiters around the world, as a defender of science and labor, and as an enemy of ignorance and obscurantism. Furthermore, I promise and swear to strictly and scrupulously follow the statutes and regulations of this degree, in accordance with the decrees and the directives of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, and to fulfill the duties of a Craftsman diligently, faithfully, and without partiality, favoritism, or prejudice. I swear and promise obedience to the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry without equivocation or mental reservation, and with no hope of escaping the penalty that our Order reserves for the traitors of the Second Degree.

After the aforementioned Solemn Obligation, the attendees stop pointing their outstretched right index finger at the Candidate's heart, and they form the Fraternal Chain: all attendees, with the exception of the D.C. (and the Candidate), give their right neighbor the left hand, and their left neighbor the right hand, so that the Candidate is included in this Fraternal Chain between the two Overseers. The Fraternal Chain remains.

V.M.: Rise, newly obliged Craftsman among Craftsmen of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.

The Candidate rises with the assistance of the D.C. All the attendees cut the Fraternal Chain, and, with the exception of the D.C., they resume their seats. The D.C. turns left and, holding the Candidate's right hand, conducts the Candidate to the north side of the Venerable Master's pedestal, so that they both are in a position parallel to the pedestal and about two feet from it, facing South towards the Venerable Master. The D.C. then releases the Candidate's hand.

V.M. (addressing the Candidate): Brother/Sister . . . *(name and surname of the Candidate)*, the working tools of a Fellow Craft are the Square, which teaches morality, the Level, which teaches equality, and the Plumb Rule, which teaches justness and uprightness of life and actions.

I shall now proceed to entrust you with the marks by which, in our Order, the Fellow Crafts are known to each other and distinguished from the other people. As in the previous degree, your body, being considered a symbol of your mind, must be perfectly erect *(the D.C. assists the Candidate to comply)*, and your feet, symbolizing the rectitude of your actions, must

form a Square *(the D.C. instructs the Candidate in whisper to unite his/her heels so that they form a Square).* You will now take a short pace towards me with your right foot, bringing the left heel into the hollow of the right foot *(the D.C. assists the Candidate to comply).* It is in this position that the Sign, the Token, and the Word of the Second Degree are communicated. *(The V.M. rises, faces the Candidate, and takes Step).*

The first part of the threefold Sign of a Fellow Craft is known as the Sign of Fidelity, and it is given by placing the right hand on the left breast with the thumb extended in the form of a square and the rest of the fingers joined together *(the V.M. illustrates, and the D.C. ensures that the Candidate copies),* and it symbolizes the protection of our repository of secrets against the insidious. The second part is known as the Sign of Perseverance, and it is given by lifting your left arm with the elbow forming a right angle, the forearm extended vertically to the ground, the thumb extended in the form of a square, and the rest of the fingers joined together facing inward *(the V.M. illustrates, and the D.C. ensures that the Candidate copies).* The third part is known as the Penal Sign, and it is given by dropping the left hand, drawing the right hand smartly across the breast and dropping it to the side with the arm at full length *(the V.M. illustrates, and the D.C. ensures that the Candidate copies).* The Penal Sign alludes to the symbolic penalty of the Second Degree, which implies that a Fellow Craft Freemason would rather have had his/her heart torn from his/her breast than profane his/her passing to the Second Degree. By giving the Sign of Fidelity and then the Sign of Perseverance and retaining both of them, a Freemason is "in Order" in the Second Degree.

The Grip or Token is given as follows *(the V.M. takes the Candidate's right hand and adjusts the grip by placing the Candidate's thumb in position before placing his/her own; the D.C. assists the Candidate):* by five equal, distinct pressures of the thumb on the second joint of the hand. This Grip or Token demands a Word. In our Order, the Word of the Second Degree is communicated as follows: the first Brother/Sister says "Id," the second Brother/Sister responds by saying "Superego," and then they both say "Ego."

After the end of this ceremony, the Secretary will upgrade your level of clearance, thus enabling you to have access to more resources in order to accomplish your duties and enjoy your benefits as a Fellow Craft in our Order.

The V.M. places the Candidate's right hand in the left hand of the D.C. and sits. The D.C. turns right, controlling the Candidate by holding his/her right hand, regains the floor of the Lodge, turns left, instructs the Candidate in whisper to step off with the right foot, and passes in front of

the Venerable Master's pedestal to the southeastern corner, where they halt for a moment in order to emphasize the practice of squaring the Lodge. The D.C. then leads the Candidate to the southwestern corner, where they halt for a moment in order to emphasize the practice of squaring the Lodge. When this is done, the D.C. leads the Candidate to the south side of the Junior Overseer's pedestal, where they stand parallel to the pedestal and a convenient distance from it. The D.C. releases the hand of the Candidate. Afterwards, the D.C. taps once on the floor with his/her wand.

D.C.: Brother/Sister Junior Overseer, I present to you Brother/Sister . . . *(name and surname of the Candidate).*

J.O.: Brother/Sister . . . *(name and surname of the Candidate),* advance to me as a Fellow Craft Freemason.

The D.C. instructs the Candidate in whisper to take Step only (thus bringing the left heel into the hollow of the right foot) and ensures that the Candidate does not give Sign at this stage.

J.O.: What is that?

Candidate (prompted by the D.C.): The second regular step in Freemasonry.

J.O.: Have you learned anything else?

Candidate (prompted by the D.C.): I have.

The D.C. instructs the Candidate in whisper to give the Fellow Craft's Sign and cut it.

J.O.: What is that?

Candidate (prompted by the D.C.): The Sign of a Fellow Craft Freemason.

J.O.: Have you anything to communicate?

Candidate (prompted by the D.C.): I have.

J.O. rises, faces the Candidate, takes Step, and offers hand. The D.C. places the Candidate's right hand in that of the J.O., and, with the left hand, he/she adjusts the grip from above. The J.O. gives the Fellow Craft's Grip after the D.C. has adjusted the Candidate's right thumb, and he/she retains the grip.

J.O.: What is this?

Candidate (prompted by the D.C.): The Grip or Token of a Fellow Craft Freemason.

J.O.: What does it demand?

Candidate (prompted by the D.C.): A Word.

J.O.: Give me that Word.

Candidate (prompted by the D.C.): Id.

J.O.: Superego.

J.O. and Candidate (prompted by the Expert): Ego.
J.O.: Pass.
The J.O. replaces the Candidate's right hand in the left hand of the D.C. and remains standing. The D.C. takes the Candidate by the right hand to the south side of the Senior Overseer's pedestal, and he/she places the Candidate's right hand in the Senior Overseer's left hand, and, standing on the right side of the Candidate, he/she ensures that they are both facing East. Then, the S.O. takes Step, and gives the Sign of Fidelity, which he/she retains.
S.O.: Venerable Master, I present to you Brother/Sister . . . *(name and surname of the Candidate)* in order to be invested with the distinguishing badge of a Fellow Craft Freemason.
V.M.: Brother/Sister Senior Overseer, I delegate you to invest him/her with the distinguishing badge of a Fellow Craft Freemason.
The D.C. divests the Candidate of the Entered Apprentice's apron and sash, and places them on the Senior Overseer's pedestal. The S.O. then cuts the Sign, releases the hand of the Candidate, and, with the Candidate facing him/her, he/she puts on the Candidate the apron of a Fellow Craft Freemason (the D.C. assists as necessary). Moreover, the S.O. invests the Candidate with the sash of a Fellow Craft Freemason, which is worn over the right shoulder and under the left arm. Thereupon, the D.C. takes the Candidate by the right hand and turns him/her towards the East, so that they both stand facing East, and then the D.C. releases the Candidate's right hand.
V.M.: Brother/Sister . . . *(name and surname of the Candidate)*, your progress in Freemasonry is marked by the position of the Square and the Pair of Compasses. In the First Degree, both points of the Pair of Compasses are hidden by the Square, whereas, in the Second Degree, one is disclosed, implying that a Fellow Craft Freemason is superior to an Entered Apprentice but inferior to a Master Mason.
The apron of a Fellow Craft Freemason is of white fabric having a triangular flap, all bordered with two-inch ribbon of light blue. The apron also has two light blue goffer rosettes at the lower corners. A Fellow Craft's apron is sixteen inches wide and fourteen inches deep. In our Order, the sash of a Fellow Craft Freemason is of light blue fabric, five inches wide, edged with orange-red braid, and embroidered with a gold rayed circle, having the number 2 in orange-red at its center. The sash is worn over the right shoulder and under the left arm. In our Order, the distinctive symbol of a Fellow Craft Freemason is a circle as it is defined in Euclidean geometry—that is, a shape consisting of all points in a plane that are equidistant from a given point, which is the center. In sacred

geometry, the circle symbolizes totality, wholeness, and the self in its entirety.

As a Fellow Craft Freemason, it is your duty to test and assess every aspect of the Entered Apprentices' works, character, and behavior. With regard to the Masons under your rule, you will act with justice, ensure that they stand to and abide by our Order's Ritual and Constitution, and that they demonstrate due obedience to their Venerable Master's commands. If you find any Entered Apprentice who will not work, or who does bad work, it is your duty to judge him/her with candor and to instruct him/her with friendship. If he/she remains obdurate, you must report him/her to the Senior Overseer and the Junior Overseer.

Brother/Sister Director of Ceremonies, you will place Brother/Sister . . . *(name and surname of the Candidate)* at the southeastern part of the Lodge to mark the progress that he/she has made in the Craft.

The D.C. takes the right hand of the Candidate and leads him/her up North to the southeastern part of the Lodge, and instructs him/her to sit there. When this is done, the D.C. leaves the Candidate sitting on a chair at the southeastern part of the Lodge, and resumes his/her regular seat in the Lodge. Everyone is sitting while the Charge is delivered.

Charge:

Orator: In general, philosophers are preoccupied with methodic and systematic investigations of the problems which originate from the reference of consciousness to the world and to itself. In other words, philosophers are preoccupied with the problems that originate from humanity's attempt to articulate a qualitative interpretation of the integration of the consciousness of existence into the reality of the world. The aforementioned problems pertain to the world itself, to consciousness, and to the relation between consciousness and the world.

It goes without saying that scientists are also preoccupied with similar problems. However, there are two important differences between philosophy and science. Firstly, from the perspective of science, it suffices to find and formulate relations and laws (generalizations) that, under certain conditions and to some extent, can interpret the objects of scientific research. Philosophy, on the other hand, moves beyond these findings and formulations in order to evaluate the objects of philosophical research and, ultimately, to articulate a *general method* and a *general criterion* for the explanation of every object of philosophical research. Whereas sciences consist of images and explanations of these images, philosophies are formulated by referring to wholes and by inducing wholes from parts. Hence, for instance, a philosopher will ask what is "scientific" about

science, or what is the true meaning of science? Therefore, philosophy and science differ from each other with regard to the level of generality that characterizes their endeavors. Secondly, as the French philosopher Pierre Hadot pointed out in his book *Philosophy as a Way of Life*, unlike the various scientific disciplines, philosophy is not merely a science, but it is a "way of life." More specifically, philosophy implies a conscious being's free and deliberate decision to seek truth for the sake of knowledge itself, since a philosopher is aware that knowledge is inextricably linked to the existential freedom and the ontological integration and completion of the human being.

Beyond the similarities between philosophy and science, philosophy is an impetus for the creation of a world of meanings (in Greek, "noēmata") that express human creativity. Moving beyond those approaches which understand "meaning" as a constituent element of language, Edmund Husserl used the Greek term "nōema" (plural: "noēmata") to designate the intentional object, meaning that element due to which an intention of the human being—such as one's intention to say something, to move one's hand, etc.—acquires content and becomes significant. In particular, in his book *Ideas: General Introduction to Pure Phenomenology*, where he introduced the Greek term "nōema" (meaning "thought," or "what is thought about"), Husserl argued that any conscious experience is directed towards an object, and that, corresponding to all points in the manifold data of the real mental content, there is a variety of data displayable in pure intuition and in a correlative "noematic content," or briefly "nōema." According to Husserl, every intentional act has noematic content, or briefly "nōema," and "nōema" is a mental act-process (such as an act of judging, meaning, liking, etc.) that is directed towards the intentionally held object (such as, the judged as judged, the meant as meant, the liked as liked, etc.). In other words, every intentional act has, as part of its formation, a correlative "nōema," which is the object of the act.

Every philosophical activity is fundamentally concerned with the study of being. In the context of philosophy, the term "being" is almost always construed as a self-sufficient reality that is sustained either by being a closed system or by being an open system. There are two general models whereby philosophers interpret the world: one gives primacy to the reality of the world and is known as philosophical realism, whereas the other gives primacy to the reality of consciousness and is known as philosophical idealism. Another important way of categorizing philosophical theories is based on the distinction between monism and dualism. Monism attributes "oneness" or "singleness" to a fundamental kind, category of things, or principle, whereas dualism maintains that, at

least in some domains, there are two fundamental kinds, categories of things, or principles. The monistic varieties of philosophical realism are prone to oversimplifications, because they fail to identify and analyze important elements and aspects of reality. The dualistic varieties of philosophical realism give rise to contradictions and logical gaps. On the other hand, idealism is highly malleable, since, by expressing and highlighting the complexity and the diversity of the output of the functions of consciousness, it gives rise to a philosophical framework in which various philosophical differentiations can take place. However, idealism tends to underestimate the ontological autonomy of the world; it is rather oblivious of the dialectical relationship between necessity and freedom, as well as of the "dialogical" nature of consciousness.

According to philosophical realism, the fact that experience provides images (even unrelated to each other) of a reality that seems to lie outside the dominion of consciousness implies that the reality of the world is the cause of the particular images of the world that are present within consciousness. From the realist perspective, the principle of causality points us in the direction of the claim that the autonomous existence of reality is naturally and logically necessary. In addition, as I have already mentioned, the philosophical "school" of realism is subdivided into several particular views that differ from each other: the monistic variety of realism, which is further subdivided into the materialist type of monism and the spiritualist type of monism; and the dualistic variety of realism. It is worth pointing out that, in the context of Neoplatonism, Plato's dualism was transformed into a spiritualist theory, since, according to Neoplatonism, the "One" is the beingly being *par excellence*, whereas matter does not really exist. Nonetheless, during subsequent stages of Neoplatonism, specifically in the context of Proclus's philosophical work, Neoplatonism assigned being to matter.

The second fundamental "school" of ontology is idealism. According to idealism, the nature of consciousness is not totally different from or contrary to the nature of extra-conscious reality. The representatives of idealism, as it was formed in the context of modern philosophy, highlight the logical principle of identity (in contradistinction to the logical principle of causality, which is highlighted by the representatives of philosophical realism). The idealists' way of thinking can be summarized as follows: if the nature of reality were totally different from the nature of consciousness, then the human being would be unable to know reality. Thus, idealism ultimately construes and studies the world not as something reflected in consciousness, but as an extension and a projection of consciousness outside itself—or, as part of consciousness itself. The

philosophical "school" of idealism presupposes a radical form and a high degree of individuation and social autonomy. For this reason, the fundamental arguments of modern idealism were originally formulated in eighteenth-century Western Europe; they were inconceivable in the ancient and the medieval societies, which were characterized by a high degree of collectivism.

The type of dualism that prevailed in modern philosophy was formulated in the seventeenth century by the Dutch-French philosopher and mathematician René Descartes (Latinized: Renatus Cartesius). Descartes's dualism is based on the distinction between "extension" and "cognition," but Descartes assigned primary importance to cognition. Descartes is considered to be the father of modern philosophy, because he founded his spiritualist variety of dualism on his perception of the self-reliance of reason. Cartesianism, as Descartes's philosophical "school" is known, was combined with British empiricism into a new philosophical "school" known as critical rationalism by Immanuel Kant.

The study of the history of philosophy leads us to the conclusion that neither philosophical realism nor idealism can stand as a general theory of reality, but particular aspects of realism and particular aspects of idealism tend to approach truth. Philosophical realism is corroborated by the indisputable awareness that the world is different from consciousness, for which reason consciousness has to try hard in order to grasp the reality of the world. Idealism is corroborated by the indisputable awareness that, from a certain perspective, the structure of the world is not fundamentally different from the structure of consciousness, for which reason consciousness can partially and increasingly grasp the reality of the world. Consequently, reality consists of both the world and consciousness—thus, consciousness refers to both itself and the world. This is the reason why, if we want to be philosophically and scientifically rigorous, we should not dwell on the relationship between "reality" and "consciousness." Instead, we should consider the relationship between the "reality of the world" and the "reality of consciousness." The difference between the reality of a being A and the reality of a being B is determined by each of these beings' degree of ontological integration and completion.

Truth, in general, can be defined as a structure. More specifically, truth can be defined as a set of relations that determine if, and the extent to which, the representation of reality within consciousness (that is, the knowledge of reality) is in concordance with the presence of reality itself (that is, with the nature of reality).

The way in which Plato defines propositional truth in his *Sophist* can be summarized and interpreted as follows: a sentence stating that "a is x" is

true if and only if it states things about "a" as they *are*—that is, if "a" is really "x." Otherwise, the given statement is false. Thus, from Plato's perspective, "truth" implies the concordance between a being or thing and its idea (the respective beingly being), so that a being or thing is true if, and to the extent that, it is in concordance with its idea. Given that Plato's conception of truth is intimately related to a metaphysical intuition, which underpins Plato's theory of philosophical vision, Plato says nothing about "correspondence" or about "facts." It was the medieval Italian scholastic philosopher Thomas Aquinas who, through interpreting Aristotle's philosophy in his *Summa Theologiae*, defined truth as "adaequatio rei et intellectus," meaning "the correspondence between the intellect [of the knower] and the thing [the known]." During the Middle Ages and the Renaissance, most of the philosophies that affirmed the possibility of obtaining valid knowledge endorsed Aquinas's correspondence theory of truth. According to Aquinas's Aristotelianism, there is a gap between reality and consciousness, and this gap can be interpreted as the distance that determines the reflection of reality in consciousness. However, the aforementioned Thomistic perception of truth is deficient, because it does not clarify whether the aforementioned reflection gives rise to the existence of a reversed or otherwise distorted image of reality in consciousness.

Both Descartes himself and Cartesianism in general oppose Aquinas's Aristotelianism, rejecting the definition of truth as a relation and instead identifying truth with reality. According to Descartes's *Meditations*, understanding (or intellection) is the basic reality, and it is activated by conceiving itself; hence, Descartes's famous *dictum*: "cogito ergo sum," meaning "I think therefore I am." According to Nicolas Malebranche, who espouses both Cartesian philosophy and various elements of Augustine's and Thomas's philosophies, truth does not merely exist within the absolute, but it is identified with the absolute. Therefore, we partake of truth to the extent that we partake of the absolute. Thus, in his *Treatise Concerning the Search after Truth*, Malebranche thinks of truth as a transcendent object, existing independently of consciousness, and he argues that consciousness can know truth by identifying itself with truth, either through the absorption of truth by consciousness or through the absorption of consciousness by truth. In fact, the aforementioned approach to the problem of truth can give rise to various syntheses between Cartesianism and mysticism.

The French epistemologist Gaston Bachelard's approach to the problem of truth is similar to Malebranche's approach to the same problem, but Bachelard emphasizes the process of objectification that takes place in the

context of science. In particular, Malebranche refers to a truth that is being increasingly approached by consciousness as the latter is trying to remove the border between itself and its object, a border that has been drawn by consciousness itself in order to help it to take distance from the world and, thus, develop a rational stance towards the world and achieve its scientific goals. In his book *Water and Dreams*, Bachelard argues that humanity's initial contact with the world is grounded in its primitive drives, which induce reverie and dream, preceding any kind of reflection. In Bachelard's view, this oneiric aspect of humanity's primary encounter with the reality of the world is characteristic of the way in which the reality of everyday life is constructed. According to Bachelard, only when humanity overcomes the aforementioned oneiric state can a rational stance towards the world come into being. Thus, Bachelard maintains that rationality is a continuous process of overcoming primary impulses, and of trying to become rational persons.

In view of the foregoing, we realize that, in the context of modern philosophy, the conception of truth as a discovery does not imply a static conception of truth. On the contrary, it is inextricably linked to a dynamic cognitive process. This dynamic approach to truth characterizes the conception of truth as an invention, too. Truth can be construed as a creation—that is, as a reality that is continuously formed and reformed by consciousness. The conception of truth as a creation can be further clarified by Bachelard's notion of an "objective meditation." In his book *The New Scientific Spirit*, Bachelard argues that subjective, Cartesian meditation aims to attain clear and definitive knowledge, whereas objective meditation is progressive in the sense that it is characterized by an intrinsic need to go further, extending the limits of the known. Thus, Bachelard characterizes science as a *dynamic* process both guided by and striving for *rationality*.

According to Descartes, the changing qualities of the wax force us to dismiss the trustworthiness of sensory-sensuous knowledge; according to Bachelard, it is precisely the experimental revealing of the morphological diversity of the wax that allows for its objectification, and so scientific consciousness should be continuously open to experience new objects, which are constituted by the manner in which different aspects of them are experimentally revealed. Truth as an invention can be regarded as a possibility that is being increasingly actualized and specified due to the interplay between consciousness and the reality of the world. In this case, the interplay between consciousness and the reality of the world is similar, but not identical to, a Thomistic-Aristotelian notion of a relation (or "correspondence") regarding truth: from the perspective of any Thomistic-

Aristotelian notion of a relation (or "correspondence") regarding truth, consciousness is a passive mirror of reality, whereas, according to the aforementioned conception of truth as an invention, consciousness plays a much more active and responsible role in the acquisition of valid knowledge due to its rationality.

In view of the arguments that I have already put forward, truth is neither a pure essence nor a pure relation (or "correspondence")—it is a dynamic and rational contemplation of the world and of consciousness, as consciousness integrates and reintegrates itself into the world. Therefore, truth should be construed neither as a discovery alone nor as an invention alone, but as the outcome of the contact and the interaction between consciousness and the reality of the world. The integration of consciousness into the world is both a volitional act and an existential necessity. However, when conscious beings integrate themselves into the world, they do not only accept the reality of the world as a substantive presence, but they also attempt to understand and interpret the reality of the world. Even when consciousness cannot enter into and partake in the reality of a particular aspect of the world or of a particular situation, consciousness can create a pertinent concept. Hence, theoretical constructs play a necessary and major role in science. Moreover, Kant has masterfully proved that scientific laws are neither connatural to reality nor innate in it, but they are kinds of relations (specifically, hypothetico-deductive systems) through which consciousness understands and interprets reality. During the process of scientific explanation, the consciousness of a scientist creates new, more complete systems of relations (namely, hypothetico-deductive systems) in order to improve one's understanding and interpretation of reality, thus replacing older, scientifically degenerating systems of relations with new ones, which have a broader explanatory domain.

When consciousness establishes a correspondence between the intellect and the thing, it is not passive, but active. In particular, consciousness conceives structures that concur with its own structure, and it integrates and reintegrates itself into the world in accordance with these structures. Thus, we should discard both the idealist argument that reality is a mere extension of consciousness and the pragmatist argument that one should merely opportunistically seek for a congenial, even temporal, way of settling the contradiction between "success" and "failure." Furthermore, we should discard any variety of philosophical realism that assigns a passive role to consciousness in the context of the correspondence theory of truth. Consciousness does not only observe reality, but also it structures and restructures reality. For this reason, a theory is not a set of observed

occurrences and recorded associations, but it is instead an explanation of them or a means of criticizing and restructuring reality. The transition from causal speculations based on factual studies to theoretical formulations requires the following methods: firstly, isolation, or viewing particular factors and forces with certain *ceteris paribus* assumptions (assuming that other things remain equal); secondly, abstraction; thirdly, aggregation, or grouping data together according to the criteria of the corresponding theory; and, fourthly, idealization, or conceiving an ideal state or a state in which a limit has been reached. Consequently, truth should be construed as the specification of the intentionality of consciousness, as the latter attempts to impose an interpretation of the world and, thus, to structure and restructure the world in accordance with the goals of consciousness.

In the context of the Thomistic-Aristotelian variety of the correspondence theory of truth, consciousness plays a mainly passive role. By contrast, our philosophy, which we call rational dynamicity, reverses the Thomistic-Aristotelian correspondence between reality and consciousness, and it highlights not only the dynamic role of consciousness, but also the potential adaptation of reality to the intentionality of consciousness. In particular, according to the philosophy of rational dynamicity, consciousness aims to structure and restructure reality not according to some arbitrary idealistic vision, but according to a rational and dynamic strategy. Consequently, truth is continuously being created by the contact and the interaction between consciousness and reality; even though the so-obtained truth is relative and partial, it does not prohibit humanity from striving for the attainment of the absolute truth in any area of reality.

Our notion of rational dynamicity stems from the synthesis between structuralism and hermeneutics. Structuralism is a method that aims to analyze isolated events or meanings in terms of their underlying laws of interaction. It seeks to comprehend "particulars" by describing and explaining their interrelationship within the totality of general codes that govern them; and these codes constist of regulative and constitutive rules. Regulative rules are rules that regulate pre-existing forms of behavior, such as, for example, the ban of smoking in public places or the obligation to comply with traffic rules. In these cases, the public places and the streets pre-exist the rules that seek to regulate them. Constitutive rules, on the other hand, have a creative function, in the sense that they make it possible to perform particular actions or to participate in a particular practice. By being ontologically prior to the practices that they regulate, constitutive rules do not just regulate those practices but constitute them into reality and define them. The rules that determine how the chess pieces

move are typical examples of constituve rules, since they create the game of chess.

In order to understand the deepest meaning of reality, we need, in addition to structuralism, to utilize the hermeneutic method (also known as the "school" of hermeneutics), which was originally developed by the German philosopher Hans-Georg Gadamer (1900–2002), and it is aimed at a deep dialogue between consciousness and its object. Gadamer, in his book *Truth and Method*, argues that language exists genuinely only in conversation or dialogue and, therefore, we have to study language not only as a system by means of which we exchange signs, but also as a system of "linguistic togetherness." According to the hermeneutic method, the whole must be understood from the individual, and the individual must be understood from the whole. In other words, Gadamer proposes a circular model of understanding, in the sense that he argues that the movement of understanding is always from whole to part, and back to whole. In this way, the hermeneutic method aims to broaden, in concentric circles, the unity of the meaning that is understood by consciousness.

Gadamer argues that "interpretation" is a peculiar immanent approach to being, in the sense that interpretation does not objectify, nor does it seek to determine something as a neutral observer, but it seeks to acquire what is actually to be understood in "a fabric of meaning" (ibid). In particular, Gadamer argues that interpretation seeks to acquire what is actually to be understood in "a fabric of meaning," not by pursuing a mere objective determination of truth, but by making the object of consciousness "speak" and bring out what is in the structures of meaning that correspond to the given object of consciousness (ibid). Therefore, influenced by Martin Heidegger, Gadamer argues that, in a conversation, language does not only mean that someone speaks, but also speaks itself. However, the hermeneutic method is not focused on a particular means of communication, but it is focused on a particular basic stance of the human being in the world: this basic stance consists of being in conversation with one another. From this perspective, according to Gadamer himself, hermeneutics is "the art of being able to listen," and this art is one that must be taught methodically because people should learn to take back, or discard, the prejudicial effects of their own will to understand and let someone oneself or something itself speak (ibid). Hence, in hermeneutics, the actual subject is "understanding-in-the-world" (ibid).

By resorting to the analysis of the rational dynamicity of consciousness, we can interpret both ontological reality and the intentionality of consciousness, which imposes its own structures on ontological reality in order to utilize ontological reality. As a criterion of reality and action,

rational dynamicity derives from consciousness, but, since it is not intended to offer philosophical "legitimacy" to arbitrary idealistic action, it is activated only when it is possible to be applied to objective reality. Additionally, the method of rational dynamicity is based on the ontological position that objective reality is activated for—that is, made present in—consciousness when consciousness assigns meaning and significance to objective reality. Even though reality is multidimensional, it becomes significant for consciousness only when it becomes mentally updated in relation to the intentionality of consciousness. Therefore, the knowledge of reality that is based on the method of rational dynamicity is in agreement with both the nature of consciousness and the nature of reality.

Closing Ceremony

V.M.: Brothers and Sisters, you will rise and assist me to close the Lodge in the Second Degree.
All rise.
V.M.: Brothers and Sisters, join me in giving the Sign of a Fellow Craft Freemason and speaking the Word.
All take Step, give the Fellow Craft's Sign, say "Id–Superego–Ego," and then they cut the Sign.
V.M.: Brothers and Sisters, our esoteric work is completed. Everything is now perfect. Let us close this Lodge of Fellow Crafts. Essentially philanthropic, philosophical, and progressive, our Order has as its objects the search for truth, the systematic inquiry into reality, the study of philosophy and science, and the practice of solidarity. It works for the material, the moral, and the intellectual improvement of humanity. "Per scientiam et philosophiam vincimus!" *(the Latin phrase "Though science and philosophy we conquer!").*
All say: "Per scientiam et philosophiam vincimus!" The Lodge is closed in the Second Degree.

Lectures for the Further Training of Fellow Craft Freemasons

After the Ceremony of Passing, the following three lectures, articulated in the form of questions and answers between the Venerable Master, the Senior Overseer, the Junior Overseer, and the Orator, should be read and discussed in the next three meetings of a Lodge of Fellow Crafts, one Lecture at each meeting, for the further training of the new Fellow Craft.

Lecture I

V.M.: When we travel through the corridors of the illustrated history of humanity, or, put simply, when we study the history of art, we see that ancient agricultural civilizations worshiped nature. It is unsurprising, therefore, that their gods usually had animal characteristics. Ancient nomadic civilizations worshiped the natural bond of blood, the power of the race. On the other hand, ancient Greek gods had human form. The human form of the ancient Greek gods was characterized by exceptional beauty because it was expressing the human quest for perfection, and, according to ancient Greek mythology, the end of the human being's existence is humanity's participation in the divine attributes. Thus, through their mythology, Greeks gave primacy to a personalistic approach to reality over the impersonal commands of nature and race. In this way, they created a metaphysically grounded anthropocentric civilization. As a matter of fact, their personalistic tradition is the main reason due to which ancient Greeks decided to adopt Christianity. From the Greeks' perspective, Christianity, identifying the classical Greek value of beauty with the person of Christ, implies the transformation of classical Greek aesthetics into a theological system. Given that personalism plays a major role in our Order's humanistic strategy, what do we mean by the term "person"?

S.O.: An existential otherness in communion, or a socialized individual. Therefore, a person is not constrained by one's substance, since it is one's communion with one's reason—in Greek, "logos"—which enables one's substance to exist.

V.M.: How can we understand the world from a personalistic perspective?

J.O.: The world can be understood as a structure of reasons. This way of understanding the world has been reinforced by quantum physics, which has scientifically disclosed the logical constitution of matter: matter is constituted not only by atoms of matter itself, but also by behavioral qualities or energies.

V.M.: How can we overcome the contradiction between individualism and collectivism from a personalistic perspective?

S.O.: The concept of a person has two constituent components or dimensions: the first is existential otherness, which corresponds to individualism; the second is communion, which corresponds to collectivism. Hence, the concept of a person represents a synthesis between individualism and collectivism, or between otherness and communion, respectively.

V.M.: The lack of a deep awareness of the meaning of the term "person" underpins several contradictions and problems that have been manifested

in the history of civilization. Explain the contradiction between individualism and collectivism in the context of Orientalism.

J.O.: By the term "Orientalism," we refer to a family of civilizations that constitute a geo-cultural entity known as the historical "East." The major constituent components of Orientalism are the Hindu civilization zone, the Chinese civilization zone, and the Islamic civilization zone. Orientalism has two complementary distinctive characteristics. On one hand, society imposes relations of tight interdependence among its members, thus giving rise to coercive relations that contradict individualism. On the other hand, Oriental systems of mysticism allow mystics to pursue their independence from their social environment through the renunciation of the world. Oriental mystics can be characterized as "otherworldly individuals," in order to be distinguished from Western "worldly individuals." However, due to their individualist characteristics, many "schools" of Oriental mysticism are popular in the West.

In the context of Hindu political thought, collectivism is founded on the concepts of *dharma* (meaning right and duty) and *danda* (meaning force and punishment). According to Hindu political thought, human society is an integral part of the universe, which, for the Hindus, is characterized by *rta*—that is, an inviolable cosmic order brought about by the operation of laws that represent divine intelligence. Hindu political thinkers argue that human society reproduces the cosmic order when all humans keep to their proper place and discharge their relevant *dharma*. The major sources of *dharma* are the Vedas, the Smritis, and *Vyavahara* (which means custom). Furthermore, according to Hindu political thinkers, humans can fall victim to illusions and temptations, so *danda* becomes necessary to keep them on the right path. Thus, from the perspective of Hindu political thought, governing people consists of using *danda* to maintain *dharma*. However, Hindu mystics may follow an individualistic path, which is *samnyasa*. In Sanskrit, *samnyasa* means renunciation or abandonment, and it is composed of the term "sam," which means "collective," the term "ni," which means "down," and the term "asa," which means "to throw" or "to put"; a literal translation of *samnyasa* would be "laying everything down." Of the 108 *Upanisads*, which are contained in the *Muktika*, 23 are focused on *samnyasa* as the path to *moksa*, meaning liberation. Moreover, Buddhism represents an important stage in the development of Hindu political thought, since it rejected the caste system, founded monasteries, gave rise to a non-theological religion, and enjoyed the support of inferior social classes (such as traders, craftsmen, merchants, and foreign settlers), thus invigorating the individualistic mystical traits of the Hindu civilization.

In the context of Chinese political thought, collectivism is founded on the following political traditions: Confucius's theory of virtue, which emphasizes rule by moral example rather than by military supremacy or according to hereditary succession; Han Fei Tzu's Legalism (or "school of the method"), which emphasizes the rule of law and administrative efficiency; and Tung Chung-shu's attempt to amalgamate Confucian political and moral philosophy with the metaphysical and cosmological speculations which are contained in the *Book of Changes* (*I Ching*), according to which the universe is an organic entity in which *yin* (representative of all that is dark, submissive, and female), *yang* (representative of all that is bright, aggressive, and male), and the five elements (water, fire, wood, metal, and earth) are dialectically interrelated and subject to a predetermined order. Thus, Han Confucianism, of which Tung Chung-shu is the preeminent representative, grafted Confucian political and ethical notions on an organic conception of society, and adopted an eclectic attitude towards the incorporation of Legalist practices in the administration of the empire. However, the religious and political teachings of Taoism, whose preeminent representatives are Chuang Tzu and Lao Tzu, pave a mystical way to individualism through renunciation, which, in this case, has the form of inaction. Since the *tao*, or way (a term used by Confucians), is all-encompassing and beyond human comprehension, and its movement proceeds unchecked by the activity of humanity, the Taoist mystic finds enlightenment in inaction. In the context of Taoism, inaction leads to the relativization of social life, and it allows the Taoist mystic to take distance from the social world.

In the context of Islamic political thought, collectivism is founded on the concept of the Caliphate. In the Islamic world, the religious and juristic genre is available in two versions: Sunni Islam holds that political and religious authority should be vested in the person of an imam-caliph elected by the *Ummah* (namely, the Muslim community), whereas Shia Islam (whose main groupings are the Twelvers, the Ismailis, and the Zaidis) limits the legitimacy to Muhammad's son-in-law and cousin Ali, who was ruling over the Islamic Caliphate from 665 to 661 C.E. In the political sphere, both the Sunni and the Shia emphasize the maintenance of the *Sharia* (the Islamic law) and of social order. They are also deeply concerned with justice, which they understand as the maintenance of things in their proper station and as the regulation of practical life in accordance with the requirements of social order. However, Islamic mystics—specifically, the Sufi Orders—follow an individualistic approach to Islam, emphasizing the transmission of the divine light from the teacher's heart to the heart of the student, independently of the worldly

society and its values. Despite the Islamic tradition's general discomfort with parable, allegory, and metaphor, many Sufi masters make extensive use of such devices; as the French philosopher, theologian, and Iranologist Henri Corbin has pointed out, "creative imagination" plays a central role in the work of many Sufi masters. In the eighth and the ninth centuries C.E., great Sufis argued that they achieved the illumination of their individual souls by individual exertion, mortification, and austerity. Thus, there are several Sufi paths, or "Orders," each of which reflects the mentality and the intellectual attainments of its founder.

V.M.: Explain the contradiction between individualism and collectivism in the context of Occidentalism.

J.O.: By the term "Occidentalism," we refer to a family of civilizations that constitute a geocultural entity known as the historical "West." The cornerstone of Occidentalism is the argument that truth is structurally united with consciousness. Hence, the individual has an intrinsic value as a conscious being, which is historically prior to one's social relations. From the aforementioned perspective, individuals enter into and constitute a society as conscious beings bearing intrinsic value, stemming from their nature. This argument is the major underpinning of individualism in the West. However, the manifestation of individualism outside society is meaningless; a totally private process of individuation becomes a shadow of itself, if not a path to insanity. The very fact that all humans partake of the same mental faculties and, especially, of a common reason socializes them—or, in other words, underpins the sociality of the human soul. The latter argument is the major underpinning of collectivism in the West.

V.M.: Without the concept of a person, the contradiction between individualism and collectivism remains a source of discontent and suffering. Nevertheless, things start to change when we analyze the principles of individualism and collectivism. The principle of individualism is centered on the following argument: the fact that each individual human being has a rational capacity and, generally, possesses the mental faculties that characterize humanity as such underpins and safeguards the intrinsic value of the human individual, and it implies that the individual sample of the human species bears intrinsic value independently of its social relations. The principle of collectivism is centered on the following argument: the fact that all individual human beings have a rational capacity and, generally, possess the mental faculties that characterize humanity as such underpins and safeguards the sociality of the human soul; and the social nature of the human mode of being implies that society bears intrinsic value and is the necessary context in which the manifestation of existential otherness, or individuality, becomes

meaningful. The synthesis of the aforementioned two principles is the essence of our Order's personalism. Moreover, the aforementioned conception of personalism affirms historicity and, in fact, implies that history is the fullest expression of the creativity of the human being.

Personhood, as a philosophical-anthropological concept, was invented and methodically studied by the ancient Greco-Roman civilization. Firstly, Greeks invented the public character of political power. The emergence of the concept of a citizen freed people from elements of origin that exclude them from the commons of the city and leads to the original institution of the rule of law. The value system of the old nobility was largely replaced by a new value system which focused on the value of prudence, and religion was integrated into the institution of the state, which oversees social order. The aforementioned developments signal and underpin the emergence of politics, that autonomous discourse which is a bearer of critique of every aspect of social life, including the fundamental civil and political rights themselves. Hence, the rules of social order are emancipated from the natural order, and the social world is the outcome of the differentiation of a constructed, "artificial," order from the "supreme and necessary order" of the natural world.

The ancient Roman civilization developed the aforementioned Greek "miracle" even further. In particular, Romans invented private law, which facilitates the definition of private property and, thus, contributes to the emergence and the reinforcement of the individuality of the human being. Henceforth, the human being has two elements: the human nature common to all people, and the individual nature, the individual "ego," which is an irreducible type of inner life.

Brother/Sister Orator, explain the way in which our Order thinks about and comprehends human creativity and historical becoming.

Orator: The way in which our Order thinks about and comprehends human creativity and historical becoming is based on the five-fold dialectic of rational dynamicity. The dialectic of rational dynamicity, as a method for the operation of consciousness and as a model of the operation of reality in general, consists of the following five stages:

Stage I: Vision and Teleology. Consciousness forms a clear intellectual image of an existential state that it wants to achieve—or, in Ernst Bloch's terms, a "utopia"—and it is clearly oriented towards that intellectual image. Thus, in this stage, consciousness determines the teleology of its action.

Stage II: Strategy. In general, "strategy" refers to the orientation of a conscious being in the long term, within its environment. Consciousness makes the strategic decision to act upon the reality

of the world and upon itself in accordance with its teleology—that is, in order to bring about intended changes.

Stage III: Planning. Consciousness articulates a plan: a method of deliberate, self-conscious activity, involving the consideration of outcomes before choosing among alternatives. The primary functions of planning are the following: (i) optimization (namely, improving efficiency of outcomes); (ii) balancing the agent's teleology (which is aimed at restructuring reality) and the goal of maintaining the continuity of existence (namely, offsetting systemic failures); (iii) widening the range of decision-making (namely, enhancing the consciousness of choice); and (iv) organizing and enriching codes and networks of communication.

Stage IV: Control. Consciousness continuously tries to maintain control over its action (and its consequences) in two ways: firstly, by intensifying its action (its intervention in the reality of the world and in itself) whenever its action is unreasonably sub-optimal (whenever it can improve its existential conditions even more, according to its strategic plan); secondly, by counterbalancing its original action (specifically, by reversing its original action and by following alternative paths of action) whenever the "negative externalities" of its original action, the costs of its original action for the world (or the "environment") in general, and/or for itself in particular, tend to exceed a critical value that represents the maximum existential risks that consciousness is determined to undertake in order to continue acting in the same way. Additionally, it should be mentioned that the term "dialectic," in general, implies a transition from one state to another without the total elimination of the previous state, in the sense that the previous state leaves its traces in the new one. Therefore, according to the dialectic of rational dynamicity, an agent of change does not bring about a totally new state, which would be uncontrolled by the agent of change. In general, change cannot go beyond certain limits without running the risk of systemic collapse. For this reason, the dialectic of rational dynamicity highlights the importance of preventing uncontrolled systemic turbulence and of continuously maintaining control over the consequences of our actions.

Stage V: Development. Consciousness seeks to ensure and enhance its capabilities and to create favorable conditions for the continuation of its action in the future. However, consciousness realizes that the achievement of its ultimate goals is a work in progress. Thus, consciousness seeks to restructure the world according to the

intentionality of consciousness—without, however, jeopardizing the possibility of future interventions in the reality of the world.

Lecture II

V.M.: We strive to expand the intellectual and moral horizons of humanity and to build a better human being within a better society. But, prior to undertaking any relevant mission, a Fellow Craft of our Order must have a rigorous understanding of history, politics, and our adversaries. Brother/Sister Senior Overseer, what is the State?

S.O.: The term "State" has two connatural meanings. The first meaning refers to any organized society that has its own distinct government. From this perspective, people tend to believe that they belong to a State and to identify themselves with said State. The second meaning refers to the operational center, "apparatus," or mechanism which exerts power. This mechanism is a specialized organization. In particular, it performs a service called "command." The members of this mechanism manage power to varying degrees. Moreover, the power of the State is a distinct type of power, because it is considered to be legitimate. It goes without saying that the State's commands are essentially based on coercion. This is a necessary attribute of the State's commands. Nonetheless, it is not the only one, because even a gang can coerce and enforce its will, but it cannot claim to be a State. In the case of a State, coercion goes hand in hand with legitimacy—more specifically, coercion goes hand in hand with the citizens' belief in the legitimacy of the State's commands. Moreover, there is a belief in the benevolent character of the State, and in the significance and the superiority of the purposes pursued by the State. In modern societies, the aforementioned belief in the benevolent character of the State leads to the identification of the citizen with the State. The identification of the citizen with the State, the belief in the legitimacy of the State's commands, and the awareness that the State is capable of enforcing its commands all give rise to, and consolidate, the citizen's obedience and, indeed, submission to the State.

Of the aforementioned three motivations that determine the citizen's obedience to the State's commands, the belief in legitimacy is the most subtle one, since it has a mythological background. In other words, the belief in the legitimacy of the State's commands requires the creation of an appropriate myth. Historically, there have been two kinds of such myths—however, both of them have been based on the idea of sovereignty. In one case, God is the supreme authority. In the other case, the supreme

authority is society itself. Therefore, the State is considered to be the trustee of the corresponding supreme authority and is bound by rules.

According to the myth of divine sovereignty, the State must act in harmony with God's will, which is represented on earth by some religious authority. According to the myth of popular sovereignty, the State must function according to the commands of the People's representatives (or, according to the commands of the Parliament). In both cases, there is a fundamental contradiction between the principal and the trustee. For instance, the political history of medieval Europe is marked by the conflict between the Church and the Monarchy; and the political history of modern Europe is marked by the conflict between the executive and the legislative branches of government. In medieval Europe, the Monarchy was ultimately imposed on the Church; in modern Europe, the executive branch of government was ultimately imposed on the legislative one.

In the context of modernity, the fading of the myth of divine sovereignty and its subsequent replacement by the myth of popular sovereignty gave rise to the modern idea of the nation-state. This event caused a new change in the idea of sovereignty. As a result of the institution of the modern nation-state, popular sovereignty mingled with national sovereignty. From this perspective, the legitimacy of the State's commands derives from the fact that the State has become the trustee of the nation, and the benevolent character of the State is evaluated with regard to the interests of the corresponding nation. The interests and, generally, the purposes of the modern nation are determined by a privileged minority, which has a monopoly of knowledge and understanding regarding the determination of the national interest because of the fact that it holds the seat of State power.

A fourth motivation for the citizen's obedience to the State is the manipulation of power by the State. In his book entitled *The New Industrial State*, the Canadian-American economist, diplomat, and public official John Kenneth Galbraith argued that, in the context of an organization, adaptation is inextricably linked to an impulse to acquire power. In state affairs, this motivation is very strong in political opportunists, and it encapsulates their profession.

V.M.: Brother/Sister Junior Overseer, what is political authority, and what is the social establishment?

J.O.: Political authority is the capacity or the power to directly or indirectly influence the decisions that the National Assembly, the Cabinet, and the bureaucracy—meaning the agents and the organizations that constitute the state mechanism—make, derived from their officially instituted roles and functions within the State. This power can be either *de*

facto or *de jure*. *De facto* political authority is the actual ability of a political subject to rule and to win the consent of those over whom one rules. *De jure* or normative political authority is the moral power, or right, of a political subject to claim obedience to one's laws and to enforce them. The methods of influencing the political process are inextricably linked to the value system and the technology of the corresponding society. The influence of the political process is the central issue in the study of political authority, which often does not flow according to the channels of the formal institutional system. Indeed, in the twentieth century, it became amply clear that the classical notion of parliamentary democracy—according to which the citizens are supposed to have equal power, or to be equally powerless, like the firms and the consumers under perfect competition, and they are represented in political affairs by elected representatives—is not even a caricature of the actual state of affairs.

Modern societies, like the previous ones, are pluralistic, in the sense that they comprise distinct departments of power, or, in other words, distinct power oligopolies. In modern capitalist societies, old "social authorities" were replaced by "private authorities." However, because of the prevalence of the concepts of popular sovereignty and national interest, which are supposedly served by the State, it became difficult for the aforementioned private authorities to defend partial interests against the overwhelming power of the State without obstruction.

As a consequence of the monopolization of legitimate power by the State, and as a consequence of the elimination of old social authorities, the major socio-political conflicts are centered on the exercise of direct or indirect influence over the organs or the institutions that constitute the State, these being the parliaments, governments, and bureaucracies. These power enclaves constitute the ruling social coalition, or, in other words, the social establishment.

V.M.: Brother/Sister Senior Overseer, explain the role that Cardinal Richelieu's political thought and political action played in shaping the ideological underpinnings of the modern nation-state.

S.O.: Cardinal Richelieu (Armand Jean du Plessis, Duke of Richelieu, 1585–1642), a politician first and churchman second, gave France its first formally and rigorously instituted system of espionage.

Armand Jean du Plessis was born in Paris. His father, François du Plessis, seigneur de Richelieu, was given command of the Maréchaussée, a precursor of the police, by King Henry IV of France in 1578 (Henry IV of the House of Bourbon was King of Navarre, as Henry III, from 1572 and King of France from 1589 to 1610). The Maréchaussée was charged with maintaining order and justice regarding serious crimes at the French Court

and throughout the kingdom. Armand Jean du Plessis's mother, Susanne de La Porte, was the daughter of a famous jurist. When Armand Jean du Plessis was five years old, his father died of fever in the French Wars of Religion. King Henry IV of France had rewarded Armand Jean du Plessis's father for his participation in the Wars of Religion by granting his family the Bishopric of Luçon.

Armand Jean du Plessis received a remarkable philosophical, theological, and military education, and he was consecrated Bishop of Luçon in 1605. Under the patronage of the Queen Mother Marie de Medici (the second wife of King Henry IV of France), he was appointed a Minister of State seven years later, but he was driven from office shortly after the assassination of King Henry IV of France (Henry IV was assassinated by François Ravaillac, a Catholic zealot). Marie de Medici was Regent of the Kingdom of France between 1610 and 1617, during the minority of her son Louis XIII of France (King Louis XIII of France was the eldest son of King Henry IV and Marie de Medici, and he reigned from 1610 to 1643). Her mandate as regent legally expired in 1614 when her son reached the age of majority, but she refused to resign and continued as regent until she was removed by a *coup* in 1617. Her trusted adviser Concino Concini, Marquis d'Ancre, exercised strong influence on her, and he treated Louis XIII with contempt. However, Marie de Medici was ultimately banished from the country by her son, and she died in the city of Cologne in the Holy Roman Empire.

Armand Jean du Plessis was appointed Cardinal in 1622, and, in 1624, he was recalled to State office by King Louis XIII of France. He was confirmed in the post of chief minister in 1629, and he was awarded the Dukedom of Richelieu in 1631; hence, he is best known as Cardinal Richelieu. From then to his death, Cardinal Richelieu virtually ruled France, masterminding the French intelligence service and creating the powerful Cabinet Noir.

The Cabinet Noir (a secret police organization) was designed to intercept and analyze correspondence within the French Court and between the lesser nobility, thus preserving the (sometimes tenuous) security of the throne of King Louis XIII of France. Moreover, Cardinal Richelieu used his secret police to whittle away and ultimately destroy the power of the great nobles. In particular, the constant attempts (first of Monsieur Gaston Duke of Orléans, the eldest surviving brother of King Louis XIII of France, later of the Marquis de Cinq-Mars, a favorite of King Louis XIII of France) to usurp the throne were discovered and eventually countered by the Cabinet Noir.

Cardinal Richelieu created an intelligence agency of gigantic proportions, with tentacles in every part of France and in every country in which France had an interest. To a large extent, the operation of the Cabinet Noir was paternalistic for the entire political system of the country. In particular, Cardinal Richelieu was keen to smother domestic disorder within the nobility, which might lead to a scandal at Court, and to suppress acts of patent treason. He died in 1642, but the influence of the Cabinet Noir lived on for a further 150 years, until destroyed by the French Revolution, which gave rise to its own intelligence machines.

In order to understand the ethos and the policy of Cardinal Richelieu, we must bear in mind that, in medieval Europe, the Monarchy was ultimately imposed on the Church, and the executive branch of government was ultimately imposed on the legislative one in modern Europe. In the context of modernity, as I mentioned earlier, the fading of the myth of divine sovereignty and its subsequent replacement by the myth of popular sovereignty gave rise to the modern idea of the nation-state. This event caused a new change in the idea of sovereignty. As a result of the institution of the modern nation-state, popular sovereignty mingled with national sovereignty. From this perspective, the legitimacy of the State's commands derives from the fact that the State has become the trustee of the nation, and the benevolent character of the State is evaluated withregard to the interests of the corresponding nation. The interests and purposes of the modern nation are determined by a privileged minority, which has a monopoly of knowledge and understanding regarding the determination of the national interest because of the fact that it holds the seat of State power. Cardinal Richelieu was fully harmonized and in line with the aforementioned historical reality and way of thinking. Thus, being a statesman first and a man of the Church second, he implemented a policy aimed at making France great through the absolute power of its monarchy. In this, Cardinal Richelieu and Hanoverian England followed different methods. Richelieu's policy as chief minister was to make France great through absolute monarchy, in the context of which the French monarch embodies the personalization of State power. Under the Hanoverians, on the other hand, the policy of the English intelligence apparatus was to make England great through a system of constitutional monarchy, representing and serving a viable and pragmatic compromise between the English monarch, the nobility, and the bourgeois-capitalist elite, ultimately making England the cradle of a system of reigning liberal oligarchy.

In order to secure and strengthen the power of the French monarch, Cardinal Richelieu introduced an internal policy of religious tolerance and wrested control of foreign policy from the Habsburgs. Thus, he managed

to frustrate the aspirations of Monsieur Gaston Duke of Orléans and the Marquis de Cinq-Mars, both of whom had conspired to take the French throne.

As already mentioned, Cardinal Richelieu employed a widespread system of espionage throughout France to ensure that he was kept fully aware of the thoughts and the deeds of both the Church and the nobility. In this, he was brilliantly served by his confidant and chief operational officer, the Capuchin father François Leclerc du Tremblay, who was the effective director of the system of espionage that had been created by Cardinal Richelieu. Superficially a mild and gentle person, François Leclerc du Tremblay possessed a mind so subtle and intriguing that it earned him the title "éminence grise" ("grey eminence"), in recognition of his great ability to operate secretly or unofficially in the service of his master.

At the height of his power, endorsing *Realpolitik*, Cardinal Richelieu forged an alliance with the Protestant King Gustavus Adolphus of Sweden, who is credited for the rise of Sweden as a great European power. The alliance between Cardinal Richelieu and King Gustavus Adolphus of Sweden weakened the position of the German princes to such an extent that it enabled France to seize Alsace. Moreover, Cardinal Richelieu weakened the power of Spain by encouraging and manipulating risings in Portugal and Catalonia, and he contrived the downfall and eventual murder of the Bohemian military leader Albrecht Wenzel Eusebius von Wallenstein, who was the supreme commander of the armies of the Imperial Army of the Holy Roman Emperor Ferdinand II (Ferdinand II was the Holy Roman Emperor from 1619 until his death in 1637). In fact, von Wallenstein was also a major figure of the Thirty Years' War (1618–48).

During the sixteenth century, the Protestant Reformation had been dividing the Holy Roman Empire (of the German nation), and, although Lutheranism had been officially recognized in 1555, the many Calvinist rulers and their citizens were not officially recognized. Hence, the resulting tensions caused Protestant and Catholic German states to form alliances. Moreover, foreign powers had vested interest in these developments in the Holy Roman Empire. France was encircled by Spanish and Austrian Habsburgs and sought to weaken them. Spain sought to retake its Dutch provinces and maintain military access along the Spanish road. The Protestant Scandinavian countries sought to expand their influence in northern Germany. The aforementioned political and geostrategic rivalries were the main causes of the Thirty Years' War, but the immediate cause of that war can be found in Bohemia (a member-state of the Holy Roman Empire), where an infighting war broke out between

Catholics and Protestants for control of the country in 1618. After 1635, the Holy Roman Empire became the geopolitical theater of competition between two geopolitical coalitions: one geopolitical coalition consisted of Catholic Bourbon France and Protestant Sweden, while the other geopolitical coalition consisted of the Catholic Emperor Ferdinand III of the Holy Roman Empire and Catholic Habsburg Spain (Ferdinand III was the Holy Roman Emperor from 1637 until his death in 1657). In 1635, the Protestant and the Catholic member-states of the Holy Roman Empire agreed to make peace. But France, under Cardinal Richelieu, feared the authority given to the Habsburgs over the empire member-states; member-states could now no longer make foreign alliances, and an imperial army was to be created. Hence, Catholic France formed an alliance with Protestant Sweden and joined in this new political war. Sweden occupied northern Germany and moved as far south as Prague, and France took Alsace and Flanders. France also helped to start separatist rebellions in Spanish Catalonia in 1640. In this context, the Catholic Habsburgs managed to gain the support of their old rival Denmark, because Protestant Denmark sought to contain Protestant Sweden. France and Sweden continued to be supported by Protestant states, such as the Dutch provinces and Prussia. It was not until the death of the chief minister of France Cardinal Richelieu in 1648 that peace was able to be achieved. The Peace of Westphalia in 1648 ended the Thirty Years' War, establishing the idea of territorial integrity, and, along with the move from mercenary to professional armies, it consolidated the institution of modern nation-states. Furthermore, the Peace of Westphalia greatly upset the balance of power within Europe; Sweden and Prussia emerged as new great powers, Bourbon France expanded its borders to the East and became the dominant land force in Europe, the Swiss had their independence guaranteed, Spain was forced to recognize Dutch independence, lost more land to France, and faced rebellions in Portugal, and the Holy Roman Empire was forced to officially recognize Calvinism and grant Calvinist states more freedom.

V.M.: A basic characteristic of capitalism is its special relationship with the state mechanism, and it has been systematically analyzed by John Kenneth Galbraith in his book entitled *The New Industrial State* and by Antonio Gramsci in his book entitled *The Prison Notebooks*. Brother/Sister Orator, explain this special relationship.

Orator: According to classical corporate capitalism, the role of the State must be limited to ensuring the legal framework for the operation of corporate capitalism, and facilitating the expansion of corporate capitalism beyond national borders in the context of imperialist adventures. However, advanced capitalism expects even more from the State. First of all,

advanced capitalism is based on social programming (specifically, on a social regulation of aggregate demand), because large corporations cannot guarantee by their actions that aggregate demand will be commensurate with their aspirations—that is, that the market will absorb their production at controlled prices. The responsibility for achieving this goal lies with the State, which tries to prevent a slump in aggregate demand by means of appropriate fiscal policy measures and monetary regulations. Given that taxation is the most effective means of achieving this goal, State revenues must represent a very large proportion of the national income in order for the State to be able to positively and effectively fulfill the aforementioned role. According to the aforementioned reasoning, and within the aforementioned context, a "big government" is acceptable and desirable by the managerial-capitalist elite. Furthermore, in the context of advanced capitalism, a significant part of the costs of the large corporations must be socialized, meaning it must be borne by the citizens. For instance, education—which, among others, provides large corporations with skilled and disciplined labor—largely falls under the responsibility of the State. In addition, significant technological efforts are made by the State, and bureaucracies whose mission is the national programming for defense and public order underpin and facilitate the maintenance and the expansion of the established capitalist system. In general, in the context of advanced capitalism, the State plans and programs, but it does so within the field of view of the established capitalist system. Hence, in accordance with the political theory of Antonio Gramsci, we have to introduce the relationship between the dominant and the dominated classes into political analysis; far from treating the State as a unitary actor, we have to analyze the dynamics of the relationships between the members of the social establishment.

V.M.: Our inquiries into politics and history have led us to the conclusion that societies are always dominated by a minority, often referred to as the "elite," which makes the major decisions within the society and which concentrates power in its own hands. Our Order strives to build, organize, and manage a better elite according to the Modern and Perfecting Rite of Symbolic Masonry. Brother/Sister Orator, explain the meaning of the term "elite" and the dynamics between different elites.

Orator: The literal and original meaning of the term "elite" is the "elect" or the best. In fact, this is the way in which the term is used in ordinary speech when policies are sometimes defended, particularly in education, as "elitist," meaning that they are intended to produce the best or most skillfully crafted. In political theory, "elitism" has a more technical reference to certain modes of explanation deriving from the works of the Italian political sociologists Gaetano Mosca and Vilfredo Pareto.

Mosca was a professor of constitutional law at the University of Turin from 1895 to 1923 and then a professor of public law at the University of Rome until 1933. In his book entitled *The Ruling Class* (translated and edited by A. Livingston, New York: McGraw-Hill, 1939), Mosca argues that, in every society, there are two basic classes of people: a class that rules and a class that is ruled. According to Mosca, the ruling class monopolizes power, enjoys the advantages of power, and it uses both legal and arbitrary, even criminal, methods in order to sustain its domination. Each ruling group possesses some resource or attribute that is esteemed or influential in the corresponding society and that it exploits in order to advance its own power or advantage. This might be the ownership of the means of production, military force, priestly status, or administrative expertise. Moreover, Mosca has argued that elites rule by violence, manipulation, ideology, and psychological operations. In particular, Mosca contends, collective "illusions," including religious or nationalist sentiments and manipulated perceptions of public order and security, hold societies together and degrade the prevailing democratic formula into a false belief.

Mosca has explained political and social change as a result of conflict between ruling classes seeking to hold on to power and new social forces, led by minorities (alternative elites) striving to displace them. Thus, according to Mosca, elites often lose power because they fail to assimilate new social forces, either by opening ranks to new claimants for power and privileges or by adapting their policies and/or ideas. An elite that is excessively introverted and inflexible gradually loses its political and ideological hold on society, and, especially if it lacks efficient social feedback mechanisms, it may itself be overthrown.

Pareto, who was originally trained as an engineer and worked in business from 1870, published his seminal book entitled *Trattato di Sociologia Generale* (translated as *The Mind and Society*) in 1916 (V. Pareto, *The Mind and Society*, translated by A. Livingston and A. Bongiorno, 4 vols., New York: Harcourt Brace, 1935). According to Pareto, societies are divided into the elite which leads, and those who are led. The elite comprises a governing and a non-governing element: the governing elite maintains its power through a combination of coercion and manipulated consent. Pareto has pointed out that government requires an effective and rational synthesis between two opposed psychological types—specifically, Class I or "foxes" and Class II or "lions." The "foxes" correspond to and are excellent in manifesting the qualities of flexibility, cunning action, and persuasiveness, thus generating consent through manoeuvres. The "lions" correspond to and are excellent in manifesting violence in order to suppress opposition, thus employing coercion in organized fashion. If a

governing elite is excessively characterized by Class I attributes and is seriously deficient of Class II attributes, then it will be overthrown by a counter-elite that has strong Class II attributes and is ready to act decisively. However, "lions" tend to become excessively conservative, stultified, and remote from the population. Therefore, sooner or later, they require the assistance from those with strong Class I attributes, who, in turn, gradually infiltrate and transform the established elite system. Apart from the analysis of the pattern of circulation between the two aforementioned psychological types, another important problem in the theory of the elite and the methodology of power is the analysis of the teleology of an elite's policy.

Our teleological approach to elitism is based on Immanuel Kant's argument that morality and politics must be related, because, as Kant contends in his essay entitled *Perpetual Peace*, "a true system of politics cannot take a single step without first paying tribute to morality" (Immanuel Kant, "Perpetual Peace," in *Kant: Political Writings*, edited by H. Reiss, second edition, Cambridge: Cambridge University Press, 1991, p. 125). However, Kant drew a clear distinction between moral motives, acting from good will or respect for the moral law, and legal motives. In his essay entitled *The Conflict of the Faculties*, he argued that moral and legal incentives must never be collapsed into each other. In particular, according to Kant, morality and public justice must be related in such a way that morality molds politics, even though it is not the motive of politics, given that politics cannot hope for "good will."

Given the aforementioned tension between a moral system and a system of public legal justice, which must be related to each other while simultaneously remaining distinct from each other, Kant concludes that the notion of "ends" can serve as a bridge between the moral realm and the political-legal realm, since public law upholds some moral ends, such as "non-murder," even though that law must content itself with a legal motive. Indeed, if "good will," in the moral realm, could mean never universalizing a maxim of action that would fail to respect persons as ends in themselves, then the realm of morality and the realm of politics and law could be connected through Kantian teleology. If every person had good will, then everyone would respect everyone else as an end, and then all humans would constitute a "kingdom of ends." However, although it ought to, this does not actually happen due to the pathological fact that humanity is radically evil.

Good will means respect for persons as ends in themselves, and public legal justice sees to it that some moral ends, such as "non-murder," get observed, if not respected. Therefore, according to Kant, public legal

justice is the partial realization of what could happen if all wills were good, and law creates a kind of environment for good will. Kant's whole position on politics as the legal realization of moral ends is best summed up in the following two passages: the first from his *Metaphysical Principles of Virtue*:

> Man in the system of nature . . . is of little significance and, along with the other animals, considered as products of the earth, has an ordinary value . . . But man as a person, i.e. as the subject of a morally practical reason, is exalted above all price. For as such a one (*homo noumenon*) he is not to be valued merely as a means to the ends of other people, or even to his own ends, but is to be prized as an end in himself (Immanuel Kant, *Metaphysical Principles of Virtue*, translated by J. W. Ellington, Indianapolis: Bobbs-Merrill, 1964, p. 96–7).

In *The Conflict of the Faculties*, Kant explained the political significance of the aforementioned passage as follows:

> In the face of omnipotence of nature, or rather its supreme first cause which is inaccessible to us, the human being is, in his turn, but a trifle. But for the sovereigns of his own species also to consider and treat him as such, whether by burdening him as an animal, regarding him as a mere tool of their designs, or exposing him in their conflicts with one another in order to have him massacred—this is no trifle, but a subversion of the ultimate purpose of creation itself (Immanuel Kant, *Political Writings*, edited by H. Reiss, Cambridge: Cambridge University Press, 1970, p. 185).

Finally, our Order's teleological approach to elitism is guided by the vision of a socialist aristocracy, which pays homage to both the spirit and the body of humanity. True aristocracy is underpinned by loyalty to reason. Loyalty to reason gives rise to meritocracy, hierarchy, and rational obedience to a structure. True socialism implies equality of rights and equality in the ownership of the factors of production in the context of a socialist model of social organization. For instance, the French socialist and aristocrat Saint-Simon advocated the rational organization of industrial production under the direction of an elite of intellect and science, and he conceived society as the ensemble and union of humans engaged in socially useful and rationally organized work.

The interpretation and presentation of socialism as the rational use of economic resources led, after Saint-Simon, to the technocratic elitism of the Fabian Society and, after the First World War, to the advocacy of planning as a method of achieving both economic efficiency and economic equality. In the twentieth century, planning drew much strength from the

development of mathematical programming and cybernetics, which, according to the great Soviet scientist Andrey Kolmogorov, means "the study of systems of any nature which are capable of receiving, storing, and processing information so as to use it for control" (see: Stuart Umpleby, "Definitions of Cybernetics," *American Society for Cybernetics*, 1982, revised 2000, online: https://asc-cybernetics.org/definitions/).

Lecture III

This lecture is presented by the Venerable Master himself/herself using the necessary logistical infrastructure—for example, a whiteboard on which he/she writes and explains various elements of the lecture, a slide projector, PowerPoint, etc.

V.M.: By the term "economic system," we mean a means by which societies or governments organize and distribute available resources (factors of production), goods, and services across a geographic region or country. An economic system can be mathematically modelled as an input-output system, and it is based on the principle of rationality. In general, "rationality" means that social behavior can be seen in terms of actors pursuing goals.

The "rationality postulate" implies the following: (i) actors have well-ordered preference systems over the set of outcomes (of alternative actions): for all pairs c_i and c_j, there is a preference relation R such that either $c_i R c_j$ (the actor prefers c_i to c_j), or $c_j R c_i$ (the actor prefers c_j to c_i), or both (the actor is indifferent); (ii) each actor's preference system is substantively independent of the other social variables; (iii) each actor acts to maximize one's utility index. In particular, one can formulate a decreasing sequence of numbers (these numbers are called "utilities," u_n) where the largest number is assigned to the most preferred outcome, the second largest number to the next outcome in the preference order, etc. The function that maps consequences to numbers representing an actor's preference over those outcomes is said to be a "utility function." The most well-known utility function is the von Neumann–Morgenstern utility function, which is defined as follows: the actor considers a set of all conceivable states of the world and assesses the likelihood of each state S by assigning a probability $p(S)$ to it, so that the expected utility $U_e(A)$ for an action A can be calculated by multiplying the probability $p(S)$ of each state's occurring by the utility $u\big(C(S,A)\big)$ of the outcome that results from the given state of the world and the given action, and then summing these products over all the possible states:

$U_e(A) = \sum_{allS} p(S)u\big(C(S,A)\big)$;

the actor chooses A such that $U_e(A)$ is maximized.

In economics, "marginal utility" is the additional satisfaction or benefit that an economic actor derives from buying or consuming an additional unit of a commodity or service. Therefore, "marginal utility" (MU) can be defined as the derivative of "total utility" (TU) with respect to the quantity bought or consumed (Q), symbolically $MU = \frac{dTU}{dQ}$. Hence, given a marginal utility function $MU(Q)$, the total utility is given by $TU(Q) = \int MU(Q)dQ$, where integration is carried out over a certain interval of bought or consumed quantity Q. In economics, the term "marginal" generally corresponds to the mathematical notion of differentiation, and the term "total" corresponds to the mathematical notion of integration.

In political economy, the term "free competitive market" means a market that has the following fundamental characteristics (see: Paul A. Samuelson and William D. Nordhaus, *Economics*, fourteenth edition, New York: McGraw-Hill, 1992, pp. 54–57, 286–91, 732–36): (i) the number of actors in it is so large that none of them can decisively influence prices by changing their supply or demand, so any actor in such a market is forced to consider prices as given variables independent of their behavior; and (ii) entry into any profession or economic sector and exit from any profession or economic sector are free. A free competitive market is in a state of equilibrium if and only if the following three conditions are met (ibid):

(i) *Subjective equilibrium condition.* All members of the economic system (households, firms, public capital, etc.) achieve the maximization of the utility, the profit, or the income that they derive from the ownership of factors of production (i.e., land, labor, and capital) based on equilibrium prices. In particular, the consumer's effort to maximize total utility, subject to a number of constraints, the most important of which are the consumer's income and the prices of the goods and services that the consumer wishes to consume, is referred to as the "consumer's problem." The solution to the consumer's problem is referred to as "consumer equilibrium." Assume that a consumer cares about consuming n goods: $good\ 1, good\ 2, \dots, good\ n$. This consumer knows the prices of these n goods and has a fixed income or budget that can be used to purchase quantities of these n goods. The consumer will purchase quantities of goods $1,2, \dots, and\ n$ so as to completely exhaust the corresponding budget. The actual quantities purchased of each of these n goods are determined by the condition for consumer equilibrium, which is:

$$\frac{marginal\ utility\ of\ good\ 1}{price\ of\ good\ 1} = \frac{marginal\ utility\ of\ good\ 2}{price\ of\ good\ 2} = \cdots = \frac{marginal\ utility\ of\ good\ n}{price\ of\ good\ n}.$$

This is subject to the constraint that the consumer's purchases do not exceed his/her budget. As long as consumer income and prices are given (in order to determine the quantity of an economic good that can be purchased with one unit of income), the demand for consumer goods can be determined. With regard to producers, the subjective equilibrium condition means that producers maximize their profits by optimizing the combination of factors of production and by optimizing the scale of production. The optimization of the combination of factors of production is achieved by combining them in such a ratio that the marginal productivity of the quantity of each factor of production that can be purchased for one unit of money is equal to the marginal productivity of the quantity of any other factor of production that can be purchased for one unit of money. Because the prices of the factors of production are assumed to be given, this condition determines the minimum production-cost curve. Given the minimum production-cost curve, production scale optimization is achieved when the marginal cost equals the price of the product (the price of the product is assumed to be given in the market). In general, producers will continue to produce for as long as they can sell the corresponding commodity at a price that exceeds the cost of producing an additional unit of output (i.e., the marginal cost of production), and consumers will continue to consume for as long the satisfaction that they derive from consumption exceeds the price that they pay (i.e., the marginal benefit of consumption). In this way, the production of each producer can be determined, but also the demand of each producer for factors of production. The aforementioned determination is inextricably linked to the first fundamental feature of a free competitive market (namely, to the assumption that the number of actors in it is so large that none of them can decisively influence prices by changing their supply or demand, so any actor in such a market is forced to consider prices as given variables independent of their behavior). The determination of the total demand of an entire sector of the economy is based on the second fundamental feature of a free competitive market—that is, on the assumption that entry into any profession or economic sector and exit from any profession or economic sector are free. Therefore, the total output of any sector of the economy satisfies the following condition: the

price of the corresponding good or service is equal to the average cost of production (per unit of production). Given each producer's output (quantity of production) and demand for factors of production, and given the total output of each sector of the economy, the demand of each sector of the economy for factors of production can be determined. Hence, if the prices of products (goods and services) and the prices of the factors of production are given, then the supply of products and the demand for the factors of production can be determined. The owners of factors of production (i.e., of land, labor, and capital) maximize their income when they sell the services (specifically, the economic result of the employment and utilization) of the corresponding factors of production to those who offer the highest price. If the demand for the factors of production is given, then the distribution of the factors of production between the different sectors of the economy can be determined.

(ii) *Objective equilibrium condition*. Equilibrium prices are determined by the condition that demand equals supply for all goods and services. Classical economists such as Adam Smith (1723–90) have argued that the free market will always be in equilibrium. This is because a shortage of any economic commodity would cause a higher price in general, which would reduce demand, leading to an increase in supply, given the right incentive, and a similar path to equilibrium would occur in the event of oversupply in a market.

(iii) *Organizational equilibrium condition*. Consumers' income is equal to their revenues from the sale of the factors of production (i.e., land, labor, and capital) that they hold plus business profits. In a state of equilibrium, business profits are considered to be equal to zero if they are understood to be substantively distinct from the money generated from the sale of factors of production. However, this does not mean that profits disappear, since, in a state of equilibrium, profits are comprehended and counted as revenues from the sale of business-managerial skills. According to the aforementioned organizational equilibrium condition, ultimately, the only variables that determine the supply of and the demand for commodities (goods and services) are prices. In this way, different price sets correspond to different supply and demand scales.

Based on the aforementioned objective equilibrium condition (according to which, equilibrium prices are determined by the condition that demand equals supply in every market), we can determine and select the set of

prices that ensures that the plans of consumers and the plans of producers agree. This condition means that the supply of and the demand for each good/service are equal to each other. Any price that satisfies this condition is said to be an "equilibrium price."

The aforementioned economic model is the *theoretical* solution to the problem of economic equilibrium under the economic regime of the free competitive market. However, the *practical* solution to the problem of economic equilibrium under this economic regime is an approximation of the solution corresponding to the aforementioned model; in practice, the solution to the problem of economic equilibrium is based on the "method of successive approximations."

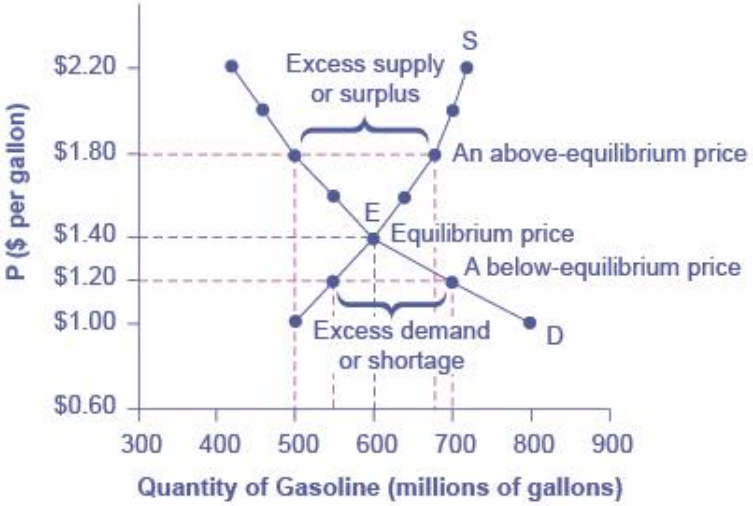
The solution to the problem of economic equilibrium with the method of successive approximations is based on the "parametric price function," which means that prices derive from the behavior of all economic actors in the market, but each economic actor individually considers the actual market prices as given elements—that is, as structural elements—to which economic actors must adapt themselves. In other words, in a free competitive market, every economic actor is faced with a market situation that every economic actor tries to exploit but no economic actor can individually control. From this viewpoint, market prices are parameters of an economic reality that no economic actor can individually control, and they determine the economic adjustment policy of economic actors. According to the French mathematician and economist Léon Walras (1834–1910), the equilibrium value of these parameters is determined by the objective equilibrium condition (according to which, equilibrium prices are determined by the condition that demand equals supply in every market) through a series of successive attempts (Léon Walras, *Elements of Pure Economics*, translated by William Jaffé, Homewood, Illinois: Richard D. Irwin, Inc., 1954).

According to Walras's analysis, economic actors receive (from the market) a random set of prices, and, based on this random set of prices, they fulfill their subjective equilibrium condition, thus maximizing their utility function (as mentioned above, the subjective equilibrium condition is the assumption that all members of the economic system maximize the utility, the profit, or the income that they derive from owning factors of production based on equilibrium prices). Since every economic good/service is associated with a quantity that represents supply and a quantity that represents demand, the objective equilibrium condition also plays a decisive role (as mentioned above, the objective equilibrium condition is the assumption that demand equals supply for all goods and services). Hence, if the demand for an economic good/service is equal to

the supply of this economic good/service, then the overall equilibrium is restored, and market prices are equilibrium prices. If the quantity demanded differs from the quantity supplied, on the other hand, then the competition of the sellers will change the price, pushing it towards a new equilibrium price.

When supply exceeds demand for a good/service, the price of this good/service tends to fall. When demand exceeds supply of a good/service, the price of this good/service tends to rise. Therefore, a new set of prices emerges, which is the new framework in which economic actors will try once more to satisfy the subjective equilibrium condition. As the effort to satisfy the subjective equilibrium condition is carried out, economic actors receive (from the market) a new set of quantities supplied and quantities demanded. If supply and demand are not equal to each other for each good/service, then prices will change again. Another set of prices will emerge, which will be a new framework in which economic actors will try to satisfy the subjective equilibrium condition again. In this way, a new set of quantities supplied and quantities demanded will emerge. The aforementioned process will continue in the same way until the objective equilibrium condition is fully met and, finally, general economic equilibrium is reached. Hence, in practice, historically given prices constitute the framework for applying the method of successive approximations.

Figure 3-1: Market Equilibrium Price: in this example, the supply curve (S) and the demand curve (D) intersect at the equilibrium point E, representing a price of $1.40 and a quantity of 600 (Source: Wikimedia Commons: Author: OpenStax College; https://openstax.org/details/books/principles-microeconomics).



The above-mentioned model of economic equilibrium under the economic regime of the free competitive market and the related method of successive approximations lead to only a partially useful approach to economic analysis, as they are simplifications, and, for the most part, the assumptions on which they are based are not descriptions of real conditions prevailing in the economy (see: W. Harrison Carter and William P. Snavely, *Intermediate Economic Analysis*, New York: McGraw-Hill, 1961, p. 266). In particular, the above-mentioned model of economic equilibrium (under the economic regime of the free competitive market) and the related method of successive approximations ignore the following issues and practical problems, which must be explicitly and rigorously addressed by economic analysis if the latter is to be of sufficient empirical or operational importance.

The Optimization of Income Distribution

The above-mentioned model of economic equilibrium and the related method of successive approximations ignore the fact that the existence of private ownership of factors of production implies that the distribution of income is determined by the distribution of ownership of factors of

production (i.e., land, labor, and capital). The distribution of ownership of factors of production is a historical fact that arises regardless of the requirements of maximizing social welfare. For instance, the distribution of land ownership is very different in a country where large feudal lands survive (by maintaining the feudal privileges of certain families and ecclesiastical institutions) from a country where the great feudal lands of the feudal era have been dissolved. It goes without saying that eliminating the feudal thrombosis of the economic circuit is not enough. In capitalism, the ownership of the factors of production can generally be distributed in a very unequal way, and soon a large part of society can be found to possess nothing but its labor force. In such circumstances, the value of demand does not reflect how urgent the needs of economic actors are, and the distribution of the factors of production that is determined by the value of demand for the corresponding consumer goods and services is far from achieving the maximization of social prosperity.

In order to address the aforementioned problems, a **Central Economic Planning Authority (CEPA)** is needed, which, given the freedom of consumption and the freedom to choose a profession, will calculate the optimal distribution of income and will adjust the actual distribution of income accordingly. Thus, first of all, what is meant by optimal income distribution must be defined in a mathematically rigorous way. The term "optimal income distribution" refers to that income distribution which maximizes the overall well-being of society by satisfying the following two conditions: (i) the marginal utility of income must be the same for all consumers in order to achieve that income distribution which ensures that the price at which the different consumers are prepared to purchase a good/service represents the same degree of urgency or need; (ii) the income must be distributed in such a way that the division of labor between the different professions ensures that the differences in the value of the marginal product of labor between the different professions are equal to the differences in marginal dissatisfaction that characterizes each of them. If we look carefully at the aforementioned definition of the optimal distribution of income, we observe that, since the curves of the marginal utility of income are supposed to be the same for all individuals, condition (i) is satisfied when all consumers have the same income, while condition (ii) entails a diversification of incomes, since, in order to ensure the required division of labor, the differences in the marginal dissatisfaction that characterizes the various professions must be compensated by differences in incomes. This contradiction between condition (i) and condition (ii) is not substantive, and the CEPA can easily overcome it by incorporating subjective factors into the utility functions of

individuals, so that the dissatisfaction characterizing each profession is considered an opportunity cost. Such subjective factors are, for example, the amount of leisure time that a profession allows its practitioners to have, the physical and mental energyexpended by a worker to perform a task, the degree of security that is provided by a profession, the pleasure that someone derives from his/her profession, etc. Therefore, choosing an occupation A that offers a lower monetary income than another occupation B, but at the same time offers less dissatisfaction than occupation B, can be interpreted as a purchase (on the worker's side) of a subjective factor (either more leisure, milder working conditions, higher security, or more job satisfaction, etc.) at a price equal to the difference between the monetary income earned through occupation B and the monetary income earned through occupation A. In view of the foregoing, the income differences required by the aforementioned condition (ii) represent prices paid by workers to ensure different working conditions and different objectives.

Consequently, the CEPA can construct an approximate model of optimal income distribution as follows: initially, it allocates the same monetary income to all workers. Then, it charges the pursuit of each profession with a price, having previously conducted the corresponding social research (thus knowing the attitudes and the opinions that workers have regarding each alternative professional choice). Based on the results of this approximate model of optimal distribution of income between the various professions, the CEPA can make the corresponding interventions in the market (through fiscal policy, monetary policy, and labor legislation) in order to maintain the actual distribution of income in politically acceptable deviation thresholds with regard to the theoretically optimal income distribution calculated on the basis of the above approximate model. Hence, in parallel with the scientific perfection of the work of the CEPA, it is required to operate an effective system of decentralized social analysis and programming in order for the CEPA to have effective mechanisms of social feedback at its disposal.

The Content of Economic Analysis

As pointed out by the English economist Arthur Cecil Pigou (1877–1959), who was a prominent representative of the neoclassical "school" of economics, an economic system based on private enterprise and, consequently, on free market theory is prone to serious errors in calculating the alternatives sacrificed or realized in the production process (A. C. Pigou, *The Economics of Welfare*, fourth edition, London:

Macmillan, 1932). To a large extent, the life, safety, and physical and mental health of workers are sacrificed without being included in the total production cost. Thus, free competitive market theory is based on and leads to erroneous macroeconomic accounting, consequently causing significant social waste. As Pigou has shown, social planning measures are needed to avoid much of this social waste, including appropriate legislation, appropriate taxation, and a system of social rewards (ibid).

In addition, private producers find it particularly difficult to assess the significant benefits and the significant costs associated with external economies of scale and external counter-economies of scale, respectively. External economies occur when the actions of economic actor A cause a positive change in the well-being of economic actor B, and B does not pay A and has no ability to control A's activity. External counter-economies occur when the actions of an economic actor A cause a negative change in the well-being of another economic actor B, and B is not compensated by A and has no ability to prevent the negative activity of A. External economies/counter-economies are called external economies/counter-economies of scale when they depend on the size of the corresponding sector of the economy or the corresponding enterprise. According to the above definition of a free competitive market, in this economic regime, the number and the policy of firms producing an economic good/service give rise to a situation in which the price of the product is equal to the average cost achieved by private producers. Therefore, neither that social benefit which is due to external economies nor that social cost which is due to external counter-economies is taken into account. These cases of inaccurate or even false macroeconomic accounting are exacerbated by the inherent contradictions of the capitalist system. These have been thoroughly analyzed by Karl Marx and Friedrich Engels, who have argued that the contradiction between the production and the circulation of capital is inherent in capitalism, because capitalist production is not only commodity production but also production of surplus value, which they call exploitation of labor.

Unlike private producers, the CEPA must take into account all alternatives and, therefore, all external economies and all external counter-economies. Specifically, the CEPA can construct a comprehensive economic analysis model for estimating social waste, external economies, and external counter-economies as follows: it determines that level of production of each sector of the economy at which the marginal cost of that economic sector for the production of a given quantity becomes equal to the price of the product. In this way, the CEPA can identify and estimate external economies and external counter-economies that may result from any

change in the output of the economic sector under consideration. In this model, external economies and external counter-economies will appear in the form of a discrepancy between the average cost and the marginal cost of the economic sector under consideration (given that the model equates the marginal, rather than the average, cost with the price of the product). Thus, after identifying and assessing external economies and external counter-economies, the CEPA can implement a corrective mix of fiscal and monetary policy in order to achieve the politically desired result. Once again, we realize that the scientific perfection of the work of the CEPA must be combined with the existence and the utilization (by the CEPA) of effective mechanisms of social feedback. Hence, a socialist economic model based on central planning allows private enterprise to operate—provided that, as the brilliant Russian-British economist Alec Nove (1915–94) has argued, private enterprise does not produce capitalist structures that contradict the principles of efficiency and rationality governing central economic planning (Alec Nove, *The Economics of Feasible Socialism*, London: Allen and Unwin, 1983).

According to Alec Nove, enterprises should be as small as possible to allow real participation on the part of the producers, but some must be larger because of the needs for economies of scale. Thus, Alec Nove envisages a whole range of enterprises including centralized state enterprises, state-owned enterprises directly managed by the labor force, co-operatives, small-scale private enterprises, and freelance self-employed workers, so that competition will continue to exist without the drawback of "wasteful" individual competition (ibid). However, there is no provision for any class of capitalist speculators, since there is no unearned income arising simply from ownership of capital or land, and every existing private entrepreneur *works* (ibid). Moreover, it is worth mentioning that, in the late twentieth century and in the early twenty-first century, Maoist China took particular productine aspects of capitalism from liberalism and used them as a tool to improve society, proving that the productive dynamics of capitalism is neither the exclusive preserve of democratic regimes nor a consequence of the classical free-market doctrine.

As regards monetary policy, it should be mentioned that a major flaw of capitalism is that it tends to abstract the financial system from the overall economic reality and to embolden usurers and other financial speculators. Referring to the actual state of affairs in the capitalist world, Arthur M. Okun has argued that "the task of combining prosperity with price stability now stands as the major unsolved problem of aggregate economic performance" (A. M. Okun, *The Political Economy of Prosperity*, New

York: Norton, 1970, p. 130). Irving Fisher's equation (actually, identity) of exchange is the following formula:

$$MV = PQ$$

where M is the quantity of money, V is the velocity of the circulation of money (i.e., the amount of nominal Gross National Product each year divided by the money stock), P is the price level, and Q is aggregate output (i.e., the Gross National Product is equal to PQ). Thus, according to Fisher, if both V and Q are constant, then a change in the money supply, M, results in an equal percentage change in the price level P.

The above-mentioned equation implies that

$M = \frac{1}{V}PQ.$

Since V is constant, $1/V$ can be replaced by a constant k. Additionally, when the money market is in equilibrium, the demand for money, M_d, is equal to M. Hence,

$M_d = kPQ,$

which means that, according to Fisher's model, the demand for money is a function of income or output and does not depend on interest rates.

However, the velocity of the circulation of money, V, is not constant in practice, even in the short-run, and especially during periods of recession. In fact, John M. Keynes extended Fisher's equation of exchange by pointing out that there are three motives of holding money: (i) Transactions motive: money is a medium of exchange, and, as income rises, people have more transactions and hold more money. (ii) Precautionary motive: people hold money for emergencies, and money demand is again expected to rise with income. (iii) Speculative motive: money is also a way for people to store wealth, and, under the speculative motive, the demand for money is negatively related to the interest rate. Moreover, Keynes modelled the demand for money as the demand for the real (as opposed to the nominal) quantity of money (i.e., real balances), M/P. According to Keynes, the demand for real money balances is a function of both income and interest rates:

$$\frac{M}{P} = f(Q, r)$$

where Q is output or income and r is the interest rate (and, hence, the velocity of the circulation of money fluctuates with the interest rate).

The system of critical rational socialism to which our Order adheres implies that the monetary system should be based on labor (that is, on physical and mental work), which is the real source of value. In other words, the value of a country's currency or paper money should be directly linked to labor (physical and mental work). Indeed, we propose a "labor standard", a monetary system in which the value of a currency is

based on labor. Therefore, we dismiss both the "gold standard" (i.e., a monetary system in which the value of a currency is based on gold) and any "fiat system" (i.e., a monetary system in which the value of a currency is not based on any physical commodity but is instead allowed to fluctuate dynamically against other currencies in the foreign-exchange markets, usually under the supervision of a banking elite). In view of the foregoing, we do not believe that cryptocurrencies (digital currencies based on blockchain technology) are a trustworthy and better alternative to previous monetary systems. In fact, cryptocurrencies have shown that parody can be turned into a "real asset," and they give rise to a financial system of institutionalized insanity. Dogecoin is a characteristic case in point: this is a cryptocurrency that started off as a meme, but it soon became a mainstream digital currency. In 2013, two software engineers used the image of a dog and created Dogecoin in order to make fun of how people would invest in anything, but people did exactly that; in April 2021, Dogecoin's market capitalization surged to more than fifty billion dollars. Like any other cryptocurrency, Dogecoin has value because people have been made to believe that it has value, due to online marketing campaigns and other means of carrying out psychological operations.

Rational Control of the Production Process

The huge economic progress that took place during the nineteenth and the twentieth centuries was mainly a consequence of scientific, technological, and organizational innovations that (as they were integrated into the production process) increased the productivity of a combination of factors of production, or created new economic goods and services. However, given the contradictions of the capitalist system, the results of the integration of scientific, technological, and organizational innovations into the economy are not homogeneous. Companies that innovate make a direct profit or increase their profitability, but this profit (or increase in their profitability) is a temporary phenomenon, as free competition will tend to equate the price of the product with the average cost of production, according to the above-mentioned model of the free competitive market. On the other hand, companies that use outdated factors of production or outdated production models, and companies that produce competitive economic goods that can be easily substituted with others (by competitors) in the market, suffer losses which lead to a devaluation of the capital invested in them. In the free competitive market regime, due to the parametric function of price and the freedom of entry and exit enjoyed by private companies in every sector of the economy, any innovation is

inevitably associated with a reduction in the value of some old investments, since, in principle, there is no way of reacting against a given innovation. What entrepreneurs can do to respond to their competitors' innovations is to try to innovate in their own companies, causing, in turn, losses for their competitors. Moreover, innovative companies need to constantly strive to innovate, because free competition tends to nullify the profitability of existing innovations (due to the freedom of entry of new competitors in each sector of the economy), so the more a company leads in the field of innovation the more profitable it becomes.

Nevertheless, as the prominent American economist, diplomat, and economic consultant John Kenneth Galbraith (1908–2006) has pointed out, industrial planning is inextricably linked to the size of the industrial complex, and size is not only a particular underpinning and provider of profits, but also the general underpinning and provider of technology and innovation (John K. Galbraith, *The New Industrial State*, with a new foreword by James K. Galbraith, Princeton: Princeton University Press, 2007). Furthermore, due to the inherent contradictions of capitalism, in the free competitive market there emerge several phenomena that oppose free competition, such as the following: (i) monopolies, (ii) monopsonies, (iii) oligopolies, (iv) oligopsonies, and (v) groups of companies (i.e., gentlemen's agreements, cartels, concerns, pools, and trusts; see: Clive M. Schmitthoff and Frank Wooldridge, eds, *Groups of Companies*, London: Sweet and Maxwell, 1991).

When the size of some business units increases so much that they can nullify both the efficiency of the parametric price function (thus being able to exert some control over prices) and the freedom of entry of new firms and new investors in a sector of the economy in general, such companies develop a strong tendency to prevent any development that could bring about a devaluation of the capital already invested. Therefore, when a firm is not forced by market competition to innovate, it will only innovate when the old invested capital is depreciated or if the reduction in production costs that is achieved by the immediate implementation of an innovation exceeds the devaluation of the capital already invested. As Galbraith (ibid) has explained, this delay in actualizing available possibilities to improve the economy works to the detriment of social interest. In addition, the British economist Lionel Robbins (1898–1984), who was made a life peer as Baron Robbins of Clare Market in the City of Westminster in 1959, has pointed out that the attempt of certain capitalist elites to maintain the value of their invested capital may lead them to prevent the entry of new producers who find the prospects of one economic sector more attractive than the prospects of other economic sectors, as well as to postpone or

cancel the implementation of technical improvements that reduce costs and, consequently, reduce the price paid by the consumer (Lionel Robbins, *The Great Depression*, with a new introduction by Murray Weidenbaum, London: Routledge, 2017).

In any case, the ruling capitalist elite seeks to keep the general development of innovation under control and to manage innovations according to its own particular interests, thus coming into conflict not only with the social interest, but also with a rival capitalist elite which wants to become the new ruling capitalist elite by displacing the previous one. As a result of the contradictions of the capitalist system, the protection of monopoly privileges and specific investments contradicts economic progress, in the sense that it hinders the reduction of prices and the improvement of the quality of economic goods and services, and it is a major source of imperialist rivalry between the great powers of the international system. When the pressure of scientific, technological, and organizational innovations for structural change is far greater than the tendency of some capitalist elites to maintain the value of old investments and their control over economic dynamics, an economic crisis ensues. This crisis is exacerbated, at a later stage, by the intensification of stock speculation, which manifests itself through a bear market for old investments and a bull market for new investments (innovations).

In times of great capitalist crises, capital, as a structure and as a type of relationship, attacks a large number of capitalists. This statement may sound strange at first, but it is not; the capitalist system itself is strange.The system of free competition invites and urges entrepreneurs to maximize their profits, acting according to the rationale of the definition of the free competitive market; however, if many entrepreneurs embrace and apply the definition and the rationale of the free competitive market in practice, then they will see their profits destroyed—they will realize that this economic mode of thinking is appropriate for those who live in the world of Disneyland and belongs to Ayn Rand's ludicrous tales.

Indeed, the model of the free competitive market, in its ideal form, is useful for waging an intellectual war against the capitalist establishment using the ideological weapons and criteria of the capitalist establishment itself—it can be attacked using its own theoretical arsenal. The capitalist establishment itself is inconsistent and, indeed, replete with contradictions: it espouses and wants the free competitive market as long as and to the extent that the free competitive market secures the privileges of the capitalist establishment, but the capitalist establishment opposes the free competitive market and violates its rules as soon as the free competitive market questions or threatens the privileges of the capitalist establishment.

It is important to understand that, in *real capitalism*, the system deceives the entrepreneur (as free competition tends, in principle, to nullify profits), and the entrepreneur deceives the system (as he/she seeks to violate and distort the free market system to his/her advantage). This awareness is a "secret" of capitalism, but it is widely and dramatically revealed during the great capitalist crises. Therefore, another fundamental responsibility of the Central Economic Planning Authority (CEPA) under critical rational socialism is the rational control of the production process, in order for innovations to be integrated into the production process in a way that maximizes social prosperity and, consequently, in order to optimize financial flows.

Finally, it is imperative that the Central Economic Planning Authority (CEPA) exercises full and strong control over the financial system in accordance with the above-mentioned rationale. Peter Bond, in his volume entitled *Monetary Economics* (Worcester: Northwick Publishers, 1989, p. 24), has summarized and explained the system of financial intermediation as follows:

> In any economy there will be at any given time two groups of economic agents: (i) those we term *SURPLUS UNITS*, i.e. those whose revenue exceeds their current expenditure during the period under consideration . . . (ii) those we term *DEFICIT UNITS*, i.e. those whose expenditure exceeds their current revenue in a given time period. Given the existence of surplus units and deficit units, some mechanism is required to ensure that the surplus funds are channeled to the deficit units . . . it is very often the case that the individual with surplus funds will lend them to a financial institution or *financial intermediary* which will then on-lend these funds by itself buying company shares, government stocks or whatever assets it normally invests in.

Additionally, Peter Bond (ibid, p. 28) has explained the role of banks as follows:

> Amongst the many types of financial intermediary, the banks have a special place because they are the prime providers of money in a modern economy . . . One common feature of all banks is *the taking of deposits* . . . A second common feature is that of *the encashment of deposits* . . . The third (and in many ways the most distinctive) feature of banks is the transfer of deposits to third parties, for the most part by way of cheques but also via standing orders, direct debits and other transfer mechanisms.

The purpose of the control of the financial system by the CEPA is to optimize the allocation of capital between economic agents in accordance

with the above-mentioned rationale for the rational control of the production process. To leave financial intermediation, and especially banking, to the forces of a deregulated market is equivalent to leaving the flows of capital to the forces of unfettered speculation and blind passion.

In the context of critical rational socialism, the head of government (namely, the "supreme leader" of a socialist polity) and the Central Economic Planning Authority (CEPA) represent an updated, modern version of Plato's political vision. The vertical and technocratic hierarchical system that we propose stands in stark contrast to libertarian socialism, libertarian communism, and postmodern leftism, but it should not be confused with other historical models, such as those of the tyrant, the dictator, the monarch, and similar others. The CEPA is analogous with the government of philosophers delineated by Plato in the *Republic*. If Plato's philosophers were the wisest of their times, in the era of modernity and globalization, the wisest persons will be the ones that have acquired the most knowledge in science, technology, economics, politics, ethics, and aesthetics. If the tyrants of the past exercised power as intellectually and morally deficient men equating "political realism" with their own rules of thumb, mentalities, and personal readings of history, the supreme leader of a polity structured and organized according to the system of critical rational socialism will have the highest level of expertise in every field of human endeavor, and will have fully endorsed and assimilated Alexander Bogdanov's vision of a "universal organizational science."

In our Order, we use the crossed hammer, sickle, and pencil as a symbol meant to represent the solidarity and spiritual unity among the supporters of critical rational socialism, which ideologically underpins our Order. The sickle is the symbol of the peasants, and the agricultural sector of the economy in general. The hammer is the symbol of the industrial workers, and the manufacturing sector of the economy in general. The pencil is the symbol of the intellectuals and the technocrats, including philosophy, science, technology, and art. The pencil also symbolizes the tertiary sector of the economy. Thus, the crossed hammer, sickle, and pencil is a symbol of aristocratic (not plebeian) socialism and an elitist vertical model of political authority.

Figure 3-2: The crossed hammer, sickle, and pencil: a symbol created by Dr. Nicolas Laos in order to be the distinctive emblem of an international network that endorses and promotes his theory of critical rational socialism. The hammer-sickle-and-pencil symbol implies an aristocratic and scientifically rigorous conception of socialism that utilizes and endorses several aspects of Marxism–Leninism, but its roots can be traced to Plato's political thought, practical philosophy, cybernetics, and a universal ethic inspired by Buddha, Confucius, Orpheus, Socrates, and Kant. Thus, in the context of our Order, the theory of critical rational socialism is the economic component of a broader vision of enlightened totalitarianism.

CHAPTER 4

MASTER MASON OF THE M∴P∴R∴S∴M∴

The ceremony of acceptance into the Third Degree of Freemasonry is called "Raising." Every candidate to be raised to the Third Degree of the Modern and Perfecting Rite of Symbolic Masonry must have been a Fellow Craft for a minimum of four years (this qualification is obligatory and cannot be waived by dispensation), and other specific conditions of promotion (pertaining to the candidate's performance in the Order's inner activities and projects) are often insisted upon. One of the prerequisites is that a candidate is required to present a research paper and pass an assessment test before his/her candidacy for the Third Degree can be considered (the subject of the research paper may be of his/her choosing, but it must belong to one of the following scholarly fields: the social sciences, the humanities, or the natural sciences; and it may not be of a Masonic nature).

In the Third Degree, the Lodge has the same layout as in the First Degree—but, on the west side of the Altar of Truth, on the Mosaic Pavement, and at a comfortable distance from the kneeling stool, is laid a peculiar carpet that is known as the "Tomb." This carpet comprises the following three main illustrations: (i) A tomb that is also a temple. This is in the form of a traditional coffin over which is laid a scroll illustrating the arched gate of a temple and a Mosaic Pavement before the gate. (ii) A spring of acacia is shown at the head of the tomb, above the west side of the coffin. (iii) Three working tools—specifically, a Plumb Rule, a Level, and a Maul—are shown at the foot of the tomb, on the east side of the coffin; and three more working tools—specifically, a Skirret, a Pair of Compasses, and a Pencil—are shown at the head of the tomb, on the west side of the coffin. Under the Skirret, the Pair of Compasses, and the Pencil, and above the scroll, there is a symbol of death consisting of a human skull and two long bones crossed together under the skull.

The Secretary invites only the Master Masons to attend the meetings of the Lodge whenever the latter works in the Master Mason degree, and the Outer Guard ensures that only the members of the Third Degree enter into

a Lodge of Master Masons (with the exception of a Candidate for Raising).
Before the Opening Ceremony, the Outer Guard waits outside the door of the Lodge. The O.G. places the Candidate in the Chamber of Reflection, where the Candidate remains alone during the Opening Ceremony. The Candidate wears the apron and the sash of a Fellow Craft, white gloves, and the Order's breast jewel.
Finally, before the Ceremony of Raising, the Lodge room should be discreetly scented by burning red sandalwood incense sticks or essential oil.

Opening Ceremony

V.M.: Brother/Sister Senior Overseer and Brother/Sister Junior Overseer, please confirm that all within the Lodge are Master Masons.
S.O.: Brothers and Sisters, it is the Venerable Master's command that you prove yourselves Master Masons.
All, except the V.M., the S.O., and the J.O., rise, take Step, give the Master Mason Sign and retain it. When this is done, the V.M., the S.O., and the J.O. rise, take Step, and give the Master Mason Sign. All cut the Master Mason Sign and resume their seats.
J.O.: Brother/Sister Inner Guard, act in accordance with the duties arising from your office.
The Inner Guard, holding his/her sword in his/her left hand, pointing upward, opens the door of the Lodge and instructs the Outer Guard (who was standing outside the door of the Lodge, guarding the porch of the Lodge) to take his/her seat in the Lodge. The Outer Guard, holding his/her sword in his/her right hand, pointing downward, enters the Lodge and takes his/her seat in it. The Inner Guard closes and locks the door of the Lodge, takes the key and puts it on the Junior Overseer's pedestal, and says to the Junior Overseer: "Brother/Sister Junior Overseer, we are internally and externally safe." When this is done, the Inner Guard (holding his/her sword in his/her left hand, pointing upward) takes his/her seat in the Lodge.
J.O.: The Lodge is working according to our rules of privacy and exclusivity.
S.O.: Venerable Master, the Lodge is working according to our rules of privacy and exclusivity.
At this point, the Inner Guard, who was holding his/her sword in his/her left hand, pointing upward, and the Outer Guard, who was holding his/her

sword in his/her right hand, pointing downward, put their swords in their scabbards.

V.M.: Freemasonry uses the language of symbology and allegory. Brother/Sister Senior Overseer, what is the shape of the Great Universal Temple?

S.O.: Perfect equilateral triangle, from North to South and from South to West.

V.M.: What is its depth?

S.O.: It extends from the surface to the center of everything.

V.M.: What is its length?

S.O.: From North to South.

V.M.: What is its width?

S.O.: From East to West.

V.M.: What is the Great Universal Temple covered with?

S.O.: A celestial canopy of diverse celestial bodies.

V.M.: Is this how this Temple is located under the Heaven, Brother/Sister Senior Overseer?

S.O.: Indeed, it is, Venerable Master.

V.M.: That's right, worthy Brother/Sister. Grand Master Hermes Trismegistus taught Asclepius as follows: "Do you not know, Asclepius, that Egypt is an image of heaven or, to be more precise, that everything governed and moved in heaven came down to Egypt and was transferred there? If truth were told, our land is the temple of the whole world." Brother/Sister Junior Overseer, how many Temples are there in the Universe?

J.O.: Three, Venerable Master.

V.M.: Which are they?

J.O.: The Ordinary Temple, which is the human body; the Symbolic Temple, which is the Earth; and the Perfect Temple, which is the Universe.

V.M.: What are the proportions of the Egyptian triangle?

J.O.: Its base is equal to three cubits, Venerable Master, its height is equal to four cubits, and its hypotenuse is equal to five cubits. In other words, by the term "Egyptian triangle," we refer to the right triangle whose sides have the lengths 3, 4, and 5, or one whose measurements keep these proportions.

V.M.: What is its function?

J.O.: It is the easiest right-angled triangle to build, and it was used by ancient Egyptians in order to determine and construct the right angles needed in order to build a strong Temple. A Diophantine equation is an equation in which only integer solutions are allowed. The term "Diophantine"

refers to the Hellenistic mathematician Diophantus of Alexandria, who made a systematic study of such equations and was one of the pioneers of algebraic geometry. The Egyptian triangle is the smallest Diophantine right triangle. If we multiply the proportions of the 3:4:5 Egyptian triangle by five, then we obtain a similar Egyptian triangle whose proportions are 15, 20, and 25 cubits. In ancient Egypt, the 15:20:25 Egyptian triangle was called "Isiac," and the term "Isiac" refers to the goddess Isis, who symbolizes the realm of the "sacred" and the longing for the absolute.

V.M.: Brothers and Sisters, working to the Glory of the Enlightened Humanity, I declare the Lodge duly open for the purposes of our Masonic Order in the Third Degree.

V.M. gives M.M.-gavel: nine knocks by three times three, given as follows: 3 short equal knocks, a short pause, 3 short equal knocks, a short pause, and 3 more short equal knocks.

S.O. gives M.M.-gavel.

J.O. gives M.M.-gavel.

The D.C. (holding his/her wand in the right hand) rearranges the Square and the Pair of Compasses that lay on the open Book of Constitutions so that both points of the Pair of Compasses are disclosed (and, therefore, the whole Pair of Compasses is elevated above the Square), implying that we are now in the degree of a Master Mason. When this is completed, the D.C. resumes his/her seat. Thereupon, the Lodge works in accordance with its agenda.

Ceremony of Raising

V.M.: It is well to gain knowledge, but it is best to grow wise, and we must also teach others. There are, Brothers and Sisters, some who appear to have done good work as Fellow Crafts, and they seek reception into the supreme grade of Freemasonry. It is a satisfaction to me to declare that, following a unanimous decision of this Lodge's Executive Board, consisting of the Venerable Master, the two Overseers, the Orator, and the Secretary, I have approved of the attainments of Brother/Sister . . . *(name and surname of the Candidate).* He/she has performed the necessary Masonic work, and has been selected to be raised to the sublime degree of a Master Mason. Brother/Sister Outer Guard, you will ascertain if Brother/Sister . . . *(name and surname of the Candidate)* is in attendance, and, if so, let him/her be prepared by the Expert.

At this point, all the lights in the Lodge room, except the desk lamps of the V.M., the S.O., and the J.O., are turned off (by the A.D.C., who is responsible for the lighting of the Lodge room). The I.G. takes the key

from the Junior Overseer's pedestal, lets the O.G. and the Expert leave the Lodge, closes and locks the door, and waits on the south side of the door. In this case, the Expert does not carry his/her wand. The O.G. guards the door from the outside, while the Expert goes to the Chamber of Reflection, and therein prepares the Candidate as follows: the Expert divests the Candidate of all his/her Masonic regalia, which he/she puts on the table that exists in the Chamber of Reflection, and then the Expert invests the Candidate with a tunic and a mantle. The mantle is of turquoise fabric, has a hood with green tasseled cords, and is embroidered with a symbol of death (specifically, a white human skull and two white long bones crossed together under the skull) below the left shoulder and with the same symbol of death below the right shoulder. The tunic is of orange-red fabric, about knee length, with the same symbol of death on the center of the breast. When this is done, the Expert takes the Candidate by the right hand, and leads him/her to the door of the Lodge. Once there, the Expert turns the Candidate backward, so that the Candidate stands with his/her back to the door of the Lodge, whereas the Expert faces the door of the Lodge. Then, the O.G. knocks on the door as a Fellow Craft, with 3 short equal knocks, a short pause, and 2 long equal knocks. Thereupon, the I.G. opens the door of the Lodge, and the following dialogue takes place aloud so that it can be heard in the East: The I.G. asks the O.G.: "Whom have you there?" The O.G. responds: "The Expert accompanying Brother/Sister . . . (name and surname of the Candidate), who has been regularly initiated into the Craft, passed to the degree of a Fellow Craft, and has made such further progress in the Craft as he/she hopes will entitle him/her to be raised to the sublime degree of a Master Mason." The V.M. says: "Brother/Sister Director of Ceremonies, let Brother/Sister . . . (name and surname of the Candidate) be admitted in due form." The D.C., without his/her wand, takes the Candidate by the left hand, while the Expert continues holding the Candidate's right hand. Then the I.G. lets the O.G., the Expert, the D.C., and the Candidate enter the Lodge, closes and locks the door, puts the key back on the Junior Overseer's pedestal, and resumes his/her seat. In the meantime, the O.G. resumes his/her seat, too, and the D.C. and the Expert assist the Candidate to enter the Lodge room walking backwards. Finally, the Candidate stands between the two Overseers' pedestals and faces the door of the Lodge (that is, he/she faces West), the D.C. stands on the left side of the Candidate holding the Candidate's left hand and facing East, and the Expert stands on the right side of the Candidate holding the Candidate's right hand and facing East.

S.O.: Brother/Sister . . . *(name and surname of the Candidate),* you have left the Chamber of Reflection, which symbolizes the womb, and you have

entered the Sanctum of the Mind, which is symbolized by the Lodge. The great psychoanalysts Sigmund Freud and Melanie Klein have analyzed the instinctive dualism underlying the conflicts in human emotional life, and they have identified two forces: the life instinct and the death instinct.

According to Freud, human emotional life is marked by a conflict between the life instinct, which seeks to preserve and enrich life, and the death instinct, which seeks the peace and inactivity of death. In particular, Freud argues that the death instinct has the following three major characteristics: firstly, it aims to achieve inactivity, passivity or sleep; secondly, it underpins the compulsion to repeat; thirdly, it underpins masochistic tendencies, which can be projected outward in the form of aggression. In psychoanalytic language, the death instinct consists in a phantasy that is characterized by the desire to re-enter the mother's body forever and return to a state of sleep in her womb. Consequently, the death instinct leads to masochism. Masochism is a symbolic expression of the death instinct's opposition to and attack against a potential or an achieved state of individuation, or autonomy, because individuation, or autonomy, implies separation and discretion, whereas the death instinct seeks the return to the mother's womb and to the order and the safety that prevail therein.

Studying the phantasy of returning to the mother's womb, Melanie Klein has concluded that the death instinct is activated by particular events, which can be divided into the following two categories: the first category is called separation, and it comprises frustrations away from the breast; and the second category is called envy, and it comprises frustrations at the breast. According to Klein, the infant responds to such frustrations with wishes to attack, control, and become the breast. Furthermore, Klein combines these phantasies, which can be subsumed under the concept of projective identification, with a phantasy of re-entering the mother's womb and attacking or eating rival babies which mother is thought to be producing. On the other hand, as the ego grows, the infant feels that it cannot sleep in the mother's body, becomes increasingly separate, and develops its own ego. Thus, as the ego grows, one may develop a fear of the aforementioned phantasies and wishes.

At this point, the D.C. and the Expert turn the Candidate clockwise towards the East, and they lead him/her directly as close to the west side of the "Tomb" (the peculiar carpet of the Third Degree) as possible without stepping on the "Tomb," standing on the Mosaic Pavement. The Expert releases the Candidate's right hand, and the D.C. releases the Candidate's left hand. The Candidate faces the Venerable Master and the

"Tomb." The D.C. stands on the left side of the Candidate and faces East. The Expert stands on the right side of the Candidate and faces East.

V.M.: You are standing on the verge of the abyss, where darkness grows darker and only decomposers work. Reflect upon these ghastly relics of what was once a Mason like you. In what you see lies hidden a great awareness. This somber gloom, this tomb, and these relics of a dead Mason are not vulgar appliances to excite fear, nor are they intended to teach elementary lessons about the evanescence of human life or about sacrifice. They sum up in few symbols the contradictions of human emotional life and, particularly, the problem of the death instinct.

The extroverted variety of the death instinct creates mental states that foster omnipotence, pseudo-maturity, and masturbatory attacks against dependency. In psychology, such mental states have been described as a narcissistic organization (according to Donald Meltzer's terminology), a false self (according to Donald Winnicott's terminology), or an idealized self-image (according to Karen Horney's terminology). If the extroverted variety of the death instinct tends to dominate the personality of a Freemason, then, in his/her hands, the Plumb Rule becomes an instrument and a symbol of megalomania.

The Junior Overseer gavels once.

The introverted variety of the death instinct creates mental states that foster a state of sleep and otherworldly bliss. In psychology, such mental states have been described as "morbid dependency" (according to Karen Horney's terminology) and as masochistic attacks against any attempt at individuation, independence, personal responsibility, and free thinking. If the introverted variety of the death instinct tends to dominate the personality of a Freemason, then, in his/her hands, the Level becomes an instrument and a symbol of suicide, or mortal addiction, or catatonia.

The Senior Overseer gavels once.

No one can become a true Master Mason unless he/she overcomes and controls the psychological tendency towards achieving a permanent state of non-tension, non-effort, peaceful sleep, uninterrupted peace, pure passivity, and blissful oblivion. This psychological tendency corresponds to Freud's conception of the death instinct, and the corresponding phantasy or mental representation is that of returning to the mother's womb.

The Venerable Master gavels once.

Go! Attain mastery over the instincts of life and death, and then return to us properly prepared to take the Solemn Obligation of a Master Mason.

At this point, all the lights in the Lodge room are turned on (by the A.D.C.). Thereupon, the D.C. takes the Candidate by the left hand and the

Expert takes the Candidate by the right hand. They turn him/her clockwise towards the West, and go directly to the door of the Lodge. In the meantime, the I.G. takes the key from the Junior Overseer's pedestal, lets the D.C., the Expert, and the Candidate leave the Lodge, closes and locks the door, and waits on the south side of the door. In the Chamber of Reflection, the D.C. and the Expert divest the Candidate of the mantle and the tunic, and reinvest him/her with the regalia of a Fellow Craft Freemason (namely, with his/her apron, sash, gloves, and breast jewel). When this is done, the D.C. takes the Candidate by the left hand, the Expert takes the Candidate by the right hand, and they lead him/her to the door of the Lodge, where the Expert knocks on the door as a Fellow Craft, with 3 short equal knocks, a short pause, and 2 long equal knocks. Then the I.G. opens the door of the Lodge, and the following dialogue takes place aloud so that it can be heard in the East: The I.G. asks the D.C.: "Whom have you there?" The D.C. responds: "Brother/Sister . . . (name and surname of the Candidate), on his/her return to the Lodge and properly prepared." Thereupon, the V.M. says: "Then let him/her be admitted in due form." Then the I.G. lets the D.C., the Expert, and the Candidate enter the Lodge, closes and locks the door, puts the key back on the Junior Overseer's pedestal, and resumes his/her seat. The D.C. and the Expert lead the Candidate before the kneeling stool, so that the Altar of Truth is in front of the Candidate, and the "Tomb" is behind the Candidate. The D.C. and the Expert release the Candidate's hands.

Venerable Master: Brother/Sister . . . *(name and surname of the Candidate),* you will now take the Solemn Obligation of a Master Mason. For this reason, you will kneel on both knees and place both hands on the Altar of Truth.

This is done. The V.M. gavels once, rises, and gives the Sign of Fidelity (he/she places the right hand on the left breast with the thumb extended in the form of a square and the rest of the fingers joined together). All other attendees (except the Candidate, whose hands are on the Altar of Truth) rise and give the Sign of Fidelity; the D.C. remains standing on the left side of the Candidate and gives the Sign of Fidelity; and the Expert remains standing on the right side of the Candidate and gives the Sign of Fidelity.

V.M.: Repeat your name at length and say after me: I . . . *(name and surname of the Candidate),* in the presence of the Universal Cause, often referred to as the Most High, and in the presence of this Lodge of Master Masons, promise and swear that I shall never directly or indirectly reveal the secrets and mysteries of the inner workings and the covert projects of the Third Degree of the Autonomous Order of the Modern and Perfecting

Rite of Symbolic Masonry, nor any of the secrets and mysteries that I have already received, except to an equal Master Mason who has likewise lawfully received them within this Order; and that I shall act as a responsible and conscientious Master of the Craft. Furthermore, I promise and swear to strictly and scrupulously follow the statutes and regulations of this degree, in accordance with the decrees and the directives of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, and to fulfill the duties of a Master Mason diligently, faithfully and without partiality, favoritism, or prejudice. I swear and promise obedience to the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry without equivocation or mental reservation and with no hope of escaping the penalty that our Order reserves for the traitors of the Third Degree.

After the aforementioned Solemn Obligation, the V.M. and all other attendees cut the Fidelity Sign, and they form the Fraternal Chain: all attendees, with the exception of the D.C., the Expert, and the Candidate, give their right neighbor the left hand, and their left neighbor the right hand, so that the Candidate is included in this Fraternal Chain between the two Overseers. The Fraternal Chain remains.

V.M.: Rise, newly obliged Master Mason among Master Masons of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.

The Candidate rises with the assistance of the D.C. and the Expert. All the attendees cut the Fraternal Chain and, with the exception of the D.C. and the Expert, resume their seats. The D.C. and the Expert turn left and, holding the Candidate by the left hand and the right hand respectively, conduct him/her to the north side of the Venerable Master's pedestal, so that the D.C., the Expert, and the Candidate are in a position parallel to the pedestal and about three feet from it, facing South towards the Venerable Master; and the D.C. and the Expert release the Candidate's hands.

V.M. (addressing the Candidate): Brother/Sister . . . *(name and surname of the Candidate),* the working tools of Master Mason are the Skirret, the Pencil, and the Pair of Compasses. The Skirret is a measure that ensures that the foundation of a building is straight by laying down a string as a marker. Therefore, the Skirret symbolizes the foundation of the way of life that we have to follow as Freemasons, as laid down to us in our Ritual. The Pencil is a symbol of intellectual rigor and moral accountability. The Pair of Compasses, enabling one with accuracy and precision to ascertain and determine the limits and the proportions of the different parts of a building, is a symbol of justice.

I shall now proceed to entrust you with the marks by which, in our Order, the Master Masons are known to each other and distinguished from the other people. As in the previous degree, your body, being considered a symbol of your mind, must be perfectly erect *(the D.C. and the Expert ensure that the Candidate complies)*, and your feet, symbolizing the rectitude of your actions, must form a Square *(the D.C. and the Expert instruct the Candidate in whisper to unite his/her heels so that they form a Square).* You will now advance to me as an Entered Apprentice and then as a Fellow Craft *(the D.C. and the Expert ensure that the Candidate complies).* You will now take another short pace towards me with your left foot, bringing the right heel into its hollow as before *(the D.C. and the Expert ensure that the Candidate complies).* That is the third regular step in Freemasonry, and it is in this position that the Sign, the Token, and the Word of the Third Degree are communicated. *(The V.M. rises, faces the Candidate, and takes Step).*

The Sign of a Master Mason, also known as the Penal Sign of a Master Mason, is given as follows: firstly, place the right hand on the navel with the thumb extended in the form of a square in such a way that the edge of the thumb touches the navel; this is the Sign of Order in the Third Degree; and then draw the right hand, with the thumb extended, smartly across the belly, from left to right. Finally, drop the hand to the side with all fingers extended *(the V.M. illustrates, and the D.C. and the Expert ensure that the Candidate copies).* The Penal Sign alludes to the symbolic penalty of the Third Degree, which implies that a Master Mason would rather have been severed in two than profane his/her promotion to the supreme degree of a Master Mason.

As you know, a handshake is used within Freemasons' ceremonies to show the level a member has reached. In our Freemasonic Order in particular, the Grip or Token of a Master Mason is given as follows *(the V.M. takes the Candidate's right hand and adjusts the grip by placing the Candidate's thumb in position before placing his/her own; the D.C. and the Expert assist the Candidate)*: by seven equal, distinct pressures of the thumb on the third joint of the hand. This Grip or Token demands a Word. In our Order, the Word of the Third Degree is the following: "The way up and the way down is one and the same" (Heraclitus).

After the end of this ceremony, you will have a higher level of clearance, and the Secretary will provide you with a special identification card that gives you access to our Order's Center for Advanced Studies.

The V.M. places the Candidate's right hand in the left hand of the Expert and sits. The Director of Music starts playing the first part of Hector Berlioz's "Grande Symphonie funèbre et triomphale." The D.C. and the

Expert turn right, controlling the Candidate by holding his/her left hand and right hand respectively, regain the floor of the Lodge, turn left, instruct the Candidate in whisper to step off with the left foot, and pass in front of the Venerable Master's pedestal to the southeastern corner, where they halt for a moment in order to emphasize the practice of squaring the Lodge. Then, the D.C. and the Expert lead the Candidate to the southwestern corner, where they halt for a moment in order to emphasize the practice of squaring the Lodge. When this is done, the D.C. and the Expert lead the Candidate to the south side of the Junior Overseer's pedestal, where they stand parallel to the pedestal and a convenient distance from it. The D.C. and the Expert release the hands of the Candidate. The Director of Music stops playing music.

D.C.: Brother/Sister Junior Overseer, I present to you Brother/Sister . . . *(name and surname of the Candidate).*

J.O.: Brother/Sister . . . *(name and surname of the Candidate),* advance to me as a Master Mason.

The D.C. instructs the Candidate in whisper to take Step only (thus bringing the right heel into the hollow of the left foot) and ensures that the Candidate does not give Sign at this stage.

J.O.: What is that?

Candidate (prompted by the D.C.): The third regular step in Freemasonry.

J.O.: Have you learned anything else?

Candidate (prompted by the D.C.): I have.

The D.C. instructs the Candidate in whisper to give the Master Mason's Sign and cut it.

J.O.: What is that?

Candidate (prompted by the D.C.): The Sign of a Master Mason.

J.O.: Have you anything to communicate?

Candidate (prompted by the D.C.): I have.

J.O. rises, faces the Candidate, takes Step, and offers hand. The Expert places the Candidate's right hand in that of the J.O., and, with the left hand, he/she adjusts the grip from above. The J.O. gives the Master Mason's Grip after the Expert has adjusted the Candidate's right thumb, and he/she retains the grip.

J.O.: What is this?

Candidate (prompted by the D.C.): The Grip or Token of a Master Mason.

J.O.: What does it demand?

Candidate (prompted by the D.C.): A Word.

J.O.: Give me that Word.

Candidate (prompted by the D.C.): "The way up and the way down is one and the same."
J.O.: Pass. "The way up and the way down is one and the same."
The J.O. replaces the Candidate's right hand in the left hand of the Expert and remains standing. The D.C. and the Expert take the Candidate by the left hand and the right hand respectively, to the south side of the Senior Overseer's pedestal, and the Expert places the Candidate's right hand in the Senior Overseer's left hand. The D.C., standing behind the Candidate, and the Expert, standing on the right side of the Candidate, ensure that all three of them face East. The S.O. takes Step, and gives the Master Mason's Sign, which he/she retains.
S.O.: Venerable Master, I present to you Brother/Sister . . . *(name and surname of the Candidate)* in order to be invested with the distinguishing badge of a Master Mason.
V.M.: Brother/Sister Senior Overseer, I delegate you to invest him/her with the distinguishing badge of a Master Mason.
The Expert divests the Candidate of the Fellow Craft's apron and sash, and places them on the Senior Overseer's pedestal. Then, the S.O. cuts the Sign, releases the hand of the Candidate, and, with the Candidate facing him/her, he/she puts on the Candidate the apron of a Master Mason (the Expert assists as necessary). Moreover, the S.O. invests the Candidate with the sash of a Master Mason, which is worn over the right shoulder and under the left arm. Thereupon, the D.C. and the Expert take the Candidate by the left hand and the right hand respectively, and turn him/her towards the East, so that all of them stand facing East. The D.C. and the Expert then release the Candidate's hands.
V.M.: Brother/Sister . . . *(name and surname of the Candidate)*, your further progress in Freemasonry is marked by the position of the Square and the Pair of Compasses. In the Third Degree, both points of the Pair of Compasses are disclosed and, thus, elevated above the Square. The apron of a Master Mason is of white fabric having a triangular flap, all bordered with two-inch ribbon of light blue, and the apron has two light blue goffer rosettes at the lower corners and a third light blue goffer rosette at the center of the flap. Moreover, there are two tassels of silver suspended by light blue ribbons from beneath the flap. In the apron of the Venerable Master of a Lodge, the tassels are gold, and the three aforementioned goffer rosettes are replaced by three gold Taus or Levels placed in the same way as the Master Masons' goffer rosettes. A Master Mason's apron is sixteen inches wide and fourteen inches deep. The sash of a Master Mason is of light blue fabric, five inches wide, edged with gold lace, and embroidered with a gold rayed equilateral triangle having the number 3 in

orange-red at its center. The sash is worn over the right shoulder and under the left arm, and its base is fringed with gold. In our Order, the distinctive symbol of a Master Mason is an equilateral triangle as it is defined in Euclidean geometry. A triangle, in general, is defined as a polygon with three edges and three vertices. An equilateral triangle in particular is defined as a triangle in which all three sides have the same length. In Euclidean geometry, an equilateral triangle is also equiangular, in the sense that all three internal angles are also congruent to each other, each being 60 degrees. In sacred geometry, the equilateral triangle symbolizes geometric sturdiness and resilience, balance, harmony, raising consciousness, and enlightenment. I must add that the Master Mason degree implies high responsibilities and powers, since it is the supreme degree in our initiatory system, and it makes one eligible to become a Lodge Officer. Be vigilant in every situation, be industrious and ever desirous of further progress, be ready to teach and just as ready to learn, and let me assure you that the eyes of the Grand Lodge will be upon you.
The D.C. takes the left hand of the Candidate, the Expert takes the right hand of the Candidate, and they conduct him/her to a seat in the South, instructing him/her to sit there. When this is done, the D.C. and the Expert resume their regular seats in the Lodge. Everyone is sitting while the Charge is delivered.

Charge:
V.M.: Brother/Sister . . . *(name and surname of the Candidate)*, your rise to the sublime and supreme degree of a Master Mason reflects the dedication you have displayed for our Order over a long period of time and through several tests and tasks; and I do congratulate you most heartily.
According to our Order, a true Master Mason is an emissary of Light, the epicenter of an earthquake that destroys rotten and crumbling creations of human imperfection, the strong medicine that fights dementia, and the acid that washes away the filth of perversion and irrationality. Moreover, a Master Mason of the Modern and Perfecting Rite of Symbolic Masonry is a genuine expert in Masonic history and in the comparative study of Masonic rituals. For this reason, we shall teach you every significant element of the history and the rituals of the English Symbolic Masonry of the eighteenth and the nineteenth centuries, since the United Grand Lodge of England is the oldest Grand Lodge in the world and has played a major role in the shaping of the Craft. Furthermore, we shall teach you every significant element of the history and the rituals of the Ancient and Accepted Scottish Rite, which was formed and instituted in the U.S.A.

during the eighteenth and the nineteenth centuries, since the Southern Masonic Jurisdiction of the U.S.A. is the oldest Supreme Council of the Ancient and Accepted Scottish Rite in the world and has played a major role in the attempt of several Masons to expand and enrich the basic English system of Symbolic Masonry with appendant Orders and degrees.
The Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry has reformed Symbolic Masonry in such a way that the Master Mason degree has absorbed and contains within it every important teaching of the Ancient and Accepted Scottish Rite. Thus, we shall offer you a complete, accurate, and synoptic exposition of all the wisdom of the English Symbolic Masonry as well as of all the degrees of the Ancient and Accepted Scottish Rite, given that our Order expresses a transition from old ritualism and mythology to philosophy and psychoanalysis.
S.O.: Even though Freemasonry already existed before the eighteenth century, the year 1717 signals the commencement of a new system of Freemasonic organization and government, namely, the "Grand Lodge" system. In particular, the "Grand Lodge of London and Westminster," also known as the "Premier Grand Lodge of England," was conceived and established on St. John the Baptist's day, 24 June 1717, by the following four Symbolic Lodges:

1. The Goose and Gridiron, St. Paul's Churchyard, established 1691.
2. Crown Ale House Lodge, Lincoln's Inn Fields, established 1712.
3. The Apple Tree Tavern, Covent Garden, now known as the Lodge of Fortitude and Old Cumberland.
4. The Rummer and Grapes, Channel Row, Westminster, later known as Horn Lodge.

The first three Grand Masters of the Grand Lodge of London and Westminster were the following: Anthony Sayer, "a Gentleman," who was elected in 1717; George Payne, who was elected in 1718, and he was appointed Secretary to the Tax Office in 1732 and Head Secretary in 1743; and the Rev. Dr. John Theophilus Desaguliers, who was elected in 1719, and who was later three times Deputy Grand Master.

In 1713, the Parliament of Westminster invited George, the Electorate of Hanover and a Protestant, to be crowned King George I of Great Britain and Ireland. Thus, King George I became the first British monarch of the House of Hanover, which succeeded the House of Stuart and the House of Orange.

In the Middle Ages, the Church was the quintessence of the State, and the civil authority was perceived to be a law enforcement apparatus of the Church. In Western Europe, by the end of the fourth century, the Church was using the secular arm of the Roman Empire in order to bring pagans

and Christian sectarians into the Church of Rome. Given that the Church was the main repository of educated thought and opinion at every level of society, the clergy played protagonist roles in the kingdoms and other feudal entities. However, from the eleventh century, there was a slow trend in Western Europe towards the strengthening of feudal states and the development within them of new increasingly independent elites. Thus, gradually, Christian subjects lose their sense of dual allegiance to the "regnum" (royal authority) and the "sacerdotium" (papal authority), and then civil rulers maintain that they could exercise power without having to defer to papal authority. In 1534, King Henry VIII of England established a state Church by his Act of Supremacy, the Church of England, which mainly maintained the religious doctrines of the Roman Catholic Church whilst being independent from the Pope. This change in the balance of power between civil and ecclesiastical authority was reinforced in the seventeenth century by the House of Stuart. In the seventeenth century, the House of Stuart united the Kingdoms of England and Scotland—thus, it formed the first United Kingdom. The United Kingdom was the first great European kingdom that became independent from the Pope's rule. Even though the Stuarts were Roman Catholic, they wanted to strengthen the king's power vis-à-vis the Papacy. Moreover, the Protestant Reformation marks the period in which the doctrine of the Two Swords, according to which the Pope possessed both swords but had granted the temporal sword to kings and other feudal lords, was replaced by the doctrine of the sovereign State, acknowledging the absolute, secular sovereignty of the ruler.

James VII and II of the House of Stuart, who was king of England and Ireland as James II and king of Scotland as James VII, converted to Roman Catholicism, favored a policy of religious tolerance rather than the prerogatives of the Church of England, promoted an alliance between the United Kingdom and the Catholic French royal house, questioning the established balance of power and the Papacy's interests in Continental Europe. His second wife, Mary of Modena, gave birth to a son who was to be brought up as Roman Catholic. Thus, opposition against James VII and II was raised both among the Anglican establishment and the Roman Catholic establishment. In the context of the Glorious Revolution, which broke out in 1688, the English Parliament deposed James VII and II in favor of his Protestant daughters: Mary II, who co-reigned with her Dutch husband William of Orange from 1689 until her death in 1694, and Ann, who succeeded William of Orange after his death in 1702. Neither Queen Mary II nor Queen Ann had any children who survived to adulthood. Therefore, under the 1701 Act of Settlement, which excluded all Roman

Catholics from the succession, Ann was succeeded by her second cousin George I of the House of Hannover. After the loss of the throne, the descendants of James VII and II came to be known as the Jacobites and continued to reclaim the Scottish and English throne as the rightful heirs.

George I of the House of Hannover was keen to eliminate every Jacobite element from British Freemasonry. In the context of the previous policy of Masonic "purity," the Premier Grand Lodge of Georgian England forbade all religious and political discussions within Masonic Lodges. Under the Hanoverian dynasty, every element that sustained prior, pre-Hanoverian forms of Freemasonic activity is declining or is buried deep below the surface. Hanoverian Freemasonry maintains that the keystone of the Freemasonic system is the principle of tolerance, and it mainly reflects the ethos of the Whig Party and the bourgeois establishment. In general, the Whigs were aligned with commercial interests and Protestant Dissenters (Protestants who separated from the Church of England during the seventeenth and the eighteenth centuries), whereas the Tories favored the interests of the landed nobility and were supportive of the Anglican Church.

Based on actual biblical references about the construction of King Solomon's Temple and on his Calvinist ethics, the Rev. Dr. John Theophilus Desaguliers, a French-born Huguenot clergyman closely aligned with the Hanoverian dynasty, articulated the following didactic story in order to become the major allegorical theme of the Third Degree within the Jurisdiction of the Grand Lodge of London and Westminster: being knowledgeable in architecture and in metallurgy, Hiram Abiff was sent by Hiram King of Tyre to Solomon King of Israel, to direct the construction of the Temple at Jerusalem, which the Israelites intended to erect to the glory of the Great Architect of the Universe. Hiram Abiff divided his workmen into three groups: Apprentices, Companions, and Masters. To be able to distinguish and recognize each group, he gave it its own set of secrets, precisely, a Word, a Sign, and a Grip. The works drawing to their close, three impious Companions ("ruffians"), having been unable to obtain the secrets of a Master, decided to extract the Sign, the Word, and the Grip of a Master from Hiram Abiff by any means. When Hiram Abiff finished his devotions, he moved towards the South entrance of the Temple, where he was opposed by the first impious Companion, who was armed with a heavy Plumb Rule and demanded the secrets of a Master, warning Hiram Abiff that he would die if he refused to comply with those impious and unworthy Masons' request. Hiram Abiff refused to divulge the secrets of a Master Mason without the consent and co-operation of Solomon King of Israel and Hiram King of Tyre, and he

added that patience and industry would, in due time, entitle every worthy Mason to a participation of the secrets of a Master Mason. The impious Companion struck Hiram Abiff a violent blow on the right temple. When Hiram Abiff recovered from the shock, he moved towards the North entrance of the Temple, where he was opposed by the second impious Companion, who was armed with a Level. After Hiram Abiff gave a similar answer to the second impious Companion, the latter struck him a violent blow on the left temple. Hiram Abiff, "faint and bleeding," moved towards the East entrance of the Temple, where he was opposed by the third impious Companion, who was armed with a Maul. After Hiram Abiff gave a similar answer to the third impious Companion, the latter struck him a violent blow on the forehead, which was fatal. Dear Brother/Sister . . . (name and surname of the Candidate), in tonight's ceremony, in which you became a Master Mason, you symbolized Hiram Abiff, according to the didactic fictional narratives of traditional English Freemasonry.

According to the United Grand Lodge of England, the aforementioned Hiramic legend is completed with the Order of the Holy Royal Arch of Jerusalem, which, according to the United Grand Lodge of England's Constitution, is the completion of the degree of a Master Mason. According to the Emulation Ritual, due to the untimely death of Hiram Abiff, the genuine secrets of a Master Mason are lost and cannot be communicated to any Freemason, since they could never be divulged without the consent and co-operation of the three Grand Masters (Solomon King of Israel, Hiram King of Tyre, and Hiram Abiff). However, according to the United Grand Lodge of England, the genuine secrets of a Master Mason are restored in the Order of the Holy Royal Arch of Jerusalem.

The Royal Arch degree originally appeared in 1743 in *Faulkner's Dublin Journal*, dated 10–14 January 1743, where an article reported that "Youghall" Lodge No. 21 celebrated St. John's Day with a parade in which there was "the Royal Arch carried by two excellent Masons." In 1744, in Dublin, Ireland, Fifield D'Assigny published a book entitled *A Serious and Impartial Enquiry into the Cause of the Present Decay of Freemasonry in the Kingdom of Ireland*, in which he argued that the Royal Arch degree was conferred in Dublin "some few years ago," and that it had been brought there from the city of York. The ceremony of "exaltation" to the Royal Arch is based on the legend of the rebuilding of King Solomon's Temple after the Babylonian Captivity, in the sixth century B.C.E.

Babylonian Exile, also called Babylonian Captivity, is the forced detention of Jews in Babylon, following the destruction of Jerusalem by

Nebuchadnezzar in ca. 586 B.C.E. The exile formally ended in ca. 538 B.C.E., when the Persian conqueror of Babylon, Cyrus the Great, gave the Jews permission to return to Palestine. The Bible's book of Ezra narrates the history of the Jewish exiles' return to Jerusalem and of the construction of the second Jerusalem Temple. In ca. 536 B.C.E., Zerubbabel, who was the grandson of Jechonias, penultimate king of Judah, led the people in rebuilding the altar and laying the second Jerusalem Temple's foundation.

According to the English Royal Arch's allegorical narrative, Jewish captives returning to Jerusalem, after the Babylonian Captivity, participated in the reconstruction of the Jerusalem Temple. While constructing the Second Temple of Jerusalem, they discovered a large underground vault consisting of nine arches. Within this vault, they discovered the true name of God, YEHOVAH (Jehovah), engraved on a golden plate that they found atop a triangular pedestal that was placed under the keystone of the ninth arch. This discovery marks the rediscovery of the long-lost genuine secrets of a Master Mason, according to the Order of the Holy Royal Arch of Jerusalem.

The symbolic meaning of the aforementioned discovery is the following: what was lost by King Solomon and most of his successors for the next four centuries was the genuine metaphysical secrets of biblical Judaism and, more particularly, the belief that Yehovah was the one and only true God of Israel. What was found by the time the captives returned to Jerusalem was an even broader awareness than the one which was lost: the exiles re-discovered the truth that Yehovah was the one and only true God of Israel, and that Yehovah was not merely the God of a specific nation—that is, Yehovah was not only the God of Israel among different "national" deities—but He was the one and only universal God. Thus, after the Babylonian exile, the Israelites developed a new, much deeper, and more universalistic awareness of monotheism. Furthermore, it is worth pointing out that the roots of the Babylonian Jewish community were very ancient, and that, as it grew and prospered, that community tended to emphasize its antiquity. By the time it had produced its own version of the Talmud, it articulated a kind of local patriotism, highlighting that Abraham, the Father of the Nation, was born "beyond the river" (Euphrates) and that Euphrates and Tigris were the two rivers which flowed out of Eden according to Genesis 2:14. Thus, the Jews of Babylonia considered themselves the aristocracy of the Jewish people, and, in their eyes, the land of Mesopotamia acquired an aura of sanctity. This was second to the land of Israel, of course, but holier than any other country.

In the fifth century C.E., Philostorgius, in his *Church History*, writing of the rebuilding of the Jerusalem Temple, refers to the discovery of the Vault. This is the earliest framework of the Royal Arch legend. Additionally, in the fourteenth century, Nikephoros Kallistos Xanthopoulos (Latinized as Nicephorus Callistus Xanthopulus), in his *Historia Ecclesiastica*, refers to a similar legend. Moreover, there is biblical evidence for the Holy Royal Arch legend's reference to finding "part of the long-lost Sacred Law." In particular, in 2 Kings, chapter 22, and in 2 Chronicles, chapter 34, we find references to the priest Hilkiah discovering a law book during the execution of repair work in the Temple of Jerusalem. Biblical commentators commonly identify the previous book that was found by Hilkiah with the kernel of the Deuteronomy—in fact, this is the source of the original Irish Royal Arch legend.

The didactic Masonic tale of Hiram Abiff is an attempt by the Rev. Dr. John Theophilus Desaguliers and by the Rev. James Anderson, another prominent Freemason and Protestant clergyman, to merge the biblical narratives about Hiram Abiff and the stories of old Masonic manuscripts referring to Noah and Bezalel, thus seeking to endow the Grand Lodge of London and Westminster with a distinctive Masonic identity—especially because, in those years, the Grand Lodge of London and Westminster was competing intensely with the Jacobites' Freemasonic forces. In February 1723, with the help of the Rev. Dr. John Theophilus Desaguliers, the Rev. James Anderson, a Scottish Presbyterian clergyman, compiled the *Book of Constitutions of the Free-Masons* (also known among Freemasons as *Anderson's Constitutions*), reflecting the ethos and the Masonic strategy of such prominent players in the politics of the Grand Lodge of London and Westminster as the two noble Grand Masters, the Duke of Montagu (in 1721) and the Duke of Wharton (in 1723), and the Deputy Grand Master at the time of the publication of the *Constitutions*, the Rev. Dr. John Theophilus Desaguliers.

In addition, the prominent Scottish author, editor, and Freemason William Preston, in his book entitled *Illustrations of Masonry*, originally published in 1772, highlights the significance of the two magnificent pillars that were erected at the porchway or entrance of the Temple at Jerusalem, which was completed by King Solomon. Those Pillars were made of molten brass, and the superintendent of the casting was Hiram Abiff. That on the left was called Boaz, which denotes "in strength"; that on the right Jachin, which denotes "to establish"; when conjoined, they denote "stability". Therefore, according to the rituals of the United Grand Lodge of England, the Grand Lodge of Scotland, and the Grand Lodge of Ireland, the pillar that stands on the north side of the door of a Masonic

Lodge is called Boaz, and the pillar that stands on the south side of the door of a Masonic Lodge is called Jachin.

The eighteenth century was a period of political and religious ferment in England. Therefore, the Grand Lodge of London and Westminster could not develop peacefully. In the years around 1740, there was a large number of Irish Freemasons in London, many of whom had been initiated into the Craft in Ireland. Many of those Irish Freemasons encountered difficulties in gaining entrance into London Lodges operating under the auspices of the Grand Lodge of London and Westminster. Therefore, in 1751, a group of those Irish Freemasons, together with other Jacobite Freemasons, founded a new Grand Lodge in London. According to their arguments, the Premier Grand Lodge had made innovations and had departed from "the landmarks," whereas they were practicing Freemasonry "according to the old institutions granted by Prince Edwin at York in A.D. 926." For this reason, they became known as the Ancients' Grand Lodge, and they were referring to their older rival, the Grand Lodge of 1717, as "Moderns." In their Masonic certificates issued to new members, the Ancients called themselves the "Grand Lodge of Free and Accepted Masons of England according to the Old Constitutions." The book of constitutions of the Ancients' Grand Lodge of England was written by Laurence Dermott, a distinguished Irish merchant, and its title was *Ahiman Rezon*. The first edition of the *Ahiman Rezon* was published in 1756, and a second one was published in 1764. It has often been said that the title *Ahiman Rezon* is derived from the Hebrew language and variously means "to help a brother," "will of selected brethren," "the secrets of prepared brethren," "royal builders," and "Brother Secretary"; the reason why Laurence Dermott used it and its meaning to him remain unclear.

The Ancients' Grand Lodge was oriented towards a more esoteric approach to Freemasonry, it was politically leaning towards the Tory Party, and it was associated with the Jacobites' secret attempts to restore King James VII (of Scotland) and II (of England) and his heirs to the British throne. Although the Jacobite Rising of 1715 ended in defeat, many Freemasons continued to support the Stuart dynasty. The Hanoverian King George I and the supporters of the Hanoverian dynasty were concerned about the Jacobites' influence on London Freemasonry, which was a secret organization meeting in small cabals, called Lodges, in the upper floors over inns, pubs, and coffee-houses around London. Thus, the supporters of the Hanoverian dynasty and the Whigs decided to reorganize, reform, and control Freemasonry. For this reason, the Grand Lodge of London and Westminster, which was founded in 1717, forbade

all political and religious discussions within its Lodges, and it supported the House of Hanover. In addition, in its formative years, the Grand Lodge of London and Westminster, or the Moderns' Grand Lodge, methodically cultivated and promoted the highly controversial opinion that the Craft's history officially began in 1717, and it suppressed information pertaining to the history and the ethos of other Freemasonic entities.

In short, the Moderns' Grand Lodge, or the Grand Lodge of London and Westminster, represented the ethos of the rising bourgeoisie, modern capitalism, Protestant ethics as it has been analyzed by the great sociologist Max Weber, and an attempt to achieve an efficient historical compromise between the old nobility and the bourgeoisie. The Ancients' Grand Lodge, meanwhile, represented the ethos of the traditional feudal nobility, and it promoted a type of idealism that underestimated the needs of the new capitalist system. Finally, on 27 December 1813, the day of St. John the Evangelist, the Moderns' Grand Lodge of England and the Ancients' Grand Lodge of England were amalgamated into the United Grand Lodge of England (UGLE), with the Duke of Sussex, who was the younger son of King George III and a Whig, as Grand Master. By the time of the Union, a considerable amount of revision had taken place in the ritual, and, in this context, the form of Freemasonry that was developed by the Moderns' Grand Lodge prevailed over the form of Freemasonry that was developed by the Ancients' Grand Lodge. As a result, the United Grand Lodge of England changed the underpinning strategic vision of Freemasonry from being Christian and nobility-oriented to that of non-Christian and bourgeois-oriented.

Nevertheless, the Ritual of the United Grand Lodge of England was taken from the Bible. Following the Union in 1813, a "Lodge of Reconciliation" was formed to reconcile the rituals worked under the two former Grand Lodges. In 1823, an "Emulation Lodge of Improvement" was established (under the sanction of the Lodge of Hope No. 7), teaching the ritual settled by the Lodge of Reconciliation, known as the Emulation Ritual. According to the Emulation Ritual, the First Degree takes place on the ground floor of King Solomon's Temple, the Second Degree takes place in the Middle Chamber of King Solomon's Temple, and the Third Degree takes place at the Porchway or Entrance of the Holy of Holies of King Solomon's Temple to symbolically describe a spiritual itinerary in biblical terms. Moreover, the Emulation Ritual includes several religious invocations, prayers, and oaths. Thus, the Emulation Ritual is a religious ritual, as several Fathers of the Roman Catholic Church (including Pope Clement XII, Pope Pius VII, Pope Pius IX, and Pope Leo XIII) have justly pointed out. In fact, the Emulation Ritual, with its religious references,

practices, and requirements, conflicts with the "Declaration" issued on 21 June 1985 by the United Grand Lodge of England, entitled "Freemasonry and Religion," which states in no uncertain terms that Freemasonry is not a religion. If the "regularity" of Symbolic Masonry is assumed to be necessarily dependent upon the use of religious Masonic rituals, then Symbolic Masonry should be directly overseen by the Church, like medieval Operative Masonry, unless one maintains that Freemasonry should develop and promote an alternative religious proposal of its own.

The form of Freemasonry that prevailed in the United Grand Lodge of England in the early nineteenth century became the prevailing form of Freemasonry throughout the British Isles, and, gradually, expanded throughout the world. For the Grand Lodges of England, Ireland, and Scotland, the aforementioned form of Freemasonry is the epitome of "Masonic regularity," and, in 1929, these three Grand Lodges, acting as the supreme "Guarantors" of what they have defined as "regular Freemasonry," formulated the basic principles for Grand Lodge recognition. According to the statement of the basic principles for Grand Lodge recognition that was drawn up in 1929 by the United Grand Lodge of England, the Grand Lodge of Scotland, and the Grand Lodge of Ireland, a Masonic Grand Lodge is "regular" if the following conditions are met: firstly, it has been established lawfully by a Grand Lodge that is duly recognized by the aforementioned three British Grand Lodges, or it has been established lawfully by three or more Lodges that have been constituted according to the aforementioned three British Grand Lodges' rules of regularity; secondly, a belief in the Supreme Being, referred to as the Great Architect of the Universe, and His revealed will are essential preconditions for membership; thirdly, all initiates must take their obligations on or in full view of the Volume of the Sacred Law, this being the major "Holy Book" of each one's religion, such as the Bible for Christians, the Torah for Jews, and the Quran for Muslims; fourthly, the membership of the Grand Lodge and individual Lodges must be exclusively of men, and Masonic regularity precludes any Masonic intercourse with mixed Masonic institutions, which admit women to membership; fifthly, the Grand Lodge must have sovereign jurisdiction over the Lodges that it controls, and international regular Freemasonry should be organized as a network of independent, territorially restricted, national Grand Lodges that are duly recognized by each other; sixthly, the three Great Lights of Freemasonry—the Volume of the Sacred Law, the Square, and the Pair of Compasses—must be exhibited when the Grand Lodge or its subordinate Lodges are at work, and the chief of the Great Lights of Freemasonry is the Volume of the Sacred Law—namely, a

religious book; seventhly, the discussion of religion and politics within the Lodge must be strictly prohibited; eighthly, the principles of the English Masons' "Ancient Landmarks," customs and usages must be strictly observed.

The Rituals of the United Grand Lodge of England, the Grand Lodge of Scotland, and the Grand Lodge of Ireland are based on the aforementioned didactic tale of Hiram Abiff and on the use of King Solomon's Temple as the main symbol of architecture. At this point, it is important to highlight the difference between the concept of a myth and the concept of a tale. A "myth" is a traditional symbolic narrative whose purpose is to disclose the spiritual core of beings and things. It is associated with the religious beliefs and, in general, with traditional cultural underpinnings of a human community, for which reason the historical origin of many myths is unknown. On the other hand, a "tale" is an imaginary narrative of an event, and it functions as a substitute for the lack of a genuine myth and a systematic philosophy. In particular, the tale of Hiram Abiff was adopted by the Moderns' Grand Lodge as a means of changing the mythological underpinnings of traditional societies in general and of traditional Masonic Lodges in particular, thus promoting the ethos of the Hanoverian dynasty and the Whig Party. Moreover, we must not lose sight of the fact that English Freemasonry has often functioned as a tool for cultural diplomacy in the service of British governments' foreign and domestic policy, and it is, of course, subservient to the spirit of capitalism and the British feudal institutions.

The origins of modern liberalism, of which the Whig Party is an integral part, can be seen most clearly in the thinking and politics linked to the English Revolution of 1688—more specifically, in British empiricism and constitutionalism. The principles of constitutionalism, religious tolerance, and commercial activity, which were promoted by the English Revolution of 1688, became a standard for European and American liberals in the eighteenth century. The successful American revolutionaries of the last quarter of the eighteenth century were attracted to John Locke's political philosophy, which was seen as the most important intellectual underpinning of the English Revolution of 1688. The motives and results of the French Revolution of 1789 were more mixed, but the French Revolution of 1789 was a liberal revolution in both rhetoric and institution, in the sense that it proclaimed the liberty of the individual, promoted respect for the right to private property as a means of eliminating the outrageous prerogatives of the feudal elites, and praised the "self-made" humans. However, the end of the Napoleonic era left Europe dominated by monarchies and by the spirit of restoration, which

maintained dark memories of and strong fears about previous revolutions, political disputes, and turmoil. In the aftermath of the Napoleonic era, conservatism, of which the Tory Party is an integral part, was articulated as a systematic and organized political expression of the advocates of tradition, historical continuity, and the preeminence of the institutions and the principles of the past.

In general, the transition from feudalism to capitalism marks an important historical progress. Feudalism, the dominant system in medieval Europe, was a system characterized by rigid social stratification, according to which everyone had a rigidly instituted position within an "organic whole," whose major constituent components were the class of the feudal lords, the class of the serfs, and the Church, whose major social role was to maintain a balance between the feudal lords and the serfs through religion. By the late Middle Ages, the bourgeois class (a social class of professionals who were neither feudal lords nor serfs) deprecated the political, economic, and spiritual despotism of the feudal system, revolted against feudalism, and proclaimed that the social position of an individual should not be determined by feudal institutions. Instead, it should be freely determined by individual action and by the interaction between individuals in the context of a free and fair society. One of the most characteristic examples of a bourgeois revolution in the modern era is the French Revolution of 1789, whose major motto was "Liberty, Equality, Fraternity."

However, the elite of the bourgeois class envisaged and instituted capitalism as the embodiment of human freedom in the domain of economics. For this reason, after the displacement of feudalism by capitalism, the liberty and the rights of the human individual were gradually largely displaced by and subordinated to the liberty and the rights of the capital itself and the capitalist elite. By the middle of the nineteenth century, the European peoples realized that capitalism had displaced feudalism, but, instead of ushering in liberty, equality, and fraternity among the people, capitalism tends to replace the authoritarian and exploitative relationship between the feudal lords and the serfs with a new authoritarian and exploitative relationship—that between the capitalist class and the proletariat (the working-class).

Even though capitalism is characterized by a significantly higher level of freedom in comparison with feudalism and has produced unprecedented economic wealth, it tends to substitute old contradictions with new ones, and to substitute the "Kantian subject" with the "homo economicus." Therefore, socialism emerged as a criticism of and a revolt against capitalism, just as the bourgeois ideology had previously emerged as a

criticism of and a revolt against feudalism. In fact, the term "socialism" first appeared in 1832 in *Le Globe*, a liberal French newspaper of the French philosopher and political economist Pierre Leroux. By the 1840s, socialism had already become the object of rigorous social-scientific analysis by the German economist and sociologist Lorenz von Stein. Moreover, in the nineteenth century, the English socialist intellectual and activist Thomas Hodgskin articulated a thorough critical analysis of capitalism and of the labor class under capitalism. His writings exerted a significant influence on subsequent generations of socialists, including Karl Marx. In particular, from the perspective of Thomas Hodgskin, socialism implies an attempt to create a free and fair market, in the context of which production and exchange are based on the labor theory of value, freed from exploitative institutions, as part of natural right, which endows moral consciousness, the freedom of the individual, social justice, and social autonomy with ontological underpinnings in accordance with Thomas Hodgskin's deism.

Furthermore, in the twentieth century, the rapid globalization of the international economy (especially in the areas of production and finance) and the changing nature of the interstate system in the post-Cold War era contribute to the emergence of a "global," as opposed to "international," political economy. Global political economy refers to an economic space that transcends all country borders, while co-existing with an international economy that is based on transactions across country borders and is regulated byinter-state agreements and practices. Thus, global political economy identifies three different levels of economic space: supra-regional, national, and sub-regional. In addition, global political economy identifies at least three different levels of social organization: social forces, states (that is, national societies), and global society.

Intimately related to the issue of globalization is the issue of global governance. Global governance concerns the identification and management of those issues which necessarily affect every part of the globe. For instance, the globalization of the international political economy and its management, the global ramifications of a nuclear war, environmental questions and ecological concerns, technological advances, outer space affairs, global health issues, inter-civilizational relations and inter-cultural communication, as well as the interplay between the notion of national self-interest and international common goods are all issues of global governance.

Eighteenth-century British Freemasons have bequeathed to us the brilliant idea of developing Symbolic Masonry from Operative Masonry, thus creating a fraternity whose declared goal was to build a better human

being within a better society. In the eighteenth and the nineteenth centuries, the rituals and the constitutions of the British Freemasonries had their usefulness, their value, and, to some extent, their grandeur. However, in view of the historical and sociological remarks and arguments that I have hitherto put forward, one can rightly claim that, if a Freemasonic institution is fixated on the statement of the basic principles for Grand Lodge recognition (drawn up in 1929 by the United Grand Lodge of England, the Grand Lodge of Scotland, and the Grand Lodge of Ireland), and if it focuses on rituals and catechisms that are founded on didactic tales inspired by the Bible or other religious books, and on the ethos and the needs of the elites that ruled the United Kingdom during the Georgian and the Victorian eras, then it ultimately becomes a case of atavism, obsolete and unable to provide highly cultured persons with anything culturally significant and intellectually fascinating. Such a Freemasonic institution is mainly suitable for the management—more specifically, the manipulation—of the "popolo" through glamour, networking, and vainglory. In fact, in 1872, the distinguished and highly influential English Freemason and author John Yarker, in his book entitled *Notes on the Scientific and Religious Mysteries of Antiquity* (pages 157 to 158), had already argued that, "as the Masonic fraternity is now governed, the Craft is fast becoming the paradise of the *bon vivant*; of the 'charitable' hypocrite . . . the manufacturer of paltry Masonic tinsel; the rascally merchant . . . and the masonic 'Emperors' and other charlatans," and he proposed "the appointment of a higher (not pecuniary) standard of membership and morality, with exclusion from the 'purple' of all who inculcate frauds, sham, historical degrees, and other immoral abuses."

The Modern and Perfecting Rite of Symbolic Masonry seeks to rectify the above situation and enrich and ameliorate the system of Symbolic Masonry, often referred to as the "Craft," without, however, subordinating Symbolic Masonry to Masonic rites and degrees beyond the Craft. By "Masonic rites and degrees beyond the Craft," I mean a host of Masonic rites and degrees that were fabricated during the period from 1740 to 1800, mainly in the European Continent, by those Freemasons to whom the form of Freemasonry that is inspired by Operative Masonry was not sufficient, and who devised ceremonies that enlarged and expanded the scope of the established Masonic domain to encompass symbolic chivalric degrees. Such degrees were usually inspired by the medieval Crusading Orders, Rosicrucianism, and several other elaborations characterized by a high level of mysticism. Of the degrees that appeared, many were credited by various European and American Freemasons with a Scottish title and/or origin—even though none was literally connected to Scotland—and they

included various legends and symbols related to the medieval Knights Templar.

Moreover, the development of several Masonic Rites and degrees is inextricably linked to social developments and political affairs. To name a few examples, King James I of England and VI of Scotland used Masonry in order to consolidate his rule. King James II of England and VII of Scotland, who was deposed in the Glorious Revolution of 1688, and the Chevalier Ramsay used Scottish Rite Masonry in particular as a means of restoring the Stuart monarchy in the kingdoms of England, Scotland, and Ireland. Oliver Cromwell and radical parliamentarians used Masonry in order to unify and invigorate the social forces that opposed the Stuart Dynasty and in order to promote a Protestant capitalist elite. Lastly, King George I of Great Britain used the Grand Lodge of London and Westminster in order to consolidate the rule of the Hanoverian Dynasty, to which he belonged, and in order to promote a system of liberal oligarchy under his scepter.

J.O.: By the 1740s, in Avignon, the capital of the department of Vaucluse in southeastern France, there already existed several centers of Hermetic studies, often working in the context of Freemasonry and practicing the three Craft degrees and other higher "Scottish" degrees. One of the most influential members of the Hermetic community of Avignon was Dom Antoine-Joseph Pernety, who was a Benedictine and librarian of Frederick the Great of Prussia and the author of the *Dictionnaire Mytho-Hermétique* (1758). There is an important intellectual link between the Scottish Rite of Freemasonry and the Société des Illuminés d'Avignon (Society of the Illuminati of Avignon), which was founded around 1784 by Dom Antoine-Joseph Pernety and a Polish Count called Tadeusz Leszczyc-Grabianka. The Société des Illuminésd'Avignon was derived from an esoteric society that existed in Berlin prior to 1779, when Pernety joined it, and it was organized in two classes superior to Symbolic Masonry: the Novices or Minors, and the Illuminés (Illuminated). Their head was called Magus and Pontiff. Moreover, the "Illuminés d'Avignon" were influenced by German Templar and Scottish Masonic degrees and legends related to Baron von Hund's "Rite of Strict Observance" and to the "Chevaliers Bienfaisants de la Cité Sainte" (Knights Beneficent of the Holy City) in Lyons. In 1783, Pernety left Berlin, and he took up residence at a house he called Château Mont Thabor near Avignon, provided by one of his disciples, the Marquis de Vaucroze. There, Pernety set up a lodge room for those who came to be known as the Illuminés d'Avignon. Among the Illuminés d'Avignon were the Duchess of Württemberg and Baron Erik Magnus Staël von Holstein, who was Chamberlain to Queen Sophia Magdalena of Sweden (the spouse

of King Gustav III). In 1783, Baron Erik Magnus Staël von Holstein was appointed chargé d'affaires to the Court of France, while, in 1785, he was named Ambassador of Sweden to France.

In the second half of the eighteenth century, several new Masonic and quasi-Masonic Orders and Rites were formed in France, such as the Ellus Coens, the Illuminés du Zodiaque, the Frères noirs, etc. In the 1760s, a French Freemasonic fraternity called the "Mother Lodge of Comtat-Venaissin" (in French, "Mère Loge du Comtat-Venaissin") was already working six degrees beyond the three Craft degrees. However, in 1775, the "Mother Lodge of Comtat-Venaissin" was suppressed by the Roman Catholic Inquisition.

In the 1770s, the chief seat of Scottish Masonry in Paris was the "Social Contract" Lodge (in French, "Le Contrat Social"), originally called the "Saint Lazarus" Lodge, which was working according to a "Philosophical Scottish Rite" (in French, "Rite Ecossais Philosophique") founded by the French physician and Hermeticist Dr. Boileau, who was a member of the Hermetic Rite of Avignon and a student of Pernety, under the auspices of the Grand Lodge of France. In 1766, the "Mother Lodge of Comtat-Venaissin" amalgamated with the "Social Contract" Lodge. Thus, after its suppression in Avignon by the Roman Catholic Inquisition, the "Mother Lodge of Comtat-Venaissin" was revived in the bosom of the "Social Contract" Lodge in Paris under the auspices of the English-style Grand Lodge of France and, subsequently, under the auspices of the Grand Orient of France, which is a purely French Masonic institution officially formed in 1773 and representing a French reformation of traditional English Freemasonry.

According to the English "Grand Lodge" system, the Grand Lodge of Symbolic Masonry is not only autonomous vis-à-vis the administrative authorities of the various Bodies of "higher degrees" (namely, vis-à-vis Masonic Orders beyond the Craft), but the latter depend on the Grand Lodge's approval of their operation, since the Grand Lodge of Symbolic Masonry has the exclusive authority to award the degree of a Master Mason (a necessary qualification for continuing one's Masonic journey beyond the three Craft Degrees). On the other hand, according to the French "Grand Orient" system, the supreme administrative authority of the "higher degrees"—specifically, the Supreme Council of the 33rd Degree of the Ancient and Accepted Scottish Rite—governs Symbolic Masonry, too, thus maintaining full authority over the Degrees 1º–33º.

The thirty-three degrees of the Ancient and Accepted Scottish Rite (A.A.S.R.) are the following: (1º) Entered Apprentice, (2º) Fellow Craft, (3º) Master Mason, (4º) Secret Master, (5º) Perfect Master, (6º) Intimate

Secretary, (7º) Provost and Judge, (8º) Intendant of the Buildings, (9º) Elect of Nine, (10º) Elect of Fifteen, (11º) Sublime Elect, (12º) Grand Master Architect, (13º) Royal Arch of Enoch, (14º) Scotch Knight of Perfection (or Grand Elect Perfect and Sublime Mason), (15º) Knight of the East or of the Sword, (16º) Prince of Jerusalem, (17º) Knight of the East and the West, (18º) Knight of the Pelican and Eagle and Sovereign Prince Rose Croix of Heredom (or simply Knight Rose Croix), (19º) Grand Pontiff, (20º) Venerable Grand Master of the Symbolic Lodges, or Master ad Vitam, (21º) Patriarch Noachite or Prussian Knight, (22º) Prince of Libanus, (23º) Chief of the Tabernacle, (24º) Prince of the Tabernacle, (25º) Knight of the Brazen Serpent, (26º) Prince of Mercy, (27º) Commander of the Temple, (28º) Knight of the Sun, (29º) Knight of Saint Andrew, (30º) Grand Elected Knight Kadosh or Knight of the White and Black Eagle, (31º) Grand Inspector Inquisitor Commander, (32º) Sublime Prince of the Royal Secret, and (33º) Sovereign Grand Inspector General. Many of these degrees are conferred by name only; and the major initiatory degrees of the A.A.S.R. that are always worked in full are the 18th degree and the 30th degree. The 30th degree of the Ancient and Accepted Scottish Rite (A.A.S.R.), which is called Grand Elected Knight Kadosh, is the supreme initiatory degree of the A.A.S.R., and, in Hebrew, "Kadosh" means holy or consecrated. The 31st degree, the 32nd degree, and the 33rd degree are administrative, roughly corresponding to the judiciary, the legislature, and the executive.

The characteristic jewel of the 33rd degree rests upon a Cross Potent: it is a nine-pointed star (one formed by three triangles of gold one upon the other and interlaced); from the lower part of the left side to the upper part of the right, a sword extends, and in the opposite direction is a hand of (as it is called) justice; in the center is a crowned double-headed eagle, holding a naked sword in its claws and having, on the right-hand side, a balance and, on the left-hand side, a pair of compasses united with a square; around the eagle, runs a band bearing the Latin inscriptions "Ordo ab Chao" (meaning Order out of Chaos) and "Deus Meumque Jus" (meaning God and My Right); the band is enclosed by two circles formed by two serpents, each biting its own tail; of the smaller triangles that are formed by the intersection of the greater ones, those nine that are nearest the band are of crimson color, and each of them has one of the letters that compose the word S.A.P.I.E.N.T.I.A.—that is, Wisdom. In our Order, every active member of the 33rd degree, namely, every Officer of the Supreme Council, wears this jewel on the left breast suspended from a gold bar by a white ribbon.

In 1754, in the College of Jesuits at Clermont outside Paris, the Chevalier de Bonneville established the Chapter of Clermont, honoring the Duc de Clermont, then Grand Master of the English-style Grand Lodge of France. The Chapter of Clermont may have worked as many as twenty-five degrees, known as the "higher degrees," the highest of which was called Sublime Prince of the Royal Secret. This Masonic organization was an asylum of the adherents of the Stuart cause, most of whom were Scotsmen. One of those "higher degrees" was known as Scottish Master, hence the origin of the name Scottish Rite. In 1758, the aforementioned degrees were introduced into Germany by the Marquis Gabriel de Lernay, a French officer captured during the Seven Years' War; he established a military Lodge in Berlin with the help of two Germans: the Baron von Printzen, a Mason who was a Master of the "Three Globes" Mother Lodge at Berlin, and Philipp Samuel Rosa, a disgraced former pastor. Thus, in 1758, these degrees were adopted by the Grand Lodge of the Three Globes, also known as Grand National Mother Lodge of the Prussian States. In the same year, these degrees were revived in Paris under the auspices of a Masonic Order that was called the Council of Emperors of East and West. However, in consequence of internal warfare in the Council of Emperors of East and West (arguably caused by Jesuits, who endeavored to sow dissension with the view of suppressing this newly-established Masonic Order), a new Masonic organization was formed: the Council of Knights of the East. The Council of Knights of the East practiced what was known as the Rite of Perfection, the name by which the twenty-five Clermont Degrees were originally known. The Council of Knights of the East was representing mainly the bourgeoisie and the middle class, as well as the Whig ideology, whereas the Council of Emperors of East and West was representing mainly the nobility and the old conservative ideology.

In 1761, the Council of Emperors of East and West managed to defeat the Council of Knights of the East and assume exclusive regular control over the Rite of Perfection. It granted a patent to a merchant called Stephen Morin to propagate the Rite of Perfection, and installed him as a Grand Inspector of the Rite of Perfection. According to Albert Mackey's *Encyclopedia of Freemasonry*, most probably, Stephen Morin was a member of a French-American Huguenot family. The original of the aforementioned document has not been found, but Freemasons know about it only from the copy preserved in the *Golden Book* of the Comte de Grasse-Tilly, founder of the Supreme Council of the Ancient and Accepted Scottish Rite for France. In 1761, Morin arrived in San Domingo, where he started propagating the Rite of Perfection, and, by

virtue of his patent, he appointed many Inspectors for both the West Indies and the United States of America.

Morin was by no means the proper person to act as the Grand Inspector and chief propagator of the Rite of Perfection in America, because his philosophical and Masonic education was poor, and he made wrong decisions with regard to the choice of his lieutenants. In particular, he appointed several merchants of dubious morality and rather bad reputation, even members of the American-Jewish underworld, as his lieutenants. Thus, unworthy, or at least underqualified, persons played an instrumental role in the history of the Rite of Perfection in America for approximately the next forty years.

Sometime between 1763 and 1767, Morin appointed Henry Andrew Francken, naturalized French citizen of Dutch origin, then-resident of Jamaica, and employed as a customs officer, "Deputy Inspector General of all the Superior Degrees of Free and Accepted Masons in West Indies." Henry Andrew Francken played an important role in the formation and propagation of the Rite of Perfection in the American colonies. In 1768, having previously settled in New York where he was appointed court interpreter, Henry Andrew Francken formed the so-called Ineffable Lodge of Perfection at Albany, New York, and he appointed Moses Michael Hays (a Jewish businessman who later became very wealthy) Deputy Inspector and Knight Kadosh with the power to constitute Grand Chapters of Knights of the Sun and of Kadosh in West Indies and North America. Moreover, Henry Andrew Francken wrote several ritual books. In 1781, Hays made eight Deputy Inspectors, four of whom were later important in the establishment of the first Supreme Council of the Ancient and Accepted Scottish Rite in South Carolina: Isaac da Costa (a distinguished Jewish merchant and shipping agent) Deputy Inspector for South Carolina, Abraham Forst Deputy Inspector for Virginia, Joseph M. Myers Deputy Inspector for Maryland, and Barend M. Spitzer Deputy Inspector for Georgia. In February 1783, Da Costa went to Charleston, South Carolina, where he established the "Sublime Grand Lodge of Perfection."

After Da Costa's death in November 1783, Hays appointed Myers as Da Costa's successor. In 1788, joined at Charleston by Forst and Spitzer, Myers opened a Grand Council of Princes of Jerusalem claiming jurisdiction over Lodges of Perfection. Moreover, Myers and his Masonic associates fabricated additional high-degree bodies, beyond the Rite of Perfection. Thus, the ruling bodies of the Rite of Perfection in South Carolina, which were originally established by Da Costa in 1783, became the Supreme Council of the so-called Ancient and Accepted Scottish Rite (A.A.S.R.) for the Southern Jurisdiction. The A.A.S.R. has the authority to

confer thirty-three degrees, most of which existed in parts of previous high-degree systems. The formation of the "Mother" Supreme Council of the A.A.S.R. in Charleston took place in May 1801.

During the aforementioned obscure, formative period of the American Lodges of the Rite of Perfection and of the A.A.S.R., there emerged a peculiar Masonic legend, according to which Frederick the Great, King of Prussia, was the Supreme Head of the Rite of Perfection. Additionally, according to the same legend, Frederick the Great ratified the Grand Constitutions of 1786 on his death-bed, which underpin the structure and the operation of the Ancient and Accepted Scottish Rite (A.A.S.R.). The legend goes that he also personally instituted the 33rd Degree of the A.A.S.R., delegating his powers as a Sovereign of Masonry to local Supreme Councils, each one of which would govern the A.A.S.R. in its jurisdiction. The original Grand Constitutions of Scottish Masonry had been written in French, but, in 1834, a Latin version of them alleged to have been signed by Frederick the Great was accepted as genuine by the Supreme Council of the A.A.S.R. for France. However, this document is a forgery. The previous legend, according to which Frederick the Great was the Supreme Head of the Rite of Perfection, the author of the Grand Constitutions of 1786, and the creator of the 33rd Degree of the A.A.S.R., was most probably fabricated by founding Grand Inspectors of the Rite of Perfection in America in order to increase the commercial value of the Rite's degrees, and as a marketing tool for the promotion of the 33rd Degree of the A.A.S.R.

The truth is that Frederick the Great was never actively involved in the Rite of Perfection, and that he neither ratified the Grand Constitutions of 1786 which have been, falsely, attributed to him—nor did he institute the 33rd Degree of the A.A.S.R. Nevertheless, the Grand Constitutions of 1786 constitute the fundamental law of the A.A.S.R. in every Supreme Council that has been regularly derived from the Charleston Supreme Council (known as the "Mother Supreme Council of the world"), and Albert Pike, who was the Sovereign Grand Commander of the Southern Jurisdiction of the A.A.S.R. for the U.S.A. from 1859 until his death in 1891, believed that the Grand Constitutions of 1786 were authentic and had been ratified by Frederick the Great. On 19 December 1861, the Grand Lodge of the Three Globes in Berlin published a Protocol, in which it officially stated the following:

> Frederick the Great is said to have revised, reorganized, and increased from 25 to 33 degrees the system of High Degrees in a Supreme Council held in Berlin . . . With regard to this subject, Bro. Le Blanc de Marconnay sent a letter dated 25 May 1833 from New York to the Directory of the

> Grand National Mother Lodge of the Three Globes . . . Are these historical traditions founded on truth? . . . The answer that the Directory returned, on 17 August 1833, says: "The Grand National Mother Lodge of the Three Globes was founded on 13 September 1740, under the authority of Frederick the Great, who was its first Grand Master. He never had anything to do with the organization and legislation of the Grand Lodge. Anything that concerns his having, in 1786, originated a high Masonic Senate, etc., has no historical basis" . . . [Georg Franz Burkhard] Kloss attends to this subject in a long examination in his *History of Freemasonry in France* and stamps the Constitutions and Statutes of the Ancient and Accepted Rite as "the grand lie of the Order." As harsh as this judgment may appear at a first glance, the Directory of the Grand Lodge of the Three Globes, after repeated researches in the archives and historical collections, cannot help sustaining it.

As I have already mentioned, the predominant form of the Scottish Rite was established in 1801 in Charleston, South Carolina. Colonel John Mitchell, a native of Ireland and an officer of the American Army in the Revolutionary War, served as its first Sovereign Grand Commander. Between 1813 and 1815, the second Scottish Rite Supreme Council was organized in the northern part of the United States to counter a clandestine Scottish Rite Supreme Council that had been created by the French Freemason Joseph Cerneau in New York. In 1827, a territorial agreement was reached between the Northern and the Southern Jurisdictions designating the fifteen States north of the Mason–Dixon Line and east of the Mississippi River as the boundary line between the two Jurisdictions. The degrees 4th–33rd that the Scottish Rite superimposed on the three degrees of Symbolic Masonry are a system of didactic narratives, allegories, and symbols designated to help Master Masons who follow the English tradition of Symbolic Masonry, which is centered on the building of King Solomon's Temple and on the Hiramic Legend, perfect themselves as human beings and Freemasons.

After the formation of the "Mother" Supreme Council of the A.A.S.R. in May 1801, other Supreme Councils of the A.A.S.R. were formed internationally. The A.A.S.R. was introduced by Comte de Grasse-Tilly into France (1804), and from France it passed into Italy (1805), Spain (1811), Belgium (1817), etc. In 1813, an officer from the Supreme Council at Charleston elevated several New York Masons to the 33rd degree of the A.A.S.R. and, thus, organized a Supreme Council of the A.A.S.R. for the "Northern Masonic District and Jurisdiction." The "Supreme Council, Scottish Rite, Northern Jurisdiction," based in Lexington, Massachusetts, oversees the Ancient and Accepted Scottish Rite in the following fifteen States: Connecticut, Delaware, Illinois, Indiana, Maine, Massachusetts,

Michigan, New Jersey, New Hampshire, New York, Ohio, Pennsylvania, Rhode Island, Wisconsin, and Vermont. In every other State of the U.S.A., the Ancient and Accepted Scottish Rite is overseen by the "Supreme Council, Ancient and Accepted Scottish Rite, Southern Jurisdiction, U.S.A.," which is commonly known as the "Mother Supreme Council of the World." It has been based in Washington, D.C. since 1911. Moreover, during the nineteenth century, additional Supreme Councils were established in the U.S.A. For instance, a separate Supreme Council was established in 1839 in New Orleans; the so-called King David Supreme Council, populated predominantly by men of color, was established in 1856; and the so-called United Supreme Council was formed in 1881 as a result of the amalgamation of previous Supreme Councils that were created and populated predominantly by men of color (including the King David Supreme Council).

In 1819, the aforementioned "Mother" Supreme Council of the A.A.S.R. in Charleston granted a Patent to the Duke of Sussex (the first Grand Master of the United Grand Lodge of England) to form a Supreme Council in England, but he failed to act upon it. This was probably due to his keen desire to see the UGLE maintain its "purist" policy, according to which pure Ancient Masonry consists only of the degrees of the Entered Apprentice, the Fellow Craft, and the Master Mason, including the Supreme Order of the Holy Royal Arch. Finally, the Supreme Council for England and Wales was founded in 1845, receiving its Patent from the aforementioned Northern Masonic Jurisdiction.

Orator: In our Order, affiliated to each Symbolic Lodge are the following Bodies of appendant degrees belonging to the tradition of the Ancient and Accepted Scottish Rite:

- A Chapter of Rose Croix: In our Order, the Chapters of Rose Croix are empowered to work the following degees: (4º) Secret Master, (5º) Perfect Master, (6º) Intimate Secretary, (7º) Provost and Judge, (8º) Intendant of the Buildings, (9º) Elect of Nine, (10º) Elect of Fifteen, (11º) Sublime Elect, (12º) Grand Master Architect, (13º) Royal Arch of Enoch, (14º) Scotch Knight of Perfection (or Grand Elect Perfect and Sublime Mason), (15º) Knight of the East or of the Sword, (16º) Prince of Jerusalem, (17º) Knight of the East and the West, and (18º) Knight of the Pelican and Eagle and Sovereign Prince Rose Croix of Heredom (or simply Knight Rose Croix).
- A Council of Knights Kadosh: In our Order, the Councils of Knights Kadosh are empowered to work the following degees: (19º) Grand Pontiff, (20º) Venerable Grand Master of the

Symbolic Lodges, or Master ad Vitam, (21º) Patriarch Noachite or Prussian Knight, (22º) Prince of Libanus, (23º) Chief of the Tabernacle, (24º) Prince of the Tabernacle, (25º) Knight of the Brazen Serpent, (26º) Prince of Mercy, (27º) Commander of the Temple, (28º) Knight of the Sun, (29º) Knight of Saint Andrew, and (30º) Grand Elected Knight Kadosh or Knight of the White and Black Eagle.

As you already know, in our Order, the Grand Officers of the Grand Lodge are the *ex officio* Officers of the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of our Order, and the Grand Master of the Grand Lodge of our Order is the *ex officio* Sovereign Grand Commander of the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of our Order. The Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry exercises direct control over the 31st, the 32nd, and the 33rd degrees. The 31st degree is called Grand Inspector Inquisitor Commander, the 32nd degree is called Sublime Prince of the Royal Secret, and the 33rd degree is called Sovereign Grand Inspector General. In our Order, the official name of this Masonic institution is the "Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry." Four weeks after your raising to the degree of Master Mason, the Chapter of Rose Croix that is attached to your Mother Lodge will invite you to begin your career in the degrees of the Ancient and Accepted Scottish Rite as prctised within our Order.

Brother/Sister . . . *(name and surname of the Candidate)*, the motto of a true Master Mason is Liberty with order, Equality with honor for legitimate authority, and Fraternity with justice.

D.C.: May Fraternity, Mercy, Peace, and Prosperity reign forever in the bowels of this Lodge. May the Brothers and the Sisters of this Lodge remain united for the sake of the common Work and, ultimately, for the sake of our noble aspirations.

Closing Ceremony

V.M.: Brothers and Sisters, you will rise and assist me to close the Lodge in the Third Degree.

All rise.

S.O.: Thanks be to the Grand Master and the Authorities of our Order for all the blessings we have received this day.
J.O.: As our minds and hearts are full, nothing now remains but to lock up our secrets in a safe repository.
V.M.: Brothers and Sisters, join me in giving the Sign of a Master Mason and speaking the Word.
All take Step, give the Master Mason's Sign, say "The way up and the way down is one and the same," and then they cut the Sign.
V.M.: Brothers and Sisters, essentially philanthropic, philosophical, and progressive, our Order has as its objects the search for truth, the systematic inquiry into reality, the study of philosophy and science, and the practice of solidarity. It works for the material, the moral, and the intellectual improvement of humanity. "Invenimus Secretum Regium!" *(the Latin phrase meaning "We have found the royal secret!").*
All say: "Invenimus Secretum Regium!" The Lodge is closed in the Third Degree.

Appendant Degrees for the Further Training of Master Masons

Each and every member of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, four weeks after his/her raising to the degree of Master Mason, is invited by the Chapter of Rose Croix that is attached to his/her Symbolic Lodge to receive the degrees 4º–18º. Each and every member of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, twelve months after his/her raising to the degree of Master Mason, is invited by the Council of Knights Kadosh that is attached to his/her Symbolic Lodge to receive the degrees 19º–30º. After completing one Masonic year of good standing in the office of the Senior Overseer or the Junior Overseer of a Symbolic Lodge, each and every member of the Order is invited by the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry to receive the 31st degree. After completing one Masonic year of good standing in the office of the Venerable Master of a Symbolic Lodge, each and every member of the Order is invited by the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry to receive the 32nd degree. After completing three years of good Masonic service following his/her first installation as the Venerable Master of a Symbolic Lodge,

each and every member of the Order is invited by the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry to receive the 33rd degree and, thus, become an honorary member of this Supreme Council.

The Chapter of Rose Croix

In our Order, each Rose Croix Chapter is attached to a Symbolic Lodge and is governed by a Most Wise Sovereign, who is always the same person as the Venerable Master of the Symbolic Lodge to which the given Rose Croix Chapter is affiliated. He/she is supported by the following Officers: (i) a Celebrant, who is always the same person as the Senior Overseer of the Symbolic Lodge to which the given Rose Croix Chapter is affiliated, (ii) a Chancellor, who is always the same person as the Junior Overseer of the Symbolic Lodge to which the given Rose Croix Chapter is affiliated, and (iii) a Conductor of Novices, who is always the same person as the Director of Ceremonies of the Symbolic Lodge to which the given Rose Croix Chapter is affiliated. The assemblies take place in a room that has the following typical layout:

The walls of the room are covered with bookcases, that is, the room is essentially the library of the Symbolic Lodge to which the Rose Croix Chapter is attached. The point of entry is in the middle of the West and facing East. The Most Wise Sovereign sits in the middle of the East and facing West. The Celebrant sits in the East, to the right of the Most Wise Sovereign and facing West. The Chancellor sits in the East, to the left of the Most Wise Sovereign and facing West. In front of the Most Wise Sovereign, there is an oblong table, called the "High Table," on which the characteristic symbols of the Rose Croix Chapter are placed (including the Warrant granted to the Rose Croix Chapter by the Supreme Council), as shown in Figure 4-1. The Candidate sits in the middle of the west side of the High Table, and the Conductor of Novices sits to the left of the Candidate, both facing East. In front of the Candidate's seat, the High Table has a kneeling stool to be used by the Candidate whenever he/she has to kneel according to the Ritual. Other invited members of the 18th degree may attend the ceremony, and, for this reason, there are chairs in the North facing South, and there are chairs in the South facing North.

Figure 4-1: The High Table in the Chapter of Rose Croix.

The Officers of a Chapter of Rose Croix wear the regalia that are normally worn by them in the Symbolic Lodge, and, on the left breast, they wear the breast jewels indicating their rank in the Ancient and Accepted Scottish Rite. The Candidate wears the regalia normally worn by the Master Masons of the Modern and Perfecting Rite of Symbolic Masonry. During the Opening Ceremony, the Candidate waits outside the closed door of the Rose Croix Chapter. Before the Opening Ceremony, the Chancellor meets the Candidate in the Anteroom and hands to him/her two copies of the Pledge of Fidelity (the Rosicrucian Oath). The Candidate must read and sign them. The Most Wise Sovereign has a gavel, with which he/she knocks on the High Table according to the Ritual. The Conductor of Novices carries a poignard in a sheath hanging from the belt of his/her apron.

Opening

Everyone stands in front of their seats.

Most Wise Sovereign (M.W.S.): Brothers and Sisters of the Rose and Cross, assist me to open this Chapter of Rose Croix.

Everyone stands in front of their seats.

M.W.S.: Brother/Sister Celebrant, what is the hour?

Celebrant: The ninth hour of the day.

M.W.S.: This hour is a reminder of the moment when the veil of the Temple was rent in twain and darkness overspread the earth, when the true Light departed from us, the altar was thrown down, the Blazing Star was eclipsed, the Cubic Stone poured fourth blood and water, the Word was lost, and despair and tribulation sat heavily upon us. Since Masonry has experienced such dire calamities, it is our duty, Princes and Princesses Rose Croix, to endeavor by renewed labors to retrieve our loss.

Conductor of Novices (C.N.): I pass the Mystic Word by letters to the East, I N R I.

M.W.S.: The Mystic Word being correct, I return it to the West, I N R I.

M.W.S.: Brothers and Sisters of the Rose and Cross, in the name of the Enlightened Humanity, and by virtue of the power vested in me by the the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, I declare this Chapter of Rose Croix duly open for the diffusion of Light and the advancement of Science.

All clap seven times with their right upon their left hand, and then they take their seats.

The Lodge of Perfection

The Candidate, wearing his/her regalia as a Master Mason of our Order, is admitted by the Conductor of Novices and placed before the west side of the High Table, facing East (everyone is sitting).

M.W.S.: By virtue of the power vested in me by the the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, I declare this Lodge of Perfection open.

He/she gavels nine times.

M.W.S. (addressing the Candidate): Brother/Sister . . . (*name and surname of the Candidate)*, the position of a Prince Rose Croix and the promotion to the 18th degree to which you aspire require that you have already had conferred upon you the first 17 degrees of the Ancient and Accepted Scottish Rite of Freemasonry. It has pleased the Supreme Council to direct that the 14 degrees intervening between the Master Mason degree and the 18th degree of the Ancient and Accepted Scottish Rite of Freemasonry shall be conferred by name. Thus, you will kneel on both knees, place your right hand upon this Rose Croix Chapter's Warrant lying on the table in front of you, and with your left hand hold the poignard upright by the blade with the cross-bar level with your eyes.

The Conductor of Novices gives his/her poignard to the Candidate, and ensures that the Candidate complies with the Most Wise Sovereign's orders.

Chancellor: Most Wise Sovereign, the Candidate has read and signed the Pledge of Fidelity.

The Chancellor gives the Pledge of Fidelity signed by the Candidate to the M.W.S., and the latter, after examining it, returns it to the Chancellor.

M.W.S. (addressing the Candidate): You will now rise with the Sign of Fidelity.

All rise, including the Candidate; the latter returns the poignard to the Conductor of Novices. All give the Sign of Fidelity by placing the right hand on the left breast with the thumb extended in the form of a square and the rest of the fingers joined together.

M.W.S. (addressing the Candidate): Brother/Sister . . . (*name and surname of the Candidate)*, I now confer upon you the following degrees by name:

(4º) Secret Master, (5º) Perfect Master, (6º) Intimate Secretary, (7º) Provost and Judge, (8º) Intendant of the Buildings, (9º) Elect of Nine, (10º) Elect of Fifteen, (11º) Sublime Elect, (12º) Grand Master Architect, (13º) Royal Arch of Enoch, and (14º) Scotch Knight of Perfection (or

Grand Elect Perfect and Sublime Mason). The whole group is known as the "Ineffable degrees." Brothers and Sisters, please, be seated.
All cut the Sign of Fidelity and resume their seats.

Celebrant: The 4th to the 14th degrees all constitute the Lodge of Perfection.

The 4th degree teaches that the Secret Masters are partakers of the Holy Truth, and that they can convey their virtue only through devotion and service to the world—more specifically, through historical action. In particular, according to the symbolic teachings of the 4th degree, there was only one entrance to the Holy of Holies of the First Temple of Jerusalem, in the East, and it was called "Ziza," a Hebrew word meaning "brightness." Hence, the emblem of the 4th degree is a key on which the letter Z is engraved. According to the symbolic content of the 4th degree, the Freemason acquires the symbolic key with which one can open the door of Ziza and cross it, thus entering the realm of pure truth, symbolized by the Holy of Holies of the First Temple of Jerusalem, after one's soul has overcome the obstacles posed by passions, prejudices, and illusions. As is reasonable, the Password of the 4th degree is Ziza; the Sacred Word of this degree is Yod, which is the tenth letter of the Hebrew alphabet; and, in the context of the Kabbalah, Yod is represented by a mere dot, which symbolizes a divine point of energy.

The 5th degree teaches that, for the Perfect Masters, age is not measured by units of time but by deeds, and that the monument of the Work of a person who has worked hard, sincerely, and wisely shines and feeds him/her with the life force of the Creator. The 6th degree teaches that our duty is to act with the best intentions and to judge fairly and honestly. The 7th degree teaches that we have intellect, feelings, instincts, strength, abilities, and commitment, and that we cannot achieve anything significant with these qualities if they are not properly coordinated—that is, if they are not governed by a higher law or principle. In his *Republic* (443d–e), Plato argues that one has cured his soul if he has "attained to self-mastery and beautiful order within himself, and . . . harmonized these three principles [the three parts of the soul, namely, reason, the emotions, and the appetites] . . . linked and bound all three together and made himself a unit, one man instead of many, self-controlled and in unison." The 8th degree teaches that progress cannot be achieved without learning and improvement, that important endeavors can be carried out with the help of others, and that knowledge can be lost if not preserved and passed on to others.

The "Elect" degrees—the 9th, the 10th, and the 11th degrees—recount the mode of the arrest and the punishment of the assassins of Hiram Abiff

by special task forces organized by Solomon, King of Israel. Therefore, the "Elect" degrees are a continuation of the Hiramic legend of English Freemasonry. According to the didactic story of the 9th degree, a task force composed of nine Master Masons found one of the assassins of Hiram Abiff, and the most ardent of the nine killed him. This Master Mason severed the head from the body, and taking it in one hand and his dagger in the other, returned to Jerusalem with the eight fellow Masons. According to the didactic story of the 10th degree, fifteen Master Masons, among whom were the nine Master Masons mentioned in the 9th degree, found the other villains, seized them, and, binding them in chains, conducted them to Jerusalem, where they were sentenced to death and executed. According to the didactic story of the 11th degree, King Solomon instituted this degree, both as a recompense for the zeal and constancy of the Elect of Fifteen and in order to honor other deserving Masons, who excel in science and in social and moral virtues. In brief, the "Elect" degrees teach that arbitrary ambition, fanaticism, and social oppression should be overthrown and dispelled by the sword of justice and freedom, that true and faithful Freemasons are earnest, honest, and sincere, and they protect the people against illegal impositions and exactions.

The 12th degree is alleged to have been established as a school of instruction for the more advanced Craftsmen in order to ensure uniformity in work and vigor in its prosecution, and to prepare Master Masons for promotion to the 13th degree, which is the main focal point of the Lodge of Perfection.

The 13th degree—the Royal Arch of Enoch—is devoted to the correct pronunciation of the name of the Absolute Being, the deity. According to the 13th degree, Enoch, an ancient Patriarch of knowledge and civilization prior to Noah's flood, had a divinely inspired vision in which he contemplated the Ineffable Name of God. This was, specifically, the Holy Tetragrammaton, which consists of four Hebrew letters: Yod–Heh–Vav–Heh (YHVH).

Table 4-1: The Holy Tetragrammaton

Hebrew	Letter name
י	Yod
ה	Heh
ו	Vav
ה	Heh

The Holy Tetragrammaton, YHVH, corresponds to the Sefira of Chokhmah, whereas the Hebrew divine name that corresponds to the Sefira of Kether is Ehyeh, and the Hebrew divine name that corresponds to the Sefira of Binah is YHVH Elohim.

Etymologists teach that the Holy Tetragrammaton—that is, the name "Yehovah" (or "Jehovah")—means "the One who I am." That explanation is in accordance with the Volume of the Sacred Law, which attributes the words "I am that I am" to God, as we read in the book of Exodus, 3:14. It is, in fact, the only name that we can give to the Absolute Being whenever we refer to the Absolute Being's essence, which is totally transcendent. In the case of the Absolute Being, the essence of being is identified with the presence of being. Additionally, according to the 13th degree, the Holy Tetragrammaton contains the following Trinitarian formula: by uniting Yod with the first Heh, we obtain a name of the deity that means Generator; by uniting the first Heh with Vav, we obtain a name of the deity that means Creative Word; and, by uniting Vav with another Heh, a third divine name appears, which continues from the first and the second, so that, when they are all united in one Big Name, we can find Three in One. Those Masons who have studied and understood the aforementioned secrets are ultimately proclaimed Grand Elect Perfect and Sublime Masons through being awarded the 14th degree. Therefore, the Sacred Word of both the 13th and the 14th degrees is "Yehovah" (or "Jehovah"). The Grip of the Grand Elect Perfect and Sublime Masons is given as follows: Give each the right hand; turn it alternatively three times, the first one saying "Berith" (which in Hebrew means "covenant," "treaty," and "agreement between two parties"), and the other saying "Neder" (which in Hebrew means "oath" and "pledge"), to which the first replies "Shelomoth" (which in Hebrew means "peacemaking" and "love of peace").

M.W.S.: I declare this Lodge of Perfection closed.

He/she gavels nine times.

The Council of Princes of Jerusalem

M.W.S.: By virtue of the power vested in me by the the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, I declare this Council of Princes of Jerusalem open.

He/she gavels three times.

M.W.S. (addressing the Candidate): You will now rise with the Sign of Fidelity.

All rise, including the Candidate. All give the Sign of Fidelity by placing the right hand on the left breast with the thumb extended in the form of a square and the rest of the fingers joined together.
M.W.S. (addressing the Candidate): Brother/Sister . . . (*name and surname of the Candidate)*, I now confer upon you the 15th degree, Knight of the East or of the Sword, and the 16th degree, Prince of Jerusalem.
Brothers and Sisters, please, be seated.
All cut the Sign of Fidelity and resume their seats.
Celebrant: The 15th degree and the 16th degree constitute the Council of Princes of Jerusalem. They refer to the assistance received from the Persian kings Cyrus and Darius in the task of rebuilding the Jerusalem Temple after the Babylonian captivity. The 15th degree is devoted to the exercise of freedom, and its characteristic symbol is the acronym L.D.P. This stands for both "Liberté De Penser," meaning "freedom of thought," and "Liberté De Passer," meaning "freedom to pass." The 16th degree is focused on the reward due to bravery, constancy, and perseverance.
M.W.S.: I declare this Council of Princes of Jerusalem closed.
He/she gavels three times.

The Lodge of Knights of the East and the West
M.W.S.: By virtue of the power vested in me by the the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, I declare this Lodge of Knights of the East and the West open.
He/she gavels six times.
M.W.S. (addressing the Candidate): You will now rise with the Sign of Fidelity.
All rise, including the Candidate. All give the Sign of Fidelity by placing the right hand on the left breast with the thumb extended in the form of a square and the rest of the fingers joined together.
M.W.S. (addressing the Candidate): Brother/Sister . . . (*name and surname of the Candidate)*, I now confer upon you the 17th degree, Knight of the East and the West.
Brothers and Sisters, please, be seated.
All cut the Sign of Fidelity and resume their seats.
Celebrant: The 17th degree reminds us of the amalgamation of different European nations in the context of the Order of the Knights of Malta during the Second Crusade. According to the 17th degree, Freemasonry conducts its own Crusades, which, in contrast to the medieval Crusades, are aimed at combating bigotry and superstition. The signs, tokens, and

words are exchanged thus: Place your left hand, with the fingers straight, palm downward, on my right hand, and then I cover it with my left hand, and you look over your right shoulder and say Abaddon, which refers to the Evil One. I reply by touching your left shoulder with my left hand, while you touch my right shoulder with your right hand, and then I look over my left shoulder and answer Jabulon, which means "The Lord is our help."

M.W.S.: I declare this Lodge of Knights of the East and the West closed. *He/she gavels six times.*

The Chapter of Princes Rose Croix of Heredom

M.W.S.: By virtue of the power vested in me by the the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, I declare this Chapter of Princes Rose Croix of Heredom open.

He/she gavels seven times.

M.W.S. (addressing the Candidate): You will now rise with the Sign of Fidelity.

All rise, including the Candidate. All give the Sign of Fidelity by placing the right hand on the left breast with the thumb extended in the form of a square and the rest of the fingers joined together.

M.W.S. (addressing the Candidate): Brother/Sister . . . (*name and surname of the Candidate)*, I now confer upon you the 18th degree, Knight of the Pelican and Eagle and Sovereign Prince Rose Croix of Heredom. The word "Heredom" derives from the Greek words "Hierōs," meaning "holy," and "Dōmos," meaning "house," so that the term "Heredom" signifies the Holy House of Masonry.

Brothers and Sisters, please, be seated.

All cut the Sign of Fidelity and resume their seats.

Celebrant: The 18th degree teaches that the Rose symbolizes secrecy and silence as well as Him who is the "Rose of Sharon and the Lily of the Valley," since, in the Song of Solomon, we find a reference to the Savior under the mystical title of the "Rose of Sharon." The cross represents the Cross of Calvary, red with the holy blood. The pelican symbolizes the Savior Jesus Christ who shed his blood for the salvation of humanity, and, therefore, it symbolizes Jesus Christ in his mediatorial character, while the eagle is a symbol of Jesus Christ in his divine character. According to the 18th degree Ritual, the Candidate is symbolically admitted in aChapter of Princes Rose Croix of Heredom "at the ninth hour of the day," when the earth quakes, the rocks are rent, the veil of the Temple is rent in twain, and

darkness overspreads the earth, highlighting the Passion Narrative in Matthew's Gospel (27:50–51).

The 18th degree refers to the figurative passage of man through the depths of darkness and the Valley of the Shadow of Death to the Mansions of Light, accompanied and sustained by the three theological virtues: Faith, Hope, and Charity, which are symbolically found by symbolically searching in and about the Pillars in the North, the South, and the West for the lost Word. Faith, corresponding to the North, supports the aspirant when tempted to despair; Hope, corresponding to the South, cheers the aspirant on his/her road; Charity, corresponding to the West, sustains the aspirant in every trial, until, having travelled through the abyss of darkness, he/she arrives at the Mansions of Light, corresponding to the East. The 18th degree teaches that, assisted by these three virtues, the candidate will succeed in attaining the Rosicrucians' ultimate goal, which is that Word on which the eternal salvation of humanity depends. In this degree, the aspirant arrives at the Eastern part of the Lodge, where he/she discovers the Lost Word as Emmanuel, meaning "God with us." Thus, the Password of the 18th degree is Emmanuel. The Sacred Word of the 18th degree is I.N.R.I., the initials of the Latin sentence that was placed upon the Cross of Jesus Christ: "Jesus Nazarenus Rex Judaeorum" (as we read in Matthew 27:37, Mark 15:26, Luke 23:38, and John 19:19–20). In fact, they used the letter I instead of the letter J because there was no letter J in the Roman, the Hebrew, and the Greek alphabets (and the letter J first came into use when the Gospel stories were translated into German). Moreover, the Rosicrucians interpret I.N.R.I. as the initials of the Hermetic (that is, alchemical) principles "Igne Natura Renovatur Integra" (meaning "by fire nature is perfectly renewed") and "Igne Nitrum Roris Invenitur" (meaning "by fire the nitre of the dew is discovered"), and as the initials of the Hebrew names of the ancient elements: "Iaminim" (water), "Nour" (fire), "Ruach" (air), and "Iebschah" (earth).

The Sign of Recognition is given as follows: The right hand raised with the index finger pointed towards the heavens, and in reply point to the earth with the same finger. Make these two signs alternatively.

The Sign of the Good Shepherd is given as follows: face each other and bow; place reciprocally crossed hands on breast, give the fraternal kiss, and say the Password, Emmanuel.

M.W.S. (addressing the Candidate): Brother/Sister . . . (*name and surname of the Candidate)*, Prince/Princess Rose Croix, rise to be invested with the breast jewel of the 18th degree. The jewel is a pair of compasses in gilt surmounted by a rose and celestial crown, depicting the pelican on one side and a silver eagle on the reverse. This is worn on the left breast

suspended from a red ribbon. The eagle on the jewel symbolizes Supreme Wisdom and Intelligence. In the Pharaonic civilization, the eagle used to symbolize the goddess Nekhbet, who was considered the protector of the Pharaoh, and her extended wings appeared as a sign of protection. The eagle and the pelican together symbolize, respectively, perfect wisdom and perfect devotedness. You should always wear this jewel in every assembly of our Order, including the Symbolic Lodges, since, in our Order, we interpret and offer the degrees of the Ancient and Accepted Scottish Rite as educational supplements to the degree of Master Mason, and we have restructured the Ancient and Accepted Scottish Rite in order to make it fully compatible with the ethos and the strategic vision of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry. I hope, my Brother/Sister, never to see you dishonor the jewel with which you have been invested.

The Conductor of Novices invests the Candidate with the breast jewel of the 18th degree (placing it on the Candidate's left breast), and then they resume their seats.

Figure 4-2: The front side of the jewel of the 18th degree (on the left), and the reverse side of the jewel of the 18th degree (on the right). In our Order, this jewel is worn on the left breast suspended from a red ribbon.

Closing

M.W.S.: Brothers and Sisters, having arrived at the perfection of Masonry, assist me to close this Rose Croix Chapter.

All rise.

Brother/Sister Chancellor, what is the hour of a Perfect Mason?
Chancellor: Most Wise Sovereign, the hour of a Perfect Mason is when the Word is found and the Cubic Stone is changed into the Mystic Rose. The Blazing Star has reappeared in all its splendor, our altars are renewed, and the true Light restored to our eyes.
M.W.S.: By the word Emmanuel, I close this Rose Croix Chapter.
All clap seven times with their right upon their left hand, and then they depart.

The Council of Knights Kadosh

In our Order, each Council of Knights Kadosh is attached to a Symbolic Lodge and is governed by a Thrice Puissant Master Kadosh, who is always the same person as the Venerable Master of the Symbolic Lodge to which the given Council of Knights Kadosh is affiliated. He/she is supported by the following Officers: (i) a First Free Judge, who is always the same person as the Senior Overseer of the Symbolic Lodge to which the given Council of Knights Kadosh is affiliated, (ii) a Second Free Judge, who is always the same person as the Junior Overseer of the Symbolic Lodge to which the given Council of Knights Kadosh is affiliated, (iii) a Third Free Judge, who is always the same person as the Orator of the Symbolic Lodge to which the given Council of Knights Kadosh is affiliated, and (iv) a Marshal, who is always the same person as the Director of Ceremonies of the Symbolic Lodge to which the given Council of Knights Kadosh is affiliated. The assemblies take place in the same room as the one in which the Chapter of Rose Croix meets, namely, in the library of the Symbolic Lodge to which the Council of Knights Kadosh is attached. However, when the Council of Knights Kadosh meets, the room has the following typical layout: The point of entry being in the middle of the West and facing East, the Thrice Puissant Master Kadosh sits in the middle of the East and facing West. The First Free Judge sits in the East, to the right of the Thrice Puissant Master Kadosh and facing West. The Second Free Judge sits in the East, to the left of the Thrice Puissant Master Kadosh and facing West. In front of the Thrice Puissant Master Kadosh, there is the "High Table," an oblong table on which the characteristic symbols of the Council of Knights Kadosh are placed (including the Warrant granted to the Council of Knights Kadosh by the Supreme Council). On the east side of the High Table, in front of the Thrice Puissant Master Kadosh, rests the sword of the Thrice Puissant Master Kadosh. The Candidate sits in the middle of the west side of the High Table, and the Marshal sits to the left of the Candidate, both facing

East. In front of the Candidate's seat, the High Table has a kneeling stool to be used by the Candidate whenever he/she has to kneel according to the Ritual. The Third Free Judge sits exactly next to the door, on the south (i.e., the right-hand) side of the door, facing East. Other invited members of the 30th degree may attend the ceremony, and, for this reason, there are chairs in the North facing South, and there are chairs in the South facing North.
The Officers of a Council of Knights Kadosh wear the regalia that are normally worn by them in the Symbolic Lodge, and, on the left breast, they wear the breast jewel indicating their rank in the Ancient and Accepted Scottish Rite. The Candidate wears the regalia normally worn by the Master Masons of the Modern and Perfecting Rite of Symbolic Masonry, and, on the left breast, he/she wears the breast jewel of the 18th degree. During the Opening Ceremony, the Candidate waits outside the closed door of the Council of Knights Kadosh. Before the Opening Ceremony, the Second Free Judge meets the Candidate in the Anteroom and hands to him/her two copies of the Grand Oath of the Knights Kadosh. The Candidate must read and sign them. The Thrice Puissant Master Kadosh has a long white rod, with which he/she taps on the floor according to the Ritual. The Marshal carries a poignard in a sheath hanging from the belt of his/her aapron.

Opening
Everyone stands in front of their seats.
Thrice Puissant Master Kadosh (T.P.M.K.): Sirs and Dames Kadosh, assist me to open this Council of Knights Kadosh.
Everyone stands in front of their seats.
T.P.M.K.: Brother/Sister First Free Judge, at what hour are the labors of the Knights Kadosh opened?
First Free Judge (F.F.J.): At the beginning of night.
T.P.M.K.: What is the object of our assembling?
F.F.J.: The hope of punishing crime and correcting flaws.
Marshal: I pass the Mystic Word to the East, Nekam Adonai.
T.P.M.K.: The Mystic Word being correct, I return it to the West, Nekam Adonai.
T.P.M.K.: Brothers and Sisters of this Council of Knights Kadosh, in the name of the Enlightened Humanity, and by virtue of the power vested in me by the the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, I declare this

Council of Knights Kadosh duly open for the protection of innocence, the punishment of crime, and the treatment of fallacies.
The T.P.M.K. taps seven times with his/her rod on the floor. All resume their seats.

Reception of the Candidate
The Candidate, wearing his/her regalia as a Master Mason of our Order and the breast jewel of the 18th degree, is admitted by the Marshal and placed before the west side of the High Table, facing East (everyone is sitting).
T.P.M.K. (addressing the Candidate): Brother/Sister . . . (*name and surname of the Candidate)*, Prince/Princess Rose Croix, the Hebrew word "Kadosh" means "sacred" as well as "set aside and elevated." The position of a Grand Elected Knight Kadosh or Knight of the White and Black Eagle and the promotion to the 30th degree to which you aspire require that you have already had conferred upon you the 11 degrees intervening between the 18th degree and the 30th degree of the Ancient and Accepted Scottish Rite of Freemasonry. I shall confer upon you these 11 degrees by name. Thus, you will kneel on both knees, place your right hand upon the Warrant of this Council of Knights Kadosh lying on the table in front of you, and with your left hand hold the poignard upright by the blade with the cross-bar level with your eyes *(the Marshal gives his/her poignard to the Candidate, and ensures that the Candidate complies with the T.P.M.K.'s orders).*
Second Free Judge (S.F.J.): Thrice Puissant Master Kadosh, the Candidate has read and signed the Grand Oath of the Knights Kadosh.
The S.F.J. gives the Grand Oath of the Knights Kadosh signed by the Candidate to the T.P.M.K., and the latter, after examining it, returns it to the S.F.J.
T.P.M.K. (addressing the Candidate): You will now rise with the Sign of Fidelity.
All rise, including the Candidate. All give the Sign of Fidelity by placing the right hand on the left breast with the thumb extended in the form of a square and the rest of the fingers joined together.
T.P.M.K. (addressing the Candidate): Brother/Sister . . . (*name and surname of the Candidate)*, Prince/Princess Rose Croix, I now confer upon you the following degrees by name:
(19º) Grand Pontiff, (20º) Venerable Grand Master of the Symbolic Lodges, or Master ad Vitam, (21º) Patriarch Noachite or Prussian Knight, (22º) Prince of Libanus, (23º) Chief of the Tabernacle, (24º) Prince of the Tabernacle, (25º) Knight of the Brazen Serpent, (26º) Prince of Mercy,

(27º) Commander of the Temple, (28º) Knight of the Sun, and (29º) Knight of Saint Andrew.
Brothers and Sisters, please, be seated.
All cut the Sign of Fidelity and resume their seats.
F.F.J.: The 19th to the 30th degrees all constitute the Council of Kadosh.

The major motto of the 19th and the 20th degrees is "Laus Deo," meaning "praise (be) to God." Pontiff means bridge-builder. The 19th degree, which is called Grand Pontiff, teaches that we must build bridges from one human being to another, serving the ideas of Justice, Truth, and Tolerance, thus leading humanity to a state of good spirit, or happiness. The 20th degree, which is called Venerable Grand Master of the Symbolic Lodges, or Master ad Vitam, is devoted to the protection of the pure core of Symbolic Masonry. In particular, the 20th degree teaches that the main goal of Freemasonry is that of philosophy, or finding the truth and serving the idea of goodness. However, as the 20th degree explicitly points out, ignorance, ambition, ostentation, and false beliefs have veiled the truths of Freemasonry; frequently, new and fantastic Masonic degrees were invented and unimaginably lofty Masonic titles were handed out. Thus, the 20th degree boldly assigns Freemasons the responsibility to rid Freemasonry of deception, corruption, false narratives, and vanity.

The major motto of the 21st and the 22nd degrees is "Omnia Tempus Alit," meaning "all time feeds or crushes." The 21st degree teaches that we must judge fairly and impartially, and not expect to gain benefits through a bad deed. The 22nd degree teaches that work is not an insult and a curse but a blessing, because it creates everything and, therefore, is a divine energy and the condition of every civilization.

The major motto of the 23rd, the 24th, and the 25th degrees is "Corde Gladioque Potens," meaning "powerful in heart and sword." The 23rd degree teaches that we should contemplate and approach the cause of being. The 24th degree teaches that we must follow the rules of correct reasoning, become free, and have a faith based on self-cultivation and self-confidence. The 25th degree teaches that we must free ourselves from the logic of material necessities. The biblical story of the "brazen serpent," to which the 25th degree alludes, is recounted in Numbers 21:5–9.

The 25th degree refers to the serpent Nehushtan, the biblical Brazen Serpent, which, according to the Books of Kings, Moses made when the Children of Israel were bitten by the serpents of fire in the wilderness. According to the Kabbalah, Nehushtan is the serpent of the Paths of the Tree of Life, and Moses set it on a pole; that is, Moses twined it round the Middle Pillar of the Sephiroth, because that is the reconciler between the fires of Geburah or Severity and the waters of Chesed or Mercy. Hence, it

is said in the New Testament that Nehushtan is a sign of Christ, the reconciler. Moreover, Nehushtan was made of brass, because brass is the metal of Venus, whose sphere is called Nogah or External Splendor. It is called the External or False Splendor, because it represents the union of the Paths of the Tree of Life without, however, understanding the Sephiroth. Nevertheless, Nehushtan is also the Celestial Serpent of wisdom, but the serpent of temptation is the serpent of the Tree of Knowledge of Good and Evil and not that of the Tree of Life. The Password of the Knights of the Brazen Serpent is I.N.R.I., as in the 18th degree, and their Sacred Word is Moses

The major motto of the 26th, the 27th, and the 28th degrees is "Ardens Gloria Surgit," meaning "burning glory rises." The 26th degree teaches altruism and good will. The 27th degree teaches struggle for the liberation of humanity, just as the Knights Templar fought for the liberation of the Holy Land. The 28th degree teaches that the Absolute Being, traditionally symbolized by the Sun, is transcendent; the essence of the Absolute Being is not accessible to the human intellect; reason is the supreme stage of mental development; and the principle of analogy, that is, thinking through analogies, is the key to understanding the logically inconceivable. The Password of the 28th degree is "Sun-Moon-Tetragrammaton," and the Sacred Word is given as follows: one says "Adonai"; the other replies by saying "Abra" (meaning blameless king).

The major motto of the 29th and the 30th degrees is "Architectonis Magni Dei Gloria," meaning "glory to the Great Architect God." The 29th degree teaches humility, patience, self-sacrifice, love of neighbor, meekness, magnanimity, courage, honesty, truthfulness, and dignity.Virtue, Truth, and Honor are the three most essential qualities of a Knight of Saint Andrew. The Grand Lodge of Scotland recognizes Saint Andrew as the patron Saint of Scottish Freemasonry.

T.P.M.K. (addressing the Candidate): Brother/Sister . . . (*name and surname of the Candidate),* Knight/Dame of Saint Andrew, on the table in front of you, you can see the Tracing Board of the 30th degree. This Tracing Board depicts the Mysterious Ladder of the 30th degree. In the 30th degree of the Ancient and Accepted Scottish Rite, the attention of the candidate is directed towards this mysterious ladder, which has two supports and seven steps. The first support on the right bears the Hebrew name "Oheb Eloah," meaning "love of God," and the second support on the left bears the Hebrew name "Oheb Kerobo," meaning "love of one's neighbor." The seven ascending steps symbolize the hierarchy of the liberal sciences, namely, Grammar, Rhetoric, Logic, Arithmetic, Geometry, Music, and Astronomy, through which abstract thought and

systematic knowledge of the universe become possible. The seven descending steps symbolize seven moral virtues, through which one can turn one's consciousness towards one's fellow-humans, "down here," in this world, in order to promote humanity's prosperity and progress. These seven virtues bear the following Hebrew names: "Tsedakah," which means justice and charity; "Schor Laban," which means white ox and, symbolically, goodness and innocence; "Mathok," which means gentleness and mildness; "Emounah," which means trust in truth; "Hamal Saghia," which means Great Work and emphasizes the moral significance of practical work; "Sabbal," which means burden and moral responsibility; and "Ghemoul Binah Thebounah," which means that prudence leads to wisdom.

My Brother/Sister, you have promised and sworn to protect innocence, to punish crime, and to correct flaws. You may ask yourself, where are the weapons that you are to employ? These weapons exist, my Brother/Sister, and they are within your reach. It remains only for you to seize them with the force of resolution, strengthened only by the awareness that your cause is just. These weapons are science, truth, and humanism. Fanaticism and superstition are the offspring of ignorance. To ignorance, oppose knowledge, which springs from enlightened education. Instruct the masses; teach them truth and critical thinking. To knowledge add morality, and the universe is saved. Even against ambition and fraud, your weapons are the same. The debates of the Knights Kadosh must be calm and dignified. The Knights Kadosh must act subtly and shrewdly. Harsh words, offensive personalities, superficiality, imposture, and glamour are strictly forbidden and systematically combated by Knights Kadosh, and the Third Free Judge guards the door of our Council.

At the bottom of this Tracing Board, below the Mysterious Ladder of the 30th degree, you can see a human skull wearing a crown and a human skull wearing a Tiara. My Brother/Sister, this crown is the emblem of hypocrisy and tyranny. It represents the crown of Philip the Fair, King of France, and the crown of all those who under the name of kings and monarchs have exercised authority in ways that are contrary to justice and reason. This Tiara is the emblem of the disgrace of religion. It represents the Tiara of the cruel and cowardly Pontiff, who sacrificed to his ambition the Order of the Knights Templar. Only if you discard all stupid and vulgar prejudices and imposture do you fully deserve to be a Knight Kadosh. Are you disposed to do it?

Candidate (prompted by the Marshal): I am.

T.P.M.K. (taps once with his/her rod): All rise! *(The order is executed, and all the attendees, including the Candidate, rise. Moreover, all the*

aattendees, including the Candidate, except the T.P.M.K., give the Sign of Fidelity and keep it. The T.P.M.K. leaves his/her rod, takes his/her sword from the High Table, and, pointing the point of his/her sword to the Candidate, addresses the Candidate.) Brother/Sister . . . (*name and surname of the Candidate)*, I do hereby constitute, create, and proclaim you a Grand Elected Knight Kadosh or Knight of the White and Black Eagle. *(The T.P.M.K. puts his/her sword back on the High Table, takes his/her rod, sits down, and then all those present sit down).*

F.F.J.: In the 30th degree, the candidate is pledged to contemplate over the history of Jacques de Molay, the last Grand Master of the Crusading Order of the Knights Templar, and to act according to the deeper meaning of the history of Jacques de Molay by punishing crime, protecting innocence and people's rights, opposing any type of mental and political despotism that undermines people's rational faculty and fundamental freedoms, and fighting against oppression and injustice. Thus, the Password of the 30th degree is given as follows: one says "Menachem Nekam," meaning "comfort through vengeance" or "comfort through the punishment of the guilty"; the other replies by saying "Pharasch-Chol," meaning "all is explained" or "all is understood." Moreover, the Sacred Word of the 30th degree is "Nekam Adonai," meaning "Vengeance, My Lord." In our Order, the Sign of the 30th degree is given as follows: place the right hand, with fingers apart, on the heart, let it fall on the right knee, and then extend the right arm from the shoulder into the air with the fingers closed in a fist and say "Nekam Adonai." Please, note that we honor the memory of Jacques de Molay not because we consider him an initiate of some secret knowledge, but because we consider him an important symbol, but also a victim, of those spiritual and political forces that we are fighting against. Although we recognize that King Philip IV of France acted criminally against the Templars, and although we recognize that Pope Clement V showed cowardice and selfish subservience to King Philip IV of France, we are also aware of the flaws and the corruption of the Templars, and we admit that the supreme authority over the historical, genuine Knights Templar is the Pope of Rome.

It is noteworthy that the Scottish adherents of the exiled British King James II who followed him into exile (after the landing of the Prince of Orange in 1688) brought to the English Court in St. Germain (which had been placed at the disposal of James II by the French King Louis XIV) Scottish Masonic traditions intermingled with Templarism. The development of Scottish Masonry and of Masonic Templar degrees and legends in France was significantly boosted by the seminal *Ramsay's Oration*, written in 1737. The Chevalier Andrew Michael Ramsay was a

native of Scotland, a Knight of the French Order of St. Lazarus, and tutor to the two sons of the exiled James III ("The Old Pretender"), the son of Great Britain's exiled King James II, in 1724 and 1725. In fact, Ramsay was born the son of a baker in Ayr, Scotland. He was educated at the University of Edinburgh, and, when King James II threw the mace into the Thames and fled to Paris, he went to the Netherlands, where he served with the English auxiliaries and studied mystical theology. Ramsay was attracted to the mysticism of quietism as practiced in the circle of George Garden, a Scottish Church minister and leading figure of the early Scottish Episcopal Church, at Rosehearty, in the historical county of Aberdeenshire in Scotland. In 1710, Ramsay travelled to Rijnsburg to meet Antoinette Bourignon de la Porte, a French-Flemish mystic and adventurer. Afterwards, he also met Jeanne-Marie Bouvier de la Motte-Guyon, known simply as Mme Guyon, who was a prominent French mystic and advocate of quietism. Mme Guyon emphasized and taught meditation on love. In August 1710, Ramsay went on to stay with François Fénelon, a French Roman Catholic archbishop, mystical theologian, poet, writer, and preceptor of the grandsons of Louis XIV. Under Fénelon's influence, Ramsay was converted to Roman Catholicism. In January 1724, Ramsay was sent to Rome as tutor to James III's two sons, Charles Edward and Henry. In November 1724, Ramsay was back in Paris.

In 1737, in Paris, Ramsay delivered a thought-provoking and controversial Oration before the English Provincial Grand Lodge of France, of which he was Grand Chancellor and Orator. In his Oration, Ramsay argued that the founders of Freemasonry "were not simple workers in stone," but crusader knights "who vowed to restore the Temple" in the Holy Land in imitation of the ancient Israelites, who, "while they handled the trowel and mortar with one hand, in the other, they held the sword and buckler." Thus, Ramsay attempted to integrate the institution and the ethos of medieval chivalry into Freemasonry, and to cultivate mystical theology through legends about the cultural interaction between crusading knights, especially the Templars, and Eastern mystics during the Crusaders' expeditions in the Eastern Roman Empire and the Middle East.

According to the Chevalier Ramsay's aforementioned oration, at the time of the Crusades, many princes, lords, and knights associated themselves and vowed to restore the Christians' Temple in Jerusalem and to employ themselves in restoring the traditional form and glory of their architecture. Thus, they agreed upon several secret ancient signs and symbolic words drawn from the Christian religion in order to recognize themselves among the heathens and Saracens. Furthermore, according to

the same oration, those secret signs and words were only communicated to those who promised solemnly never to divulge them. As a result, the Chevalier Ramsay maintains, the previous sacred promise, often associated with the Templars' inner circle, was not heretical, but instead it was a holy bond to unite Christians of all nationalities into one confraternity. Moreover, according to Ramsay, sometime afterwards, Masonry was somehow united with the Knights of St. John. From that time, the Masons' Lodges took the name of Lodges of St. John.

The "Poor Fellow-Soldiers of Christ and of the Temple of Solomon" (in Latin, *Pauperes Commilitones Christi Templique Salomonici*)—commonly known as the "Knights Templar," the "Order of the Temple" (in French, *Ordre du Temple* or *Templiers*), or simply as "Templars"—were among the most famous Crusading Orders. When the Holy Land was lost, support for the Order faded. Rumors about the Templars' secret initiation ceremony created mistrust, and King Philip IV of France, deeply in debt and jealous of the Templars' financial prosperity and power, took advantage of the situation. In 1307, many Templars in France were arrested, tortured into giving false confessions, and then burned at the stake. King Philip IV had Jacques de Molay, the 23rd and last Grand Master of the Knights Templar, burned at the stake in Paris, in March 1314. Moreover, under pressure from King Philip IV, Pope Clement V disbanded the Order in 1312. Since at least the eighteenth century, several Masonic Bodies have embraced Templar symbols and rituals, mainly using the historical Order of the Temple as a symbol in order to teach moral lessons emphasizing the significance of committing oneself to noble goals, fortitude, and the need for a humanistic crusade.

Both the aforementioned oration made by the Chevalier Ramsay in 1737 in Paris and the Rite of Strict Observance, which was set up in Germany in the 1760s by Baron von Hund, have propagated the following legend: after the destruction of the Templar Order and the execution of its last Grand Master, Jacques de Molay, a number of Knights Templar escaped, came together on the mysterious Mount Heredom near Kilwinning, where, to avoid persecution, they turned themselves into Freemasons and transferred the Templar secrets into the secrets of Freemasonry. However, this legend is historically unsubstantiated and should be treated as a tale for the following reasons. Firstly, the Knights Templar were never persecuted in Scotland, and the Prior of the Knights Templar in Scotland, who was the Preceptor of Torphichen, participated in the Scottish government by virtue of that office until the Protestant Reformation in Scotland. Therefore, in Scotland, there was no need for the Templars to transform themselves into anything else. Secondly, the

mysterious Mount Heredom never existed. The term "Heredom" was merely a popular name in many degrees fabricated in Europe in the eighteenth century; according to Mackey's *Encyclopedia of Freemasonry*, the word "Heredom" derives from the Greek words "Hierōs," meaning "holy," and "Dōmos," meaning "house," so that the term "Heredom" signifies the Holy House of Masonry. Thirdly, the Templars never had any "secrets" so important that would endanger one's life by simply keeping them.

It is worth pointing out that one of the most important and oldest mottos of the 30th degree of the A.A.S.R. is: "Nec proditor Nec proditus innocens foret," which means that neither the betrayer nor the betrayed can be innocent. The Order of the Knights Templar was betrayed by some of its own knights, as well as by its secular head, King Philip IV of France, and by its ecclesiastical head, Pope Clement V. However, the Order of the Knights Templar itself had betrayed the ancient pure Christian ethos, which the Templars were supposed to safeguard. In fact, the Knights Templar gave in to the temptations of bloody religious wars and avarice. Not only did the Templars conduct a ruthless religious war and loot several areas in Byzantium and the Middle East, but they also accumulated huge financial wealth by trading money. The Order of the Knights Templar created an international banking network, became a banker of monarchs for their mortgages, and was offering loans to finance wars through the Order's Paris headquarters. Moreover, the overriding majority of the Templars were generally uneducated and capable only of wielding the sword, with no qualifications for governing. When traditional Masonic rituals refer to the rebuilding of King Solomon's Temple, they do not refer to a literal, material project, but to a biblical symbol of a spiritual world order. The Templars conceived a similar project which they were unable of executing, because they were morally and intellectually deficient, and because the later Chiefs of the Order of Solomon's Temple did not fully comprehend its mission.

Leaving aside every non-Masonic neo-Templar organization, which may pursue its own peculiar agenda, I should stress that those who can understand the esoteric teachings of Freemasonry are aware that the 30th degree of the A.A.S.R. uses the history of the Knights Templar neither in order to teach the "merits" of usury and "holy wars" nor in order to "sell" defunct chivalric titles, but in order to urge Freemasons to contemplate moral values and human rights and in order to transform Freemasons into Crusaders of Humanism.

Intimately related to the 30th degree is the ancient Egyptian legend of Osiris. From the perspective of a Knight Kadosh, the legend of Osiris

firstly symbolizes the destruction of the Order as a result of the slain of Hiram Abiff, a martyr of fidelity to a sacred obligation. Secondly, it symbolizes the Freemasons' attempt to raise Hiram Abiff as "the radical intelligence," what the Kabbalists call Geburah, which symbolically corresponds to Horus, or "Horus Abiff."

T.P.M.K. (addressing the Candidate): Brother/Sister . . . (*name and surname of the Candidate)*, having been elevated to the degree of Grand Elected Knight Kadosh or Knight of the White and Black Eagle, rise to be invested with the breast jewel of the 30th degree. The jewel is a black double-headed eagle surmounted by a crown, holding a sword in its claws. This is worn on the left breast suspended from a white ribbon, next to the breast jewel of the 18th degree. You should always wear these jewels in every assembly of our Order, including the Symbolic Lodges, since, in our Order, we interpret and offer the degrees of the Ancient and Accepted Scottish Rite as educational supplements to the degree of Master Mason, and we have restructured the Ancient and Accepted Scottish Rite in order to make it fully compatible with the ethos and the strategic vision of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry. I hope, my Brother/Sister, never to see you dishonor the jewels with which you have been invested.

The Marshal invests the Candidate with the breast jewel of the 30th degree (placing it on the Candidate's left breast), and then they resume their seats.

Figure 4-3: The jewel of the 30th degree. In our Order, this jewel is worn on the left breast suspended from a white ribbon, next to the breast jewel of the 18th degree.

Closing

T.P.M.K.: Sirs and Dames Kadosh, assist me to close this Council of Knights Kadosh.

All rise.
Brother/Sister Second Free Judge, at what hour are the labors of the Knights Kadosh closed?
S.F.J.: At the break of the day, Thrice Puissant Master Kadosh.
T.P.M.K.: May we leave this Council with a firm determination to protect innocence, to punish crime, and to correct flaws. In the name of the Enlightened Humanity, and by virtue of the power vested in me by the the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, I declare this Council of Knights Kadosh closed.
The T.P.M.K. taps once with his/her rod. All depart.

The 31st, the 32nd, and the 33rd Degrees

In our Order, the *Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry* is the only Masonic Body that confers the 31st, the 32nd, and the 33rd degrees. According to our Order's policy, the 31st, the 32nd, and the 33rd degrees serve the following purposes: in the 31st degree, which is called Grand Inspector Inquisitor Commander, the training of our Order's Freemasons in administrative policies and procedures and in operations management culminates (the Supreme Council offers special training programs on these subjects); in the 32nd degree, which is called Sublime Prince of the Royal Secret, the training of our Order's Freemasons in strategic management culminates (the Supreme Council offers special training programs on these subjects); and, in the 33rd degree, which is called Sovereign Grand Inspector General, the members undertake the duty to encourage charity and fraternal love throughout the Order and preserve with due reverence the Order's ethos. The Officers of the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry are considered to be the only active members of this Masonic Body, while all other persons upon whom this Supreme Council bestows the 33rd degree are considered honorary members of the given Supreme Council. The Officers of the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, as they are also Grand Officers of the Grand Lodge of the Autonomous Order of the

Modern and Perfecting Rite of Symbolic Masonry, constitute the executive council of the Scottish Rite degrees in our Order, and they are the following:

- Sovereign Grand Commander: He/she is the Chief Executive Officer of the Supreme Council of the 33rd and Last Degree of the Ancient and Accepted Scottish Rite for the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry. This office is automatically occupied by the Grand Master of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.
- Pro Sovereign Grand Commander: Given that the Sovereign Grand Commander of the Supreme Council is also the Grand Master of the Grand Lodge, he/she may appoint a Pro Sovereign Grand Commander to be his/her principal adviser and to act for him/her when, due to other Masonic duties, he/she is unable to be present. This office is automatically occupied by the Deputy Grand Master of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.
- Lieutenant Grand Commander: He/she is the Deputy Chief Executive Officer of the Supreme Council. This office is automatically occupied by the Assistant Grand Master of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.
- Grand Chancellor: He/she is the law officer of the Supreme Council, advises the Sovereign Grand Commander and the Supreme Council on matters of Masonic law and precedent, and to him/her belongs to prepare charges against and prosecute offenders before the Supreme Council or any tribunal created by it for the administration of justice in the context of the degrees 4º–30º. This office is automatically occupied by the Senior Grand Overseer of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.
- Grand Chamberlain: He/she is the officer of foreign affairs and shall, when directed by the Sovereign Grand Commander, conduct all correspondence with foreign bodies and report upon it to the Supreme Council; and he/she shall perform such other like duties as may from time to time be required of him/her by the Sovereign Grand Commander or the Supreme Council. This office is automatically occupied by the Junior Grand Overseer of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.

- Grand Librarian and Grand Recorder: He/she is the education officer of the Supreme Council. This office is automatically occupied by the Grand Orator of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.
- Grand Master of Ceremonies: He/she is charged with the duty of causing distinguished visitors to be suitably presented and properly welcomed, and he/she shall direct the formation of all processions of the Supreme Council, escort and introduce all distinguished visitors, and perform such other duties as the Sovereign Grand Commander or the Supreme Council may direct. This office is automatically occupied by the Grand Director of Ceremonies of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.
- Grand Secretary General: He/she shall keep a faithful record of the transactions of the Supreme Council, shall keep in his/her possession and affix to such papers the official seal of the Supreme Council, and perform such other duties as the Sovereign Grand Commander or the Supreme Council may direct. This office is automatically occupied by the Grand Secretary of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.
- Grand Treasurer: He/she is the financial officer of the Supreme Council. This office is automatically occupied by the Grand Treasurer of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.
- Grand Captain of the Guard: He/she is charged with the security of the Supreme Council, he/she shall assist the Sovereign Grand Commander in opening and closing the Supreme Council, lead all processions of the Supreme Council, and perform such other duties as the Sovereign Grand Commander or the Supreme Council may direct. This office is automatically occupied by the Grand Expert of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.
- Grand Standard Bearer: He/she has in his/her charge the Great Banner of the Supreme Council; it is his/her duty to carry the Great Banner in all processions of the Supreme Council, and perform such other duties as the Sovereign Grand Commander or the Supreme Council may direct. This office is automatically occupied by the Grand Inner Guard of the Grand Lodge of the

Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.

- Grand Sword Bearer: He/she has in his/her charge and care the great cross-hilted double-edged sword of the Supreme Council; and it is his/her duty to bear the great sword in all processions of the Supreme Council, and, whenever the Supreme Council is in session, to guard and protect the Great Banner of the Supreme Council, and perform such other duties as the Sovereign Grand Commander or the Supreme Council may direct. This office is automatically occupied by the Grand Outer Guard of the Grand Lodge of the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry.

All the above-mentioned Officers of the Supreme Council 33º wear their respective Grand Lodge regalia (mentioned in Chapter 2), and, on the left breast, they wear the breast jewel of an active member of the Supreme Council 33º (Fig. 4-8), instead of the standard 33rd-degree jewel (Fig. 4-7) worn by the honorary members of the Supreme Council 33º.

In our Order, the room of the Supreme Council 33º has the following typical layout: All the walls are covered with red wallpaper, and the whole floor is covered with red carpet. The point of entry is in the middle of the West and facing East. The zone of the East is elevated by four steps. In the middle of the zone of the East, is the Sovereign Grand Commander's Throne facing West (Fig. 4-4); to the right of the Sovereign Grand Commander's Throne, are the following seats from right to left, facing West: the seat of the Pro Sovereign Grand Commander, the seat of the Grand Chancellor, the seat of the Grand Standard Bearer, and the seat of the Grand Sword Bearer; between the Grand Standard Bearer and the Grand Sword Bearer, stands the Great Banner of the Supreme Council; to the left of the Sovereign Grand Commander's Throne, are the following seats from left to right, facing West: the seat of the Grand Chamberlain, the seat of the Grand Librarian and Grand Recorder, the seat of the Grand Master of Ceremonies, and the seat of the Grand Captain of the Guard. The Sovereign Grand Commander's Throne is richly decorated, and it bears the symbols of the Supreme Council 33º (Fig. 4-4). Before the Throne of the Sovereign Grand Commander, there is a Pedestal covered with red cloth; and the Sovereign Grand Commander's Throne and Pedestal are elevated by one step in relation to the other seats that exist in the zone of the East, and, thus, given that the zone of the East is elevated by four steps, the Sovereign Grand Commander's Throne and Pedestal are elevated by five steps (Fig. 4-4). On the Sovereign Grand Commander's Pedestal, there is a three-branched candlestick (preferably an electric one),

and, on the floor, just in front of this Pedestal and on the same level as the Pedestal, stands a large seven-branched candlestick (preferably an electric one). On the north side of the Sovereign Grand Commander's Pedestal, the great sword of the Supreme Council stands upright on a base placed on the floor, while, on the south side of the Pedestal, a tall floor candlestick stands (preferably an electric one). On the west side of the Sovereign Grand Commander's Pedestal, rests an upright occult Rosicrucian Tracing Board facing West (Fig. 4-4). A short distance in front of this Pedestal, on the floor of the room, and, thus, four steps below the zone of the East, is the Altar of the Supreme Council, on which is an open volume of our Order's Ritual and Constitution, and on this open volume are a Pair of Compasses and a Square interlaced with each other as in the degree of Master Mason (Fig. 4-4). On the north side of the Altar, on the floor, there is a Rough Ashlar, and on the south side of the Altar, on the floor, there is a Perfect Ashlar (Fig. 4-4). The Grand Secretary General and the Grand Treasurer sit in the north-west corner of the room facing East. Specifically, in the north-west corner of the room is the Pedestal of the Grand Secretary General, and the Grand Treasurer sits to the right of the Grand Secretary General. On the Pedestal of the Grand Secretary General, there is a two branched candlestick (preferably an electric one). The Pedestal of the Grand Secretary General is elevated by two steps. The Lieutenant Grand Commander sits in the south-west corner of the room facing East. Specifically, in the south-west corner of the room is the Pedestal of the Lieutenant Grand Commander; and his/her Pedestal is elevated by two steps. On the Pedestal of the Lieutenant Grand Commander, there is a three-branched candlestick (preferably an electric one). There are chairs in the North facing South, and there are chairs in the South facing North. The honorary members of the Supreme Council sit in the North and in the South.

All movements in the Supreme Council Chamber are clockwise, and the Grand Master of Ceremonies guides the Candidate by standing to the left of the Candidate at all times.

Figure 4-4: The Sovereign Grand Commander's Throne.

Opening Ceremony of the Labors of the Supreme Council 33º
Sovereign Grand Commander (S.G.C.): *He/she strikes ten times with the base of his/her scepter, and addresses the Supreme Council as follows:* Most enlightened Brothers and Sisters, I am about to open the Supreme Council. Please be upstanding. *All rise.* Announce yourself!

Lieutenant Grand Commander (L.G.C.): The Pair of Compasses represents the foundation on which the Temple is built.
Grand Chancellor: The Square represents the foundation stone on which the Temple stands.
Grand Chamberlain: The Book of our Ritual and Constitution is the Word that commands the construction of the Temple whose Pillars are Reason, Realism, Truth, and our Right.
Grand Captain of the Guard: The Temple of the Supreme Council works for Freedom, Tolerance, Fraternity, Transcendence, and the Initiatory Secret; the place is consecrated and hidden, which guarantees the inviolability of our mysteries, and we can begin our work.
Grand Standard Bearer: The Great Banner is a sign of the unity and the solidarity of the Masons who make stone blocks for the Temple. Our Great Banner consists of two horizontal bands of equal width, with white on the top and black on the bottom. The white band is embroidered with a gold double-headed eagle surmounted by a crown and holding a sword in its claws. The black band is embroidered with a rayed gold equilateral triangle that is turned upward and has the number thirty-three in red at its center, while two daggers, having a half black and half white hilt, are directed, one from the right and the other from the left, towards the right side and the left side of the aforementioned triangle, respectively. These daggers, with their hilts half black, half white, as well as the two bands of our Great Banner symbolize two apparently antagonistic forces and the contest between light and darkness, good and evil, truth and error, a struggle which began with Time, and which is typified in Masonry by the efforts and anxieties of the Candidate to attain the Light.
Grand Sword Bearer: The Sword of our Supreme Council represents the thought and the analytical skills of the Masons who make stone blocks for the Temple.
S.G.C.: I declare the Supreme Council open for carrying out work in the degree of . . . *(Grand Inspector Inquisitor Commander, or Sublime Prince of the Royal Secret, or Sovereign Grand Inspector General, according to the agenda of the particular session). He/she strikes ten times with the base of his/her scepter. All resume their seats.*

Reception of the Candidate in the 31st Degree
During the Opening Ceremony, the Candidate (wearing his/her Masonic regalia) waits outside the closed door of the Supreme Council Chamber. After the opening of the Supreme Council's labors according to the aforementioned Opening Ceremony, the Grand Master of Ceremonies introduces the Candidate into the Supreme Council Chamber, and leads

him/her firstly before the Grand Secretary General, where the Candidate reads and signs the Grand Oath of the 31st degree. The Grand Master of Ceremonies then leads the Candidate before the Altar of the Supreme Council, where the Sovereign Grand Commander (S.G.C.) explains to the Candidate the special symbols and signs of the 31st degree. Specifically, the S.G.C. informs the Candidate that the Sacred Word of the 31st degree is given as follows: one says "Tsedakah," meaning justice, the other replies by saying "Mishor," meaning equity, and then they both say "Amen." Moreover, the S.G.C. addresses the Candidate as follows:

Sovereign Grand Commander: The term "Inquisitor" means one who seeks, searches for, and investigates the truth. The Grand Inspector Inquisitor Commanders must constantly bear in mind that Justice and Equity are the watchwords of the Order. In particular, your duties are the following: to administer the justice of equity, of right judgment, and of mercy; to safeguard that the recipients of the higher administrative degrees are not unnecessarily multiplied and that improper persons are carefully excluded from membership; and to see that, in their life and conversation, Freemasons bear testimony to the excellence of the Order's ethos.

Subsequently, the Grand Master of Ceremonies leads the Candidate before the Lieutenant Grand Commander, who, by order of the Sovereign Grand Commander, places on the Candidate's left breast the jewel of the 31st degree, replacing with it the breast jewels of the 18th degree and the 30th degree worn until then by the Candidate. The jewel of the 31st degree is a double-headed eagle of black having wing-tips and tail of gilt, it is surmounted by a crown, and it holds a sword in its claws. This jewel is worn on the left breast suspended from a gold bar by a black ribbon. After the investiture of the Candidate, the Lieutenant Grand Commander congratulates the Candidate and expresses the Supreme Council's wishes to the Candidate. Finally, the Grand Master of Ceremonies leads the Candidate to the North, where he/she directs the Candidate to sit down, and then the Grand Master of Ceremonies resumes his/her seat in the East. The S.G.C. closes the Supreme Council.

Figure 4-5: The jewel of the 31st degree. In our Order, this jewel is worn on the left breast suspended from a gold bar by a black ribbon.

Reception of the Candidate in the 32nd Degree

During the Opening Ceremony, the Candidate (wearing his/her Masonic regalia) waits outside the closed door of the Supreme Council Chamber. After the opening of the Supreme Council's labors according to the aforementioned Opening Ceremony, the Grand Master of Ceremonies introduces the Candidate into the Supreme Council Chamber, and leads him/her firstly before the Grand Secretary General, where the Candidate reads and signs the Grand Oath of the 32nd degree. The Grand Master of Ceremonies then leads the Candidate before the Altar of the Supreme Council, where the Sovereign Grand Commander (S.G.C.) explains to the Candidate the special symbols and signs of the 32nd degree. Specifically, the S.G.C. informs the Candidate that the Sacred Word of the 32nd degree is given as follows: one says "SALIX," the other replies by saying "NONIS," and then they both say "TENGU." The words "SALIX NONIS TENGU" are scrambled, and constitute a Masonic cipher that can be interpreted as follows: "LUX INENS AGIT NOS," which is a Latin expression meaning "THE LIGHT THAT IS IN US GUIDES US." Moreover, the S.G.C. addresses the Candidate as follows:

Sovereign Grand Commander: As a result of the revision and the restructuring of the Ancient and Accepted Scottish Rite by our Order, namely, by the Autonomous Order of the Modern and Perfecting Rite of Symbolic Masonry, the 32nd degree of the Ancient and Accepted Scottish Rite is focused on the writings of the Italian Renaissance nobleman and philosopher Giovanni Pico della Mirandola and, particularly, on his *Oration on the Dignity of Man* (*De hominis dignitate* in Latin), which has been called the "Manifesto of the Renaissance." Giovanni Pico della Mirandola's *Oration on the Dignity of Man* was presented as a

public discourse in 1486, and it was published in 1496, two years after Pico's death. Pico's *Oration on the Dignity of Man* underpins the way in which our Order interprets the "Royal Secret."

In his oration, Pico studies the humankind, and he contends that pure reason is the highest level that humanity can reach. Alone among creatures, the human being has a part of every other creature. This freedom enables every human being to raise or lower himself/herself. The higher levels of being are better than the lower levels of being. Freedom sets humanity apart from the other creatures, and even the higher beings imagined in various religious contexts should be envious of humans for their freedom. Pico maintains that humans can improve themselves through the moral sciences. By following higher ideals, humans can rise. These ideals include charity, intelligence, and justice. Acting with charity or justice requires intelligence.

When humans act well, they rise. When humans act poorly, they fall. A human being can thus become like a plant or animal. A plant is a low form, which only lies in place and consumes. An animal lacks reason. When a human being stays still or responds without reason, he/she becomes like a lower creature.

Each person has inner conflicts within himself/herself, dragging him/her down. Such conflicts produce the passions. Additionally, each person often reasons incorrectly. By limiting one's passions, both positive and negative, and improving one's reasoning, a human being can prepare oneself for the most important spiritual accomplishments.

Pico devoted himself to his philosophical studies, instead of concerning himself over money or the opinions of others. He claims that philosophy should be pursued only for the sake of truth, not for money. Additionally, Pico studied mathematics. He considered numbers among the most important methods of thought, as did Plato and other philosophers. In this, they do not refer to ordinary calculations as merchants would perform, but rather to more arcane numerical arts and to the structure of reality.

Finally, according to Pico, "magic," which refers to a deep understanding of nature and the causing of intentional change, can be enlightened or dark. Enlightened magic is like religion, art, or science. Scholars have gone to great lengths to study magic. Magic can unite the higher and the lower. However, among some people, magic has a poor reputation. Dark magic, which degenerates into mania, illusion, and delirium, can have ruinous consequences.

Subsequently, the Grand Master of Ceremonies leads the Candidate before the Lieutenant Grand Commander, who, by order of the Sovereign

Grand Commander, places on the Candidate's left breast the jewel of the 32nd degree, replacing with it the breast jewel of the 31st degree worn until then by the Candidate. The jewel of the 32nd degree is a double-headed eagle of black with the wings, legs, claws, and outer tips of feathers in gilt, it is surmounted by a crown, and it holds a sword in its claws. This jewel is worn on the left breast suspended from a gold bar by a red ribbon. After the investiture of the Candidate, the Lieutenant Grand Commander congratulates the Candidate and expresses the Supreme Council's wishes to the Candidate. Finally, the Grand Master of Ceremonies leads the Candidate to the North, where he/she directs the Candidate to sit down, and then the Grand Master of Ceremonies resumes his/her seat in the East. The S.G.C. closes the Supreme Council.

Figure 4-6: The jewel of the 32nd degree. In our Order, this jewel is worn on the left breast suspended from a gold bar by a red ribbon.

Reception of the Candidate in the 33rd Degree

During the Opening Ceremony, the Candidate (wearing his/her Masonic regalia) waits outside the closed door of the Supreme Council Chamber. After the opening of the Supreme Council's labors according to the aforementioned Opening Ceremony, the Grand Master of Ceremonies introduces the Candidate into the Supreme Council Chamber, and leads him/her firstly before the Grand Secretary General, where the Candidate reads and signs the Grand Oath of the 33rd degree. The Grand Master of Ceremonies then leads the Candidate before the Altar of the Supreme Council, where the Sovereign Grand Commander (S.G.C.) explains to the Candidate the special symbols and signs of the 33rd degree. Specifically, the S.G.C. informs the Candidate that the motto of the 33rd degree is "Deus Meumque Jus," meaning "God and My Right," and that the Sacred Word of the 33rd degree is the following: "Mikamocha Ba'elim Adonai?",

meaning "Who is like you in strength, O Lord?" (Exodus 15:11). Moreover, the S.G.C. addresses the Candidate as follows:

Sovereign Grand Commander: The members of the 33rd degree realize, and are explicitly taught, that their object is not to rebuild the material Temple of King Solomon, but a moral temple, wherein truth and love will dwell and people will live as an ecumenical fraternity abiding by the laws of eternal equity and justice. Moreover, the members of the 33rd degree realize, and are explicitly taught, that they have not to avenge the murder of Hiram Abiff, who is a symbol of Masonic wisdom—which ignorance, lust for power, and falsehood have concealed from many people—but they must go on in search of the laws that regulate the moral world.

The human being is both good and bad; at times, good prevails over evil, and, at others, evil prevails over good. Human nature is characterized by the dualistic expressions of good and evil, just and unjust, true and false, beautiful and ugly. In order to bridge the abyss of dualism, humans invented religion—once they become conscious of their own suffering, they turn to a merciful divine being and ask for divine intervention to help them resolve their problems. The humanistic approach to religion that is promoted by the 33rd degree of the A.A.S.R. comprises the following principles: (1) religion is an invention of thinking humanity; (2) religion is meaningful only if and to the extent that it helps humans to resolve their existential problems; (3) religion and science are expressions of human creativity and mechanisms of adaptation of the human species—therefore, religion and science must be reconciled; (4) the cultivation of moral consciousness, through both religious and non-religious anthropologies, leads the most developed minds to the conclusion that, even though humanity may not be ready to unite in one religious formula, it can unite morally within the framework of certain universal moral norms to which all religions conform.

Given that one of the declared goals and structural characteristics of the Modern and Perfecting Rite of Symbolic Masonry is the dereligionization of Freemasonry, the Modern and Perfecting Rite of Symbolic Masonry dismisses any religious or quasi-religious aspect or practice of the traditional rituals of the English Symbolic Masonry and of the Scottish Rite. In principle, it endorses the humanistic approach to religion that is promoted by the 33rd degree of the A.A.S.R., because the latter's attempt to refine, and indeed spiritualize, religion and to promote a type of moral ecumenism is in consonance with the type of spirituality that is promoted by the Modern and Perfecting Rite of Symbolic Masonry. However, the Modern and Perfecting Rite of Symbolic Masonry explicitly

maintains that its project for the spiritualization of religion consists in substituting the traditional "gods" of religions with the ideal of the man-god, which is focused on the idea of wisdom. Thus, the "man-god" of the Modern and Perfecting Rite of Symbolic Masonry is a perfectly wise human being, and the project of the Modern and Perfecting Rite of Symbolic Masonry for the spiritualization of religion implies that wisdom, instead of traditional practices of worship, should be the primary value towards which humans aspire.

Subsequently, the Grand Master of Ceremonies leads the Candidate before the Lieutenant Grand Commander, who, by order of the Sovereign Grand Commander, places on the Candidate's left breast the jewel of the 33rd degree, replacing with it the breast jewel of the 32nd degree worn until then by the Candidate. The jewel of the 33rd degree is a double-headed eagle of silver and gilt, it is surmounted by a crown, and it holds a sword in its claws. This jewel is worn on the left breast suspended from a gold bar by a white ribbon. After the investiture of the Candidate, the Lieutenant Grand Commander congratulates the Candidate and expresses the Supreme Council's wishes to the Candidate. Finally, the Grand Master of Ceremonies leads the Candidate to the North, where he/she directs the Candidate to sit down, and then the Grand Master of Ceremonies resumes his/her seat in the East. The S.G.C. closes the Supreme Council.

Figure 4-7: The jewel of the 33rd degree. In our Order, this jewel is worn on the left breast suspended from a gold bar by a white ribbon.

Closing Ceremony of the Labors of the Supreme Council 33º
Sovereign Grand Commander (S.G.C.): Most enlightened Brothers and Sisters, having satisfactorily labored to diffuse the Light, I am about to close the Supreme Council. Please be upstanding. *All rise. The S.G.C.*

strikes ten times with the base of his/her scepter, and then he/she says: The Temple is closed. *All depart.*

Figure 4-8: The breast jewel of the active members, namely, of the Officers, of the Supreme Council 33º in our Order.

CHAPTER 5

A COURSE OF MATHEMATICS AND NATURAL SCIENCES FOR THE MEMBERS OF THE MODERN AND PERFECTING RITE OF SYMBOLIC MASONRY

"Let no one ignorant of geometry enter."
Inscription at the entrance of Plato's Academy

In their book *What Is Mathematics?* (Oxford University Press, 1941), Richard Courant and Herbert Robbins rightly maintain that, "for more than two thousand years some familiarity with the methods and results of mathematics has been regarded as an indispensable part of the intellectual equipment of every cultured person." Moreover, Symbolic Masonry highlights the importance of the liberal arts and sciences. The Modern and Perfecting Rite of Symbolic Masonry literally and persistently cultivates and promotes the systematic study of the liberal arts and sciences, and it aims to create intellectually rigorous and creative polymaths.

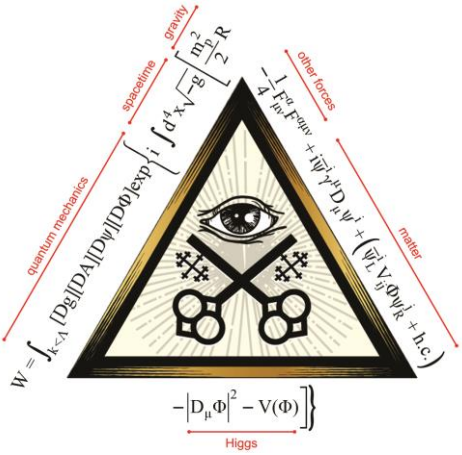
The study of mathematics is hard and takes a lot of effort, but my lecture notes, as I present them here, may even enable one who has little or no mathematics background to learn mathematics on his/her own. Do not feel defeated or stupid if you do not understand everything immediately—it is normal. My lecture notes are self-contained; so think deeply and persist in studying them. At bottom, self-study is fun and an enriching experience, because you study for the sake of learning and at your own pace.

I have written the following "Logico-Mathematical Course" in order to be taught to and assimilated by every Entered Apprentice of the Modern and Perfecting Rite of Symbolic Masonry. It presents the subject in terms of ideas and concepts. It is not intended to be a complete exposition of the subject, but it familiarizes the student with every basic aspect of algebra, geometry, calculus, and modern physics, and it makes him/her capable of having a remarkable access to scientific literature. It requires no prerequisites other than basic high-school mathematics.

The word "mathematics" comes from the Greek word "manthānein," which means "to learn." Mathematics is mainly about forming ways to see problems in order to solve them by combining logical rigor, imagination, and intuition. Furthermore, mathematics is a peculiar sense that allows us to perceive realities that would otherwise be inaccessible to us. In fact, mathematics is our sense for patterns, relations, and logical connections, and it underpins the scientific methodology, whose keystone is the distinction between "knowledge" and "opinion."

The following "Logico-Mathematical Course" covers the following topics:
Mathematical Logic.
Arithmetic and Algebra.
Matrices and applications in input-output analysis and linear programming.
The Basic Equations of Money.
Probability and Risk Analysis.
Classical Euclidean Geometry, Analytic Geometry, and Trigonometry.
Vectors, Vector Spaces, and Normed Vector Spaces.
Non-Euclidean Geometries.
Infinitesimal Calculus (limits, continuity, differentiation, integration, partial differentiation, and multiple integration).
Complex Numbers and Complex Analysis.
Vector Calculus.
Mathematical Physics (mechanics, relativity, conservation of energy and thermodynamics, electromagnetism and light, quantum physics, etc.).

The Venerable Master of each and every Lodge of the Modern and Perfecting Rite of Symbolic Masonry must make sure that suitably qualified Master Masons of his/her Lodge meticulously teach and explain the following "Logico-Mathematical Course" to every Entered Apprentice of his/her Lodge.



The Logico-Mathematical Course

Logic

By the term “deductive system,” we mean a calculus endowed with an interpretation of its terms. A “calculus” is a collection of symbols equipped with a set of rules for their manipulation. When a calculus is equipped with an “interpretation” of its terms, that is, with a set of rules that makes its terms meaningful, then it becomes a deductive system. A deductive system is called “pure” if the rules of the interpretation are sufficient to establish the truth or the falsity of its constituent statements. The statements of a pure deductive system are called “L-determinate,” where L stands for the relevant formal language (the truth value of an L-determinate statement is determined in L by an interpretation of the symbols in L). For instance, logic (the science of correct reasoning) and mathematics are pure deductive systems. Therefore, truths derived from pure deductive systems are based on reason alone, and they are certain because they can never be empirically refuted. If a statement cannot be assigned a truth value only according to the rules of interpretation in the relevant deductive system, then it is called “non-L-determinate.” A non-L-determinate statement is called true or false not only on the basis of the rules of interpretation in the relevant deductive system, but also on the basis of a rule of disposition by reference to empirical data. Non-L-determinate statements for which a rule of disposition by reference to empirical data has been established are called “factual statements,” while the deductive systems in which they appear are called “applied.”

A "scientific theory" is a deductive system (pure or applied) that explains generalizations (i.e., "scientific laws") or aims to criticize and change the structure of the world and/or consciousness.

In symbolic or mathematical logic, the following symbols are used:

∧ or &: conjunction ("and");

∨: disjunction ("or");

¬: negation ("not");

→ or ⇒: material implication ("if . . . then . . ."");

↔ or ⇔: biconditional ("if and only if");

∀: universal quantification ("for every");

∃: "there exists";

∃!: "there exists exactly one";

∄: "there does not exist";

$P(x)$: predicate letter (meaning that x (an object) has property P);

|: "such that";

⊢: turnstile ($x \vdash$ y means that x "proves" (i.e., syntactically entails) y; a sentence φ is "deducible" from a set of sentences Σ, expressed $\Sigma \vdash \varphi$, if there exists a finite chain of sentences $\psi_0, \psi_1, \psi_2, \ldots, \psi_n$ where ψ_n is φ and each previous sentence in the chain either belongs to Σ, or follows from one of the logical axioms, or can be inferred from previous sentences; $\nvdash$ denotes the negation of ⊢);

⊨: double turnstile ($x \models$ y means that x "models" (i.e., semantically entails) y; a sentence φ is a "consequence" (i.e., an ordered list) of a set of sentences Σ, expressed $\Sigma \models \varphi$, if every model of Σ is a model of φ);

$B \subseteq A$: B is a "subset" of A, meaning that every element of a set B is an element of a set A;

$B \subset A$: B is a "proper subset" of A, meaning that $B \subseteq A$ and there is at least one element of A that is not an element of B;

$x \leq y$: x is less than or equal to y;

$x < y$: x is strictly less than y;

$x \geq y$: x is greater than or equal to y;

$x > y$: x is strictly greater than y;

x^n: this operation is called "exponentiation" (pronounced as "x raised to the power of n"), and it means that x is multiplied by itself n times, where $n = 0,1,2,3, \ldots$; $x^0 = 1$, $x^1 = x$, $x^2 = x \cdot x$, $x^3 = x \cdot x \cdot x$, etc.;

$x^{1/n}$: this operation is called the "nth root," and it is the number whose nth power equals the given number ($n \neq 0$); $x^{1/2} = \sqrt{x}$ is the square root, $x^{1/3} = \sqrt[3]{x}$ is the third root, etc.;

( ): brackets; they are used for convenience in grouping terms together (there are specific rules for removing brackets).

The English mathematician and philosopher George Boole (1815–64) realized that arguments expressed in an ordinary language (e.g., in ordinary English) can be expressed in the notation of mathematical logic and then studied in the context of "propositional calculus." For instance, consider the following argument:

- If you want to learn mathematics, then you must study methodically.
- If you must study methodically, then you must be taught an effective method of studying.
- Therefore, if you want to learn mathematics, then you must be taught an effective method of studying.

The aforementioned argument involves various propositions, which we may present by letters as follows:

P: You want to learn mathematics.

Q: You must study methodically.

R: You must be taught an effective method of studying.

These propositions can be "true" or "false." The aforementioned argument can be formalized as follows:

$$P \Rightarrow Q$$
$$Q \Rightarrow R$$

$$P \Rightarrow R$$

where the two propositions above the dashed line are the "premises," and the one below the dashed line is the "conclusion." The reasoning process that leads from premises to a conclusion is called a "deductive process" or just a "deduction." A "theorem" is a formula inferred by means of a rule of inference in a finite number of steps from axioms and previously inferred formulas. Those propositions where truth value is dependent on the values of the variables in them are called "predicates" (hence, we talk about "predicate calculus").

It is important to distinguish between the terms "validity" and "truth" as they are used in logic. An argument, a reasoning process, or a deduction is said to be valid (i.e., logically correct) if the truth of the conclusion follows from the truth of the premises. Note that, if the premises are both true, then the conclusion is logically necessarily true, too. Therefore, with one or more factually incorrect premises, an argument may still be valid, although its conclusion may be false. Furthermore, a valid argument based on false premises does not necessarily lead to a false conclusion. In other words, there is a significant difference between *logical* (i.e., procedural)

correctness ("validity") and *factual* correctness. If an argument is valid (i.e., logically correct), and if its premises are true (i.e., if the facts on which it is based are true), then it is said to be "sound." In logic, we focus on the validity of arguments rather than on their soundness, and this fact explains the "instrumental" role of logic in philosophy and science.

A "Boolean algebra" is the six-tuple

$\langle A, \wedge, \vee, \neg, 0, 1\rangle$,

consisting of a set A equipped with two binary operations: $\wedge$ (called "meet" or "and") and $\vee$ (called "join" or "or"), a unary operation $\neg$ (called "complement" or "not"), and two elements 0 and 1 in A (called "bottom" and "top" respectively, and denoted by the symbols $\bot$ and $\top$ respectively), such that the truth value of a true sentence is 1, the truth value of a false sentence is 0, and, for all elements a, b, and c of A, the following axioms hold:

i. Associativity:
$a \vee (b \vee c) = (a \vee b) \vee c;\ a \wedge (b \wedge c) = (a \wedge b) \wedge c.$
ii. Commutativity:
$a \vee b = b \vee a;\ a \wedge b = b \wedge a.$
iii. Absorption:
$a \vee (a \wedge b) = a;\ a \wedge (a \vee b) = a.$
iv. Identity:
$a \vee 0 = a; a \wedge 1 = a.$
v. Distributivity:
$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c);\ a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c).$
vi. Complements:
$a \vee \neg a = 1$ and $a \wedge \neg a = 0.$

For instance, the 2-element Boolean algebra has only two elements, namely, 0 and 1, and it is defined by the following rules:

Table 5-1: Truth Tables of a 2-Element Boolean Algebra.

a	b	$a \wedge b$	$a \vee b$	a	$\neg a$
0	**0**	0	0	**0**	1
1	**0**	0	1	**1**	0
0	**1**	0	1		
1	**1**	1	1		

Arithmetic and Algebra

The attempts of nineteenth-century mathematicians to found mathematical analysis in a rigorous way were based on real numbers, which also needed

a rigorous foundation. Numbers are abstract objects, concepts. Simultaneously, they are intimately related to the world, since we organize the world with them (that is, we count, we measure, and we form scientific theories with numbers). In order to understand the concept of a number, we have to keep in mind that what we count are not "things," but "sets of things."

The history of set theory and of non-numerical mathematics, in general, can be traced back to the era of classical Greece, but the first systematic inquiry into the foundations of set theory was due to the German mathematician Georg Ferdinand Ludwig Philip Cantor (1845–1918). However, before Cantor, George Peacock (1791–1858), Augustus De Morgan (1806–71), and George Boole (1815–64) had already made significant contributions to the formalization of non-numerical mathematics.

According to Cantor, by the term "set," we should understand a well-defined gathering together into a whole of definite, distinguishable objects of perception or of our thought that are called elements of the set. By the term "well-defined," Cantor means that, given any object and any set, the given object is either an element of the given set or not an element of the given set. By the terms "definite" and "distinguishable," Cantor means that no two elements of a set are the same.

The empty set is denoted by $\emptyset$. The empty set has no elements. If a set has only one element, then it is called a "singleton."

If every element of a set B is an element of a set A, then B is said to be a "subset" of A, and we write $B \subseteq A$. Every set is a subset of itself. If A is an arbitrary set, then $\emptyset \subseteq A$; that is, the empty set is a subset of every set. Two sets A and B are "equal" if and only if $A \subseteq B$ and $B \subseteq A$, and then we write $A = B$. If two sets A and B satisfy the condition $B \subseteq A$ and there is at least one element of A that is not an element of B, then B is said to be a "proper subset" of A, and we write $B \subset A$. If $B \subseteq A$ or $B \subset A$, then A is said to be a "superset" of B. When in a particular situation all the sets under consideration are subsets of a fixed set, this fixed set, which is the superset of every set under consideration, is called the "universal set," or the "universe of discourse."

If the elements of a set are sets themselves, then the set is called a "set of sets," "family of sets," "collection of sets," or "class of sets." For instance, $\mathcal{C} = \{\{x\}, \{y, z\}\}$ is a class of sets (notice that x is something different from $\{x\}$).

If A and B are two arbitrary sets, then we define their

i. "union":
$A \cup B =$
$\{every\ x\ such\ that\ x\ belongs\ to\ at\ least\ one\ of\ A\ and\ B\}$;

and

ii. "intersection":

$$A \cap B = \{every\ x\ such\ that\ x\ belongs\ to\ both\ A\ and\ B\}.$$

Two sets are called "(relatively) disjoint" if their intersection is the empty set.

The German mathematician, logician, and philosopher Friedrich Ludwig Gottlob Frege (1848–1925) has explained that any number n can be used in order to count any n-membered set. For instance, the number two can be thought of as the set of all 2-membered sets, or as the set of all pairs, independently of the nature of the objects that constitute each pair. Similarly, the number three can be thought of as the set of all triplets, the number four can be thought of as the set of all quadruples, and so on.

In particular, in order to define the concept of a natural number $(0,1,2,3,\ldots,n,n+1,\ldots)$, Frege defined, for every 2-place relation R, the concept "x is an ancestor of y in the R-series," and this new relation is known as the "ancestor relation on R." The underlying idea can be easily grasped if we interpret Frege's 2-place relation R as "x is the father of y in the R-series." For instance, if a is the father of b, b is the father of c, and c is the father of d, then Frege's definition of "x is an ancestor of y in the fatherhood-series" ensures that a is an ancestor of b, c, and d, that b is an ancestor of c and d, and that c is an ancestor of d. More generally, given a series of facts of the form aRb, bRc, and cRd, Frege showed that we can define a relation R^* as "y follows x in the R-series." Thus, Frege formulated a rigorous definition of "precedes," and he concluded that a "natural number" is any number of the predecessor-series beginning with 0.

Using the concept of a "predecessor," the American mathematician John von Neumann (1903–57) has proposed an even more accurate definition of a "natural number." According to von Neumann, instead of defining a natural number n as the set of all n-membered sets, a natural number n should be defined as a particular n-membered set—more specifically, as the set of its predecessors. For instance, the number two having two predecessors, zero and one, we can think of the number two as the set $\{0,1\}$, where zero has no predecessor. Therefore, zero can be thought of as the empty set, denoted by $\emptyset$. The number one has only one predecessor, zero. Therefore, we can think of the number one as $\{\emptyset\}$, namely, as the singleton of the empty set. Thus, von Neumann formulated the modern definition of "ordinal numbers." In particular, given the "successor operation," which is defined as

$$succesor(n) = n \cup \{n\},$$

the set of von Neumann natural numbers, the ordinal numbers, denoted by ω, is defined as follows:

i. $\emptyset \in \omega$.
ii. If $n \in \omega$, then $succesor(n) \in \omega$.
iii. Nothing belongs to ω unless it can be constructed using the preceding rules.

Thus, we obtain the following definitions:

$0 = \emptyset$.

$1 = successor(0) = \emptyset \cup \{\emptyset\} = \{\emptyset\} = \{0\}$.

$2 = successor(1) = \{\emptyset\} \cup \{\{\emptyset\}\} = \{\emptyset, \{\emptyset\}\} = \{0,1\}$.

$3 = successor(2) = \{\emptyset, \{\emptyset\}\} \cup \{\{\emptyset, \{\emptyset\}\}\} = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} = \{0,1,2\}$.

$\vdots$

Let X be a set of elements $a, b, \ldots$ Suppose that there is a binary relation expressed by $a \prec b$, defined between certain pairs (a, b) of elements of X, and satisfying the following properties:

$a \prec a$;

if $a \prec b$ and $b \prec a$, then $a = b$;

if $a \prec b$ and $b \prec c$, then $a \prec c$ (transitivity).

Then X is said to be "partially ordered" (or "semi-ordered") by the relation $\prec$.

Let X be a partially ordered set with elements $a, b, \ldots$ If $a \prec c$ and $b \prec c$, then c is said to be an "upper bound" for a and b. If, furthermore, $c \prec d$ whenever d is an upper bound for a and b, we call c the "least upper bound," or the "supremum," of a and b, and we write $sup(a, b)$. This element of X is unique if it exists. In a similar way, we define the "greatest lower bound," or the "infimum," of a and b, and we denote it by $inf(a, b)$.

A partially ordered set X is said to be "linearly ordered" (or "totally ordered") if, for every pair (a, b) in X, either $a \prec b$ or $b \prec a$ holds. A subset of a partially ordered set X is itself partially ordered by the relation that partially orders X; and the subset may even be linearly ordered by this relation. If X is a partially ordered set and A is a subset of X, then an element $m \in X$ is said to be an upper bound of A if $a \prec m$ for every $a \in A$. An element $m \in X$ is said to be "maximal" if the relations $m \in X$ and $m \prec x$ imply that $m = x$ (the maximum is the largest number of the set, while the supremum is the smallest upper bound of the set). In a similar way, we define a "minimal element."

The major sets of numbers are the following:

The Natural Numbers

$\mathbb{N}$: the "natural numbers," namely, the positive integers 1,2,3, ..., which are used to count objects, and 0. For any natural numbers m, n, and k, the following equalities hold true:

i. $m + n = n + m$,
ii. $m + (n + k) = (m + n) + k$,
iii. $mn = nm$,
iv. $m(nk) = (mn)k$,
v. $m(n + k) = mn + mk$,
vi. $m \cdot 1 = m$.

Equalities (i) and (iii) express the "commutative law" of addition and multiplication respectively; equalities (ii) and (iv) express the "associative law" of addition and multiplication respectively; and equality (v) is known as the "distributive law" of multiplication over addition. The aforementioned laws underlie all computations. If a natural number m is divisible by a natural number n, then m is said to be a "multiple" of the number n, and n, in turn, is said to be the "divisor" of the number m. If m is a multiple of the number n, then there is a natural number k such that $m = kn$. For instance, 18 is divisible by 3, and we write $18 = 6 \cdot 3$. In this case, $m = 18$ (the "dividend"), $n = 3$ (the "divisor"), and $k = 6$ (the "quotient"). If a natural number m is not exactly divisible by a natural number n, that is, if there is no natural number k such that $kn = m$, then we consider "division with a remainder." For instance, 33 divided by 2 equals 16 ("partial quotient") with a remainder of 1, and therefore $33 = 16 \cdot 2 + 1$. If a natural number has only two divisors, a unity (one) and the number itself, then it is called a "prime number"; if it has more than two divisors, then it is called a "composite number."

The Italian mathematician and glottologist Giuseppe Peano (1858–1932) has organized the natural numbers as an axiomatic system on the basis of the following axioms, known as the "Peano axioms":

i. 0 is a natural number, symbolically: $0 \in \mathbb{N}$.
ii. If n is a natural number, then the successor of n (i.e., $successor(n) = n + 1$) is also a natural number.
iii. If two natural numbers have the same successor, then the two natural numbers are identical.
iv. 0 is not the successor of any natural number.
v. "Induction Axiom": If X is a set containing both 0 and the successor of every natural number belonging to X, then every natural number belongs to X.

The "Induction Axiom" gives rise to and underpins the principle of "Mathematical Induction," which is a mathematical proof technique for

propositions: Suppose that P is a proposition defined on the natural numbers $\mathbb{N}$, such that:

i. $P(1)$ is true, that is, P holds true for 1;
ii. $P(n+1)$ is true whenever $P(n)$ is true.

Then P is true for every natural number. In this case, P is the "inductive hypothesis." By completing the aforementioned two steps of mathematical induction, we prove that P is true for every natural number.

Example: Let P be the proposition that the sum of the first n natural numbers is $\frac{1}{2}n(n+1)$, namely: $P(n) = 1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n+1)$. We can prove that P is true for every natural number $n \in \mathbb{N}$ using mathematical induction as follows:

Basis step: The proposition holds for $n = 1$, because $1 = \frac{1}{2}(1)(1+1)$. Hence, $P(1)$ is true.

Induction step: We assume that $P(n)$ is true, and we add $n+1$ to both sides of $P(n)$, obtaining

$$1 + 2 + 3 + \cdots + n + (n+1) = \frac{1}{2}n(n+1) + (n+1) =$$
$$\frac{1}{2}[n(n+1) + 2(n+1)] = \frac{1}{2}[(n+1)(n+2)],$$

which is $P(n+1)$. Hence, $P(n+1)$ is true whenever $P(n)$ is true. By the principle of mathematical induction, P is true for every natural number $n \in \mathbb{N}$.

The Integral Numbers

$\mathbb{Z}$: the "integral numbers," or the negative integers, zero, and the positive integers:

$$\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$$

The notation $\mathbb{Z}$ for the set of integers derives from the German word "Zahlen," which means "numbers."

From the perspective of ancient mathematicians, numbers are things by means of which we count, but modern mathematical analysis, founded on Cartesianism, understands numbers mainly as positions on the number line. Let us draw a straight line l and mark on it a point 0 that will be taken as the origin. Then we choose a unit segment $0P$, where P is a natural number that lies to the right of 0, and, in this way, we specify the positive direction. In other words, the unit segment $0P$ determines the direction of the number line and corresponds to the positive unity $+1$ (or simply 1). Let us, for instance, take the number 4. Laying off the unit segment from the point 0 in the given direction four times, we obtain the point Q that corresponds to the number 4. Let us now lay off four unit segments from

the zero point in the direction opposite to the specified. We then get the point Q', which is symmetric to the point Q about the origin 0. The point Q' corresponds to the number -4. Thus, the numbers 4 and -4 are said to be "opposite." By analogy, we can define any other integer (positive or negative). In general, the numbers situated on the number line l in the specified direction are said to be "positive," whereas the numbers located on the number line in the direction opposite to the given one are said to be "negative." Hence, the natural numbers and their opposites (the opposite of the number zero being the same number) form together the set of integral numbers (integers), which is denoted by $\mathbb{Z}$.

If a point X of the line l corresponds to some number r, then this number is said to be the "coordinate of the point X," and, in this case, we write $X(r)$.

The "absolute value" of the number r is denoted by $|r|$. The absolute value of any positive number is the number itself. The absolute value of any negative number is equal to its opposite number. The absolute value of the number zero is zero.

The sum of two negative numbers is a negative number. In order to find the absolute value of a sum, it is necessary to add together the absolute values of the addends. The sum of two numbers having unlike signs is a number that has the same sign as the addend with greatest absolute value. In order to find the absolute value of their sum, it is necessary to subtract the smaller value from the larger one.

In order to subtract one number from another, it is necessary to add to the minuend a number that is the opposite of the subtrahend.

The product (resp. quotient) of two negative numbers is a positive number. The product (resp. quotient) of two numbers having unlike signs is a negative number. In order to find the absolute value of a product (resp. quotient), it is necessary to multiply (resp. divide) the absolute values of these numbers.

The Rational Numbers

$\mathbb{Q}$: the "rational numbers," namely, the set of all numbers of the form $\frac{p}{q}$ such that the numbers p and q are integers, $q \neq 0$, and the greatest common divisor of the integers p and q is ± 1 (that is, p and q are relatively prime integers). In other words, the integral and the fractional numbers (both positive and negative) form together the set of rational numbers, which is denoted by $\mathbb{Q}$. The notation $\mathbb{Q}$ for the set of rational numbers derives from the Italian word "quoziente," which means "quotient."

By the term "common fraction," we refer to a number of the form $\frac{m}{n}$, where m and n are integral numbers, and $n \neq 0$. The number m is called the "numerator" of the fraction, and the number n is called the "denominator" of the fraction. In particular, n may be equal to 1. In this case, we usually write m rather than $\frac{m}{1}$. In other words, any integral number can be represented in the form of a common fraction whose denominator is 1.

Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are regarded to be equal if $ad = bc$. The "basic property of fractions" states the following: the fractions $\frac{a}{b}$ and $\frac{am}{bm}$ are equal. Therefore, if the numerator and the denominator of a given fraction are multiplied or divided by the same natural number, then an equivalent fraction is obtained (namely: $\frac{a}{b} = \frac{am}{bm}$). Taking advantage of the basic property of fractions, we may sometimes replace a given fraction with another equivalent fraction but with a smaller numerator and a smaller denominator by dividing all common factors out of the numerator and the denominator. This operation is called "reduction of a fraction to its lowest terms," or simply "reduction of a fraction." In general, reduction of a fraction is always possible if its numerator and denominator are not relatively prime numbers. If the numerator and the denominator are relatively prime numbers, then the fraction is called "irreducible."

The addition of common fractions is defined in the following way:
$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$.

The subtraction of common fractions is defined in the following way:
$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$.

The multiplication of common fractions is defined in the following way:
$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

The division of common fractions is defined in the following way:
$\frac{a}{b} \div \frac{c}{d} = \frac{a/b}{c/d} = \frac{ad}{bc}$.

A fraction $\frac{m}{n}$ is called a "proper fraction" if its numerator is less than the denominator; and it is called an "improper fraction" if its numerator is greater than the denominator.

Let us consider an improper fraction $\frac{m}{n}$. Since m is greater than n, there are two numbers p and r (with r less than n) such that $m = pn + r$, so that: $\frac{m}{n} = \frac{pn+r}{n} = \frac{pn}{n} + \frac{r}{n} = p + \frac{r}{n}$. Since the remainder is always less than the divisor, $\frac{r}{n}$ is a proper fraction. Hence, we have succeeded in representing

the improper fraction $\frac{m}{n}$ in the form of a sum of a natural number p and a proper fraction $\frac{r}{n}$. This operation is called the "reduction of an improper fraction to a mixed number." A number consisting of an integer and a fraction is called a "mixed number." For instance, in order to locate the mixed number $3\frac{1}{8}$ on the number line, we think as follows: laying off the unit segment ($0P = +1$) from the point zero in the given (positive) direction three times and then $\frac{1}{8}$th part of this unit segment, we obtain the point Q that exactly corresponds to the mixed number $3\frac{1}{8}$ (the coordinate of the point Q is $3\frac{1}{8}$).

The Irrational Numbers

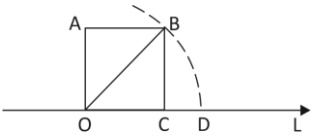
$\mathbb{Q}^{\sim}$: the "irrational numbers," or the set of all numbers that cannot be written as the quotient of two relatively prime integers. For instance, we can prove that $\sqrt{2} \in \mathbb{Q}^{\sim}$ by *reductio ad absurdum* as follows: For the sake of contradiction, suppose that $\sqrt{2} = \frac{p}{q}$ where $p, q \in \mathbb{Z}$, the greatest common divisor of the integers p and q is ± 1, and $q \neq 0$. Then

$$\sqrt{2} = \frac{p}{q} \Rightarrow 2 = \frac{p^2}{q^2} \Rightarrow p^2 = 2q^2 \Rightarrow p = 2k,$$

where k is an appropriate integer; therefore $4k^2 = 2q^2 \Rightarrow q^2 = 2k^2$; but then the greatest common divisor of the integers p and q is 2, which contradicts the hypothesis.

The German mathematician Richard Dedekind (1831–1916) observed that there exist infinitely many points on the straight number line L that correspond to no rational number. Thus, the domain of rational numbers is insufficient if we want to arithmetically follow up all phenomena on the straight line. Therefore, new numbers must be created in such a way that the domain of all numbers will gain the same "completeness" or "continuity" as the straight line. In fact, Dedekind observed that there exist infinitely many cuts that are not produced by rational numbers. For instance, as shown in Figure 5-1, construct a square $OABC$ on the unit segment OC (i.e., the length of OC is equal to one) and lay off in the positive direction a line segment OD equal in length to the diagonal OB; then (according to the Pythagorean Theorem, which we shall study shortly) it is clear that D is a point that does not correspond to any rational number—it, in fact, corresponds to $\sqrt{2}$.

Figure 5-1: Irrational numbers.



The history of irrational numbers goes back to the Pythagorean mathematicians, who had demonstrated that there exist lengths incommensurable with a given unit of length. In the seventh century B.C.E., Thales of Miletus (a Greek mathematician, astronomer, and philosopher from Miletus, in Ionia, Asia Minor) officially initiated a new approach to mathematics. In contrast to the mathematics of other civilizations, such as the Babylonians and the Egyptians, Thales's approach to mathematics is based on the thesis that scientific propositions are not recipes for practical tasks—that is, techniques whose validity is determined by the method of trial and error—but they should be explained and proved. In other words, Thales attempted to endow mathematics with rigor—which, in this case, means logical validity.

In the context of Thales's rigorous mathematics, by the term "line segment," we mean a part of a line that is bounded by two distinct endpoints, and contains every point on the line between the endpoints. Let us consider the line segments $a_1, a_2, a_3, \dots, a_n$ and the non-zero line segments $b_1, b_2, b_3, \dots, b_n$. The line segments $a_1, a_2, a_3, \dots, a_n$ are said to be "proportional" to $b_1, b_2, b_3, \dots, b_n$, respectively, if

$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \cdots = \frac{a_n}{b_n}$.

Thus, two arbitrary line segments a and c are proportional to two other arbitrary line segments b and d respectively, if and only if b and d are non-zero, and it holds that

$$\frac{a}{b} = \frac{c}{d}. \quad (1)$$

Any equality between two ratios, such as (1), is said to be a "proportion" with terms a, b, c, and d, as shown above.

Assume that AB is a non-zero straight line segment, and that P is a point on AB. Then we say that the point P "divides internally" the straight line segment AB in a ratio λ, where $\lambda \geq 0$, if it holds that

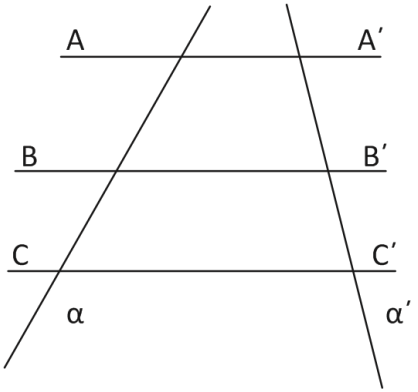
$\frac{PA}{PB} = \lambda$.

If this is the case, then

$\frac{PA}{PA+PB} = \frac{\lambda}{\lambda+1} \Leftrightarrow PA = \frac{\lambda}{\lambda+1} AB$, which implies the uniqueness of P.

Similarly, we say that a point Q "divides externally" the straight line segment AB in a ratio λ, where $\lambda \geq 0$, if the points A, B, and Q are collinear, Q is external to AB, and it holds that
$\frac{QA}{QB} = \lambda$.
If this is the case, then $\frac{QA}{|QA-QB|} = \frac{\lambda}{|\lambda-1|}$ (given that $QA \neq QB$, it holds that $\lambda \neq 1$), so that
$QA = \frac{\lambda}{|\lambda-1|}$ AB, which implies the uniqueness of Q.
Thales's Theorem: If parallel straight lines intersect two straight lines, then they define proportional straight line segments on them. For instance, if parallel straight lines l_1, l_2, and l_3 intersect straight lines a and a' at points A, B, C and A', B', C' respectively, as shown in Figure 5-2, then
$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$.

Figure 5-2: Thales's Theorem.



Corollary 1: Every straight line that is parallel to the bases of a trapezoid divides, internally or externally, the non-parallel sides of the given trapezoid in equal ratios.
Corollary 2: Every straight line that is parallel to one side of a triangle divides, internally or externally, the other two sides of the given triangle in equal ratios.
Corollary 3: If two triangles have a common angle, and if they have parallel opposite sides, then they are said to be in Thales position, and then they are similar and have proportional sides.
In the sixth century B.C.E., Pythagoras and his school (the so-called "Pythagoreans") endorsed Thales's approach to mathematics. From the

Pythagorean perspective of mathematics, the relations between the objects of the world (e.g., magnitudes) correspond to the relations between natural (and, generally, integral) numbers. However, it was soon realized that things are not so simple, since it was realized that there exist magnitudes that do not have a common measure. According to the Pythagoreans, two objects (magnitudes) are "commensurable" (that is, they have a common measure) if and only if there is a magnitude of the same kind that is contained an integral number of times in both of them. In other words, two magnitudes are "commensurable" if and only if their ratio is a rational number. However, the Pythagoreans encountered "incommensurable" magnitudes: magnitudes whose ratio is an irrational number. For instance, as shown in Figure 5-1, the length of a diagonal of a unit square, specifically of a square whose sides have length 1, is, according to the Pythagorean Theorem, equal to $\sqrt{2}$, which is an irrational number. Similarly, a circle's circumference and its diameter are incommensurable (that is, π, the ratio of a circle's circumference to its diameter, is an irrational number). The awareness that there exist incommensurable magnitudes compelled ancient Greek mathematicians to inquire into the relations between incommensurable magnitudes. This event marked a major crisis in ancient mathematics.

According to ancient Greek mathematicians, quantities (magnitudes) are continuous and uniform objects, which are best represented by straight line segments. Their division into parts, or their measurement in terms of a "unit of measurement" (i.e., a definite magnitude of a quantity), meanwhile, represents the notion of discreteness. Ancient Greek mathematicians used the term "ratio of magnitudes" in order to refer to the relation between two magnitudes that can be measured in terms of a common unit of measurement. Thus, the ancient Greek concept of a ratio is most similar to the more abstract modern concept of a number. In the context of ancient Greek mathematics, the objects of mathematics were quantities (represented by straight line segments), and the ratio between two quantities was a meta-object, or something that was used in order to study mathematical objects without being treated as a mathematical object itself. In other words, in the context of ancient Greek mathematics, a ratio (a number) was construed as a measuring relationship between two quantities, and such a measuring relationship could be built up (and hence proved) in finitely many steps, using a common unit of measurement. Nevertheless, the discovery of incommensurable ratios demonstrated that a ratio could not be interpreted as a measuring relationship in the aforementioned way. In fact, as a result of the discovery of incommensurable ratios, the concept of a ratio (or a number) acquired its

conceptual autonomy, and, instead of being treated as a meta-object, it started being treated as an object of mathematics. Therefore, ancient Greek mathematicians had to transcend the system of mathematics that was based on commensurable ratios. Notice that a commensurable ratio could easily become an object of mathematical theory, since it is a rational number, and therefore can be constructed in finitely many steps, whereas the decimal representation of an irrational number neither terminates nor infinitely repeats but extends forever without regular repetition.

In the fourth century C.E., Theon, one of the most important Greek mathematicians and commentators of Euclid's and Ptolemy's works, attempted to solve the problems that were generated as a result of the aforementioned crisis in the foundations of ancient Greek mathematics. In particular, Theon started from an extremely small (infinitesimal) unit square such that the ratio between any of its sides and any of its diagonals is equal to 1 (given that it is infinitely small); symbolically, if a_1 is the length of each of the sides of the given infinitesimal unit square, and if δ_1 is the length of each of the diagonals of the given infinitesimal unit square, then $\frac{\delta_1}{a_1} = 1$. Subsequently, Theon formulated a recursive sequence of unit squares defined by

$a_n = \delta_{n-1} + a_{n-1}$ and $\delta_n = 2a_{n-1} + \delta_{n-1}$,

so that the ratio between a diameter and a side approaches its actual value (meaning the real relationship between a diameter and a side of these unit squares according to the Pythagorean Theorem),

$\frac{\delta_n}{a_n} \to \sqrt{2}$.

He explained that he started from the case in which $\frac{\delta_1}{a_1} = 1$ because, just as the sperm of a living organism encompasses all the subsequent properties of the given organism, any ratio (including the ratio between a diagonal and a side of a unit square) spermatically (at the infinitesimal level) encompasses the unit.

Theon's aforementioned reasoning is underpinned by Aristotle's concept of a "potential infinity." The concept of modern mathematics that is semantically most similar to Aristotle's concept of a "potential infinity" is the convergence of a sequence of natural numbers. Thus, from the perspective of ancient Greek mathematics, infinity is not a being (i.e., it is not an actual state); it cannot be simultaneously considered in its whole extension, but it can only be considered as a becoming (i.e., a process). In this way, the concept of an infinite approach helps us to overcome the contradiction between incommensurable ratios and commensurable ratios, since we can think of an incommensurable ratio infinitely approaching a

commensurable ratio (and vice versa). Similarly, the concept of an infinite approach helps us to overcome the contradiction between broken lines and curves, as well as the contradiction between continuity and discreteness. This reasoning is endorsed by Euclid; in his *Elements*, he does not consider infinitely extended straight lines, but he always works with straight line segments which, as he says, can be extended as much as one needs.

However, several intellectuals have used infinite processes in a way that is not rigorous. For instance, they have attempted to compute the length of the circumference of a circle by considering an inscribed polygon whose number of sides increases indefinitely. Therefore, the length of each side of such a polygon decreases indefinitely, so that a triangle whose base is a side of the given polygon and whose vertex (i.e., the "top" corner opposite its base) is the center of the given circle could become such that its base coincides with the given circle's circumference. To what extent is such a shape a triangle, then, and beyond which point does a straight line segment (in this case, the base of a triangle) become a chord? One may argue that these changes happen when a straight line segment becomes infinitely small, but then one may counter-argue that, by becoming infinitely small, a straight line segment is not "something" any more, and it becomes "nothing." Hence, how is it possible that an infinite series of "nothing" ("no-things") gives "something," such as a circle? The aforementioned example indicates the problems that are generated as a result of the use of infinite processes in computations.

The aforementioned crisis in the foundations of mathematics was overcome by Eudoxus's theory of proportions and by the method of exhaustion, which derives from Eudoxus's theory of proportions, and it was used by Archimedes. The method of exhaustion was originally developed in the fifth century B.C.E. by the Athenian scholar Antiphon, and it was put in a rigorous scientific setting shortly afterwards by the Greek mathematician and astronomer Eudoxus of Cnidus, who used it in order to calculate areas and volumes. The Greek mathematician and acknowledged father of "Euclidean geometry" Euclid, and the Greek mathematician, physicist, and engineer Archimedes, made extensive use of the method of exhaustion in order to prove several mathematical propositions. For instance, Archimedes used the method of exhaustion in order to compute the area of a circle by approximating the area of a circle from above and below by circumscribing and inscribing regular polygons of an increasingly larger number of sides (so that sides become "infinitesimals," or infinitely small): each of the polygons is a union of triangles, so it is easily verified that the area of a circle of radius r and

circumference C is equal to the area of a triangle whose altitude is equal to r and whose base is equal to $C = 2\pi r$. Then, given that the area of a triangle is equal to half of the product of its base and altitude, we obtain the formula for the computation of the area of a circle: $\frac{1}{2}(rC) = \frac{1}{2}(r2\pi r) = \pi r^2$. Moreover, Archimedes was able to calculate the length of various tangents to the spiral (i.e., to a curve emanating from a point moving farther away as it revolves around the point).

Archimedes was very careful in the use of infinite processes; he approximated π by using the fact that the circumference of a circle is bounded by the perimeter of an *inscribed* polygon and the perimeter of a *circumscribed* polygon. According to Eudoxus and Archimedes, there is always a ratio between any two magnitudes, and we can always make any magnitude smaller or greater than a given magnitude, so that the ratio between two magnitudes a and b is the same as the ratio between two other magnitudes c and d if and only if, for any natural numbers m and n, it holds that

$$ma \lessgtr nb \Rightarrow mc \lessgtr nd, \tag{2}$$

meaning that both of these ratios are characterized by the same placement property (i.e., ordering) with regard to other numbers. In (2), the equality sign ($=$) refers to commensurable ratios, whereas the inequality signs ($\lessgtr$) refer to incommensurable ratios. These ideas of Eudoxus and Archimedes indicate that ancient Greek mathematicians discovered not only incommensurable magnitudes but also incommensurable numbers. Eudoxus's aforementioned theory of proportions underpins Archimedes's method of exhaustion for the solution of geometric problems, and Archimedes's method of exhaustion underpins modern infinitesimal calculus.

It is important to notice that the way in which Eudoxus solved the problem of the existence of incommensurable ratios (specifically, his attempt to study the conundrum of irrationality that appears to exist in elementary geometry in a scientifically rigorous way) marks a shift away from the traditional constructivist approach to mathematics towards formalism. In other words, Eudoxus does not explain what a ratio is (as a mathematical object), but he states only when two ratios are similar to each other. The constructivist approach to mathematics allows us to determine what an object is by being able to construct it, whereas the formalist approach to mathematics is not concerned with the substance of the mathematical object under consideration, and is concerned only with the relations of the mathematical object under consideration to other mathematical objects.

Moreover, the ideas of Eudoxus and Archimedes are conceptually very similar to Dedekind's cuts.

Fusing geometry and arithmetic is an arduous task. In order to understand the difficulties that originate from fusing geometry and arithmetic, let us consider, for instance, the famous irrational number $\sqrt{2}$, which was discovered by Pythagoreans when they attempted to compute the length of a diagonal of a unit square.

The Pythagoreans realized that the diagonal of a unit square is not commensurable with the side of the given square, but, by keeping geometry and arithmetic separate from each other (that is, by refusing to identify numbers with lengths of straight line segments), ancient Greek mathematicians could argue as follows: given a straight line segment whose length is one, we can construct a straight line segment whose length is $\sqrt{2}$ (as shown in Figure 5-1). In general, irrational numbers are geometrically constructible (and, hence, geometrically explicable and manageable), even though, from the perspective of arithmetic, irrational numbers are ideal quantities, in the sense that the calculation of irrational numbers (such as $\sqrt{2}$) is an infinite process (namely, irrational numbers have infinitely many decimal digits).

On the other hand, having endorsed the Cartesian approach to mathematics, mathematicians in the nineteenth century realized that they had to clarify some still ambiguous fundamental concepts (such as that of a real number), to formulate new methods of doing mathematics in a logically rigorous way, and to create a rigorous theory of the arithmetic continuum—specifically, a rigorous theory of real numbers and their arithmetic.

The Real Numbers

$\mathbb{R}$: the "real numbers," or the set that is formed by the union of the set $\mathbb{Q}$ of all rational numbers and the set $\mathbb{Q}^{\sim}$ of all irrational numbers; symbolically: $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}^{\sim}$.

Richard Dedekind made an in-depth study of real numbers and continuity. He began with the following three properties of rational numbers:

i. If $a > b$ and $b > c$, then $a > c$.
ii. If a and c are two distinct (rational) numbers, then there exist infinitely many distinct numbers lying between a and c.
iii. If a is any definite (rational) number, then all numbers of the system $\mathbb{Q}$ fall into two classes, A_1 and A_2, each of which contains infinitely many individuals; A_1 contains all numbers a_1 that are $< a$, while A_2 contains all numbers a_2 that are $> a$; the number a

itself may be assigned at will to A_1 or A_2, being, respectively, the greatest number of A_1 or the least number of A_2.

Then Dedekind stated three properties of the points on a straight number line L:

i. If p lies to the right of q and q to the right of r, then p lies to the right of r; and q is said to lie between p and r.

ii. If p and r are two distinct points, then there always exist infinitely many points lying between p and r.

iii. If p is a definite point on L, then all points on L fall into two classes, P_1 and P_2, each of which contains infinitely many individuals; P_1 contains all the points p_1 that lie to the left of p, while P_2 contains all the points p_2 that lie to the right of p; the point p itself may be assigned at will to P_1 or P_2. In any case, every point of P_1 lies to the left of every point of P_2.

Each such division (or partition) of the set $\mathbb{Q}$ of all rational numbers defines a "cut," called the "Dedekind's cut." However, after having observed that every rational number effects a "cut" in the set of rationals, Dedekind considered the inverse question: if, by a given criterion, the set of rationals is divided into two subsets A and B so that every number in A is less than every number in B, is there always a greatest rational in A or a smallest rational in B? Dedekind immediately realized that the number line should be "continuous," or unbroken, in the intuitive sense. Like Eudoxus and Cantor before him, he developed theoretical concepts for the purpose of filling the gaps in the ordered set of rationals so that the final geometric picture is a continuous, straight number line. However, the answer to the last question is in the negative: when A has no maximum rational and B has no minimum rational, there is, indeed, a gap in the rational series (or a puncture in the number line) which must be filled. In that case, the cut (A, B) is said to define (or to be) an irrational number (as shown, for instance, in Figure 5-1). Hence, the set $\mathbb{R}$ of all real numbers is called the "(arithmetic or geometric) continuum" or the "straight line of real numbers."

In modern mathematical notation, the set of all real numbers x such that $a \leq x \leq b$ is said to be a "closed interval," denoted by $[a, b]$, of the real line $\mathbb{R}$, while the set of all real numbers x such that $a < x < b$ (which does not include its endpoints) is said to be an "open interval," denoted by (a, b), of the real line $\mathbb{R}$. The intervals $[a, b) = \{x \in \mathbb{R} | a \leq x < b\}$ and $(a, b] = \{x \in \mathbb{R} | a < x \leq b\}$ are neither open nor closed, but they are sometimes called "half-open" or "half-closed." Notice that $(a, a) = \emptyset$, and $[a, a] = \{a\}$. Moreover, we define the intervals:

$(a, \infty) = \{x \in \mathbb{R} | a < x\}$,

$[a, \infty) = \{x \in \mathbb{R} | a \leq x\}$,
$(-\infty, a) = \{x \in \mathbb{R} | x < a\}$,
$(-\infty, a] = \{x \in \mathbb{R} | x \leq a\}$.
By the term "interval," we generally mean a set of points with the property that, if x and y are distinct points of the set, every point between x and y is also a point of the set (if the points x and y are included, then the interval is closed; otherwise, it is open).
Assume that ε is a positive real number—that is, $\varepsilon > 0$. Moreover, consider the open interval $N = (p - \varepsilon, p + \varepsilon)$. Hence, $p \in (a, b) \subseteq (p - \varepsilon, p + \varepsilon)$. If this is the case, then $(p - \varepsilon, p + \varepsilon)$ is called the ε-neighborhood of the point p, and it is denoted by $N_\varepsilon(p)$. In other words, the ε-neighborhood of a point p on the real line is the set of all those real numbers which are within an ε distance of p on either side of it; p is the midpoint or the center of $N_\varepsilon(p)$; ε is the radius of $N_\varepsilon(p)$. We shall use the notation $N'_\varepsilon(p)$ in order to denote the "deleted neighborhood," consisting of $N_\varepsilon(p)$ with the point p deleted. In terms of the real line $\mathbb{R}$, a deleted neighborhood is an interval on $\mathbb{R}$ with the center point removed.
Given a set S, a real number p is said to be an "interior point" of S if S is a neighborhood of p; symbolically: if $p \in (a, b) \subseteq S$. Obviously, an interior point of a set S belongs to S. The set of all interior points of a given set S is called the "interior" of S, and it is denoted by $Int(S)$. In general, a point $p \in \mathbb{R}^n$ is said to be an "interior point" of U if some neighborhood (open ball) $N_\varepsilon(p)$ with center p is contained in U. For instance, if $S = [2,5]$, then $\frac{7}{2}$ is an interior point of S, whereas neither 2 nor 5 is an interior point of S, because $[2,5]$ is not a neighborhood of 2 and 5. The interior of the closed interval $[2,5]$ is the open interval $(2,5)$.
A real number p is called a "closure point" of a set $S \subseteq \mathbb{R}$ if every neighborhood of p contains a point of S. The set of all closure points of S is called the "closure" of S, and it is denoted by $Cls(S)$. Therefore, every point of $S \subseteq \mathbb{R}$ is a closure point of S.
A real number p is called an "accumulation point," a "limit point," or a "cluster point" of S if every deleted neighborhood of p contains at least one point of S; symbolically: if $S \cap N'_\varepsilon(p) \neq \emptyset \; \forall \varepsilon > 0$ (in other words, every neighborhood of p contains at least one point of S other than p). For instance, if $A = [a, b]$ and $B = (a, b)$, then every member of A is an accumulation point of A and of B, since, for instance, $\forall \varepsilon > 0$, the neighborhood $(a - \varepsilon, a + \varepsilon)$ of a contains infinitely many elements of A and of B. Moreover, every real number is an accumulation point of the set $\mathbb{Q}$ of all rational numbers as well as of the set $\mathbb{R}$ of all real numbers, since, for instance, given an arbitrary real number p, $\forall \varepsilon > 0$, the neighborhood

$(p-\varepsilon, p+\varepsilon)$ contains infinitely many real numbers as well as infinitely many rational numbers. On the other hand, the set $\mathbb{N}$ of all natural numbers, the set $\mathbb{Z}$ of all integral numbers, and the empty set have no accumulation point. Furthermore, no finite set has any accumulation point, because, if, for instance, $A = \{a_1, a_2, a_3, \dots, a_n\}$, and if p is an arbitrary real number, we can construct a sufficiently small neighborhood N with center p such that N contains no point of A; therefore, p, which is an arbitrary real number, is not an accumulation point of A.

A "real number" is a quantity x that has a "decimal expansion":

$x = n + 0.d_1 d_2 d_3 \dots$,

where n is an integer, each d_i is a digit between 0 and 9 ($i = 1,2,3, \dots$), and no infinite sequence of 9 's appears. The aforementioned representation implies that

$$n + \frac{d_1}{10} + \frac{d_2}{100} + \cdots + \frac{d_k}{10^k} \leq x < n + \frac{d_1}{10} + \frac{d_2}{100} + \cdots + \frac{d_k}{10^k} + \frac{1}{10^k},$$

for all positive integers k.

Let a be a real number. Then the product $a \cdot a \cdot a \dots$ (n times) is denoted by a^n, where n is called the "exponent," and a is called the "base." Therefore, the following results hold $\forall a, b \in \mathbb{R}$:

i. $a^n a^m = a^{n+m}$,
ii. $(a^n)^m = a^{nm}$,
iii. $\frac{a^n}{a^m} = a^{n-m}$,
iv. $a^0 = 1$, and
v. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

A "factorial" is a function in mathematics denoted with the symbol ! that multiplies a positive integer n by every number that precedes it:

$n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 2 \cdot 1$.

For instance, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Notice that $0! = 1$, and $1! = 1$. In fact, $n!$ is the number of "permutations" of n elements. The number of "arrangements" of n elements taken m at a time is $A_m^n = \frac{n!}{(n-m)!}$, and the number of "combinations" of n elements taken m at a time is $C_m^n = \frac{n!}{(n-m)!m!} = \frac{A_m^n}{m!}$.

Intimately related to the concepts of an exponent and an index is the concept of a logarithm, which is the inverse function to exponentiation. The "logarithm" of an arbitrary real number a is the exponent to which another fixed real number, the base b, must be raised to produce the real number a; symbolically:

$log_b a = x \Leftrightarrow b^x = a$.

For instance, $log_{10}1{,}000 = 3$, since $10^3 = 1{,}000$, and $log_3 81 = 4$, since $3^4 = 81$. The method of logarithms was originally developed by the Scottish mathematician, physicist, and astronomer John Napier (1550–1617), who published his book *Mirifici Logarithmorum Canonis Descriptio* (*Description of the Wonderful Rule of Logarithms*) in 1614.

In case the base $b = e = \sum_{n=0}^{\infty} \frac{1}{n!} \approx 2.718$, which is known as Euler's number (in honor of the Swiss mathematician Leonhard Euler), then $log_e a$ is written as lna, and it is said to be the "natural logarithm" of a. Euler's number e is irrational, and it was originally derived from the study of compound interest: if one places $1USD$ into a deposit account at a banking institution with 100% interest, and the compounding period is n, as a fraction of a year, then the formula of the compound interest $(1 + \frac{r}{n})^n$, where, in our case, $r = 1$ (annual interest rate as a decimal: $100\% = \frac{100}{100} = 1$), tends to e as n tends to infinity. However, the problem of compound interest was systematically investigated by the Swiss mathematician Jacob Bernoulli (1655–1705), who studied the following question: if an account starts with $1USD$ and pays 100% interest per year, and if the interest is credited once at the end of the year, then the value of the account at the year-end will be $2USD$, but what will happen if the interest is computed and credited more frequently during the year? In fact, Bernoulli noticed that, if there are n compounding intervals, the interest for each interval will be $\frac{100\%}{n}$, and the value of the aforementioned account (which started with $1USD$) at the end of the year will be $1\,USD \times \left(1 + \frac{1}{n}\right)^n$. Furthermore, Bernoulli noticed that this sequence approaches a limit (the "force of interest"). More specifically, it approaches the number e, as n increases—that is, as compounding intervals become smaller. For instance, compounding monthly (i.e., $n = 12$) yields approximately 2.613 USD, while compounding daily (i.e., $n = 365$) yields approximately 2.7146 USD. The limit as n tends to infinity is the number $e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \approx 2.718$, meaning that, with continuous compounding, the value of the aforementioned account will reach approximately 2.718 USD. Leonhard Euler proved that the number e is irrational by showing that its simple continued fraction expansion is infinite (by a "continued fraction," we mean an expression obtained through an iterative process of representing a number as the sum of its integral part and the reciprocal of another number, then writing this other number as the sum of its integral part and another reciprocal, etc.).

The following properties of the logarithm can be easily verified:

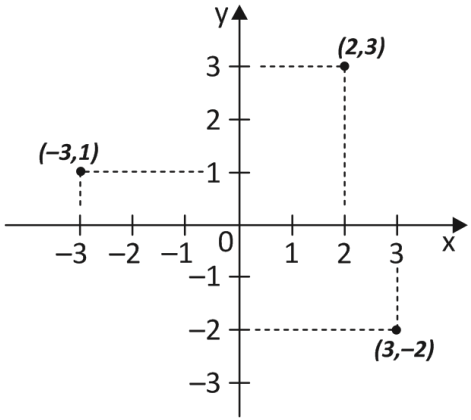
i. $log_b(xy) = log_b x + log_b y$,
ii. $log_b\left(\frac{x}{y}\right) = log_b x - log_b y$,
iii. $log_b x^k = k log_b x$,
iv. $log_b 1 = 0$,
v. $log_b b^x = x = b^{log_b x}$,
vi. $log_b x = \frac{log_a x}{log_a b}$ (change of base rule).
vii. If x, y, and b are positive real numbers with $b \neq 1$, then $x = y \Rightarrow log_b x = log_b y$, and, conversely, $log_b x = log_b y \Rightarrow x = y$. Hence, we can solve exponential equations (i.e., equations in which the unknown is in the exponent) by taking the logarithm of both sides of the equation. For instance, let us solve the exponential equation $5^{2x} = 21$ using log base of e:
$5^{2x} = 21 \Rightarrow ln(5^{2x}) = ln21 \Rightarrow 2x \cdot ln5 = ln21 \Rightarrow 2x = \frac{ln21}{ln5} \Rightarrow$
$x = \frac{\frac{ln21}{ln5}}{2} \approx 0.9458.$

Ordered Pairs and the Cartesian Product

The Fundamental Property of Ordered Pairs: For any ordered pairs $(w, x) and\ (y, z)$, it holds that:
$(w, x) = (y, z) \Leftrightarrow w = y\ \&\ x = z$,
and, in this case, the two ordered pairs are called "equal."
The "Cartesian product" (also known as the "direct product") $A \times B$ of two sets A and B is the set of all ordered pairs (a, b) such that $a \in A\ and\ b \in B$; symbolically:
$A \times B = \{(a, b) | a \in A\ \&\ b \in B\}$.
For instance, if $A = \{1,2\}$ and $B = \{1,3\}$, then the Cartesian product $A \times B$ is the set $\{(1,1), (1,3), (2,1), (2,3)\}$. In general, the Cartesian product of the sets $A_1, A_2, \ldots, A_n$, denoted by $A_1 \times A_2 \times \ldots \times A_n$ is the set of all ordered n-tuples of the form $(a_1, a_2, \ldots, a_n)$, where a_i is an element of $A_i (i = 1,2, \ldots, n)$.
Remark: It is easily checked that, for any sets A, B, and C, we have:
$A \times (B \cup C) = (A \times B) \cup (A \times C)$,
$A \times (B \cap C) = (A \times B) \cap (A \times C)$.
If $A = \emptyset$ or $B = \emptyset$, then $A \times B = \emptyset$.
$A \times B = B \times A \Leftrightarrow A = B$.
Let $A \times B = \{(a, b) | a\ \&\ b\ are\ real\ numbers\}$. Then $A \times B$ is the set of all points in a plane whose coordinates are (a, b). Thus, $A \times B$ is the Cartesian plane

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$,
as shown, for instance, in Figure 5-3. In this case, each point P in the plane represents an ordered pair (a, b) of real numbers, and vice versa. In other words, the vertical line through P meets the x-axis at a, and the horizontal line through P meets the y-axis at b. Thus, we can understand the relationship between set theory, mathematical analysis, and geometry. In other words, a two-dimensional coordinate system consists of the horizontal axis (namely, the x-axis) and the vertical axis (namely, the y-axis), and the intersection of the two axes is the origin $O(0,0)$ of the coordinate system (by the term "axis," we mean a straight line with respect to which a body or structure is symmetrical). By analogy, we can define an n-dimensional coordinate system for $n \geq 3$ ($n = 3,4,5, ...$), using n axes of reference at right angles to each other.

Figure 5-3: The Cartesian Plane.



As noted above, the set $\mathbb{R}$ of all real numbers is called the real line, or the continuum. A set of pairs of real numbers is called a "number plane," and it is denoted by $\mathbb{R}^2$. As already mentioned, the set $\mathbb{R}$ can be represented geometrically as a horizontal number line. A geometric representation of the set $\mathbb{R}^2$ is the coordinate plane xOy, defined as two perpendicular number lines with a common origin O and the same scale (the number of units represented by a unit length along an axis is called the "scale"). The point $O(0,0)$ is called the "origin of coordinates." If P_0 is a point in the coordinate plane, then, by projecting it on the coordinate lines Ox and Oy, we find the coordinates of the projections x_0 and y_0 (notice: if you drop a perpendicular from a point to a line or plane, then the point you reach on

that line or plane is called the projection of the point onto the line or plane). The coordinates are called respectively the "abscissa" (i.e., the x-coordinate) and the "ordinate" (i.e., the y-coordinate) of the point P_0, and the straight lines Ox and Oy are respectively called the "axis of abscissas" and the "axis of ordinates". Hence, to the point P_0 there corresponds one pair of numbers (x_0, y_0); conversely, given a pair of numbers (x_0, y_0), we mark the points x_0 and y_0 on the coordinate lines (axes) Ox and Oy respectively, and, drawing through these points straight lines parallel to the coordinate lines (axes) Ox and Oy, we find the point of their intersection P_0. By analogy we work in $\mathbb{R}^n$.

In general, the use of coordinate systems implies that space itself is encoded by n-tuples (i.e., by sequences, ordered lists, of n numbers), and, specifically, that the two-dimensional space, the "plane," is encoded by pairs of numbers, so that the conception of space becomes subordinate to the conception of arithmetic.

Relations and Functions between Sets

Let A and B be two arbitrary sets. Then a "relation" between A and B, denoted by R, is defined to be a subset of the Cartesian product $A \times B$; symbolically: $R \subseteq A \times B$. The "domain" of relation R is defined by $D_R = \{a|(a,b) \in R\}$, and the "range" of relation R is defined by $R_R = \{b|(a,b) \in R\}$. If R is a relation from A to B, then the relation from B to A is called the "inverse" of R, and it is defined by $R^{-1} = \{(b,a)|(a,b) \in R\}$. A relational proposition is often denoted by aRb, where R relates a term a to a term b. Hence, a relation of two terms proceeds, somehow, from one to the other.

If R_1 is a relation from a set A to a set B, and if R_2 is a relation from B to a set C, then their "composition," denoted by $R_2 \circ R_1$, is a relation from A to C, symbolically:

$R_2 \circ R_1 = \{(a,c)|for\ some\ b \in B, (a,b) \in R_1\ \&\ (b,c) \in R_2\ with\ a \in A, c \in C\}$.

If R_1 and R_2 are relations such that $R_1 \subseteq R_2$, then R_2 is said to be an "extension" of R_1, and R_1 is said to be a "restriction" of R_2.

A relation R on a set A is "reflexive" if (a,a) is an element of R for every $a \in A$; it is "symmetric" if (a,b) is an element of R whenever (b,a) is an element of R; and it is "transitive" if (a,c) is an element of R whenever (a,b) and (b,c) are elements of R. A relation R on a set A is "antisymmetric" if, whenever a and b are distinct, then (a,b) is an element of R only if (b,a) is not an element of R. For instance, if $A = \{u,v,w\}$ and R is a relation on A, then:

$R = \{(u,v),(v,u),(u,u),(v,v),(v,w),(w,w)\}$ is a reflexive relation on A;
$R = \{(u,v),(v,u),(w,w)\}$ is a symmetric relation on A;
$R = \{(u,v),(v,w)(u,w),(v,v)\}$ is a transitive relation on A;
$R = \{(u,w),(v,v),(u,v),(u,u)\}$ is an antisymmetric relation on A.
Let A and B be two arbitrary sets. A relation $f \subseteq A \times B$ is called a "function," "mapping," or "transformation," denoted by $f: A \to B$, if it assigns to each element $a \in A$ exactly one element $b \in B$. The set A is called the "domain" of the function f and is denoted by D_f, while the set B is called the "codomain" of the function f. The set of all elements of B that are related to the elements of A via f is called the "range" of the function f, and it is denoted by R_f, meaning that the range of f is the image of A by f:
$f(A) = \{f(a) | a \in A\}$.
By the term "graph" of a function $f: A \to B$, we mean the set $\{x, f(x)\}$, where $x \in A$. If c is a positive constant, then:

i. The graph of $y = f(x) + c$ is the graph of f raised by c units.
ii. The graph of $y = f(x) - c$ is the graph of f lowered by c units.
iii. The graph of $y = f(x + c)$ is the graph of f shifted c units to the left. In fact, if we analyze the x-values, we can see a pattern, and we realize that the new x that we need in order to obtain $f(0)$ is the one that makes $f(x + c) = f(0)$, namely, $-c$. We can generalize this result as follows:
$f(x_{new} + c) = f(x) \Rightarrow x_{new} + c = x \Rightarrow x_{new} = x - c$,
meaning that the new x-values are the old x-values translated $-c$ units (that is, c units to the left, since that direction is the negative direction).
iv. The graph of $y = f(x - c)$ is the graph of f shifted c units to the right.

The graph of $y = -f(x)$ is the graph of f reflected about the x-axis.
If $c > 1$, then the graph of $y = cf(x)$ is the graph of f stretched by a factor of c. If $0 < c < 1$, then the graph of $y = cf(x)$ is the graph of f flattened out by a factor of c.
Vertical line test: Imagine a vertical line sweeping across a graph. Assume that the vertical line at any position intersects the graph in more than one point. Then the graph is not the graph of a function.
Two functions $f: A \to B$ and $g: A \to B$ are called "equal" if $f(x) = g(x), \forall x \in A$, and they are called "different" if there is at least one $x_0 \in A$ such that $f(x_0) \neq g(x_0)$.
A function f is said to be "odd" if $f(-x) = -f(x)$ for every x in the domain of f. The graph of an odd function has symmetry about the origin.

A function f is said to be "even" if $f(-x) = f(x)$ for every x in the domain of f. The graph of an even function has symmetry about the y-axis.

A function $f: X \to Y$ is called "one-to-one" (or "injective," or an "injection," or a "monomorphism") if

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2, \forall x_1, x_2 \in X$;

that is, a function is "one-to-one" if each x value in the domain is assigned a different y value, so that no two ordered pairs have the same second component. If more than one element of X has the same f-image in Y, then the function $f: X \to Y$ is said to be "many-to-one."

Horizontal line test: Imagine a horizontal line sweeping down the graph of a function. Assume that the horizontal line at any position intersects the graph in more than one point. Then, the function is not one-to-one, and its inverse is not a function.

A function $f: X \to Y$ is called "into" if there exists at least one element of Y that is not the f-image of an element of X. In other words, for any into function $f: X \to Y$, the range set $f(X)$ is a proper subset of Y; symbolically, $f(X) \subset Y$.

If the range of a function f is the whole codomain of f, then f is said to be "onto" (or "surjective," or a "surjection," or an "epimorphism"). In other words, for any onto function $f: X \to Y$, $f(X) = Y$.

If a function is both one-to-one and onto, then it is called "bijective," or a "bijection," or an "one-to-one correspondence."

For instance:

i. If A is a subset of X, then the restriction to A of the identity mapping id_x, defined by $A \ni x \to x \in A$, is an injection j_A, called the "natural injection."
ii. The identity mapping of any set is bijective.
iii. The function $f: X \times Y \to Y \times X$ defined by $(x, y) \to (y, x)$, where $x \in X \; and \; y \in Y$, is bijective.
iv. The function $f(x) = x^2$, where $x \in \mathbb{R}$, is not injective, since $f(-x) = f(x) = x^2$, but the restriction to $\mathbb{R}^+$ (the set of all positive real numbers) of f is injective.
v. $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3$ is an one-to-one and onto mapping, that is, a bijection from $\mathbb{R}$ to $\mathbb{R}$.

Sequences and Series

A "sequence" is a function whose domain is the set of positive integers (i.e., 1,2,3, ...). The functional values (i.e., the range elements) are called the terms of the sequence. In other words, a sequence is a set of numbers arranged in a definite order.

An "arithmetic progression" is a sequence of numbers in which each term after the first is found by adding a constant to the preceding term. This constant is called the "common difference" and is symbolized by d. Thus, the formula for the nth term in an arithmetic progression with first term a_1 and common difference d is:

$a_n = a_1 + (n-1)d.$

A "geometric progression" is a sequence of numbers in which each term after the first is found by multiplying the preceding term by a constant. This constant is called the "common ratio" and is symbolized by r. Thus, the formula for the nth term in a geometric progression with first term a_1 and common ratio r is:

$a_n = a_1 r^{n-1}.$

Associated with any sequence $a_1, a_2, a_3, \ldots$ is a "series"

$$a_1 + a_2 + a_3 + \cdots$$

which is the sum of all the terms in the sequence. A series that is associated with an arithmetic progression is called an "arithmetic series." A series that is associated with a geometric progression is called a "geometric series."

The sum of the first n terms of an arithmetic series is given as the following formula:

$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n-1)d].$

The sum of the first n terms of a geometric series is given as the following formula:

$S_n = \frac{a_1 - a_1 r^n}{1-r} = \frac{a_1 - a_n r}{1-r}.$

Real Equations and Algebra

By the term "equation," we mean a statement that two quantities are equal. For instance, $1{,}000m = 1km$. More often, an equation contains an unknown quantity that is represented by a symbol, and we try to find the value of this unknown quantity. By the term "algebra," we refer to methods and techniques for solving equations. In fact, the core of the study of structures in mathematics consists of taking numbers and putting them into equations in the form of "variables"; and the rules for manipulating these equations are contained in algebra.

The word "algebra" derives from the Arabic word "al-Jabr," meaning "transformation." It refers to a methodology developed by the Persian mathematician Al-Khwarizmi, who lived in Baghdad early in the Islamic era. Al-Khwarizmi was interested in solving algebraic equations, and his method consists in applying a transformation to the given equation in

order to put it into a standard form for which the solution method is known.

Equations requiring multiplication and division:

i. We can solve the equation $\frac{x}{12} = 4$ as follows: multiplying each side by 12, we get $\frac{x}{12} \times 12 = 4 \times 12 \Rightarrow x = 48$. Check: when $x = 48$, the left-hand side of the given equation becomes $\frac{48}{12} = 4$. The right-hand side of the given equation is equal to 4. Therefore, the solution is correct.

ii. We can solve the equation $6x = 3$ as follows: dividing each side by 6, we get $\frac{6x}{6} = \frac{3}{6} \Rightarrow x = \frac{1}{2}$. Check: when $x = \frac{1}{2}$, the left-hand side of the given equation becomes $6 \times \frac{1}{2} = 3$. The right-hand side of the given equation is equal to 3. Therefore, the solution is correct.

Equations requiring addition and subtraction:

i. We can solve the equation $x - 2 = 4$ as follows: adding 2 to each side, we get $x - 2 + 2 = 4 + 2 \Rightarrow x = 6$. The operation of adding 2 to each side is the same as transferring -2 to the right-hand side, but, in so doing, the sign is changed from a minus to a plus. Hence, $x - 2 = 4 \Leftrightarrow x = 4 + 2 \Leftrightarrow x = 6$. Check: when $x = 6$, the left-hand side of the given equation becomes $6 - 2 = 4$. The right-hand side of the given equation is equal to 4. Therefore, the solution is correct.

ii. We can solve the equation $x + 18 = 30$ as follows: subtracting 18 from each side, we get $x + 18 - 18 = 30 - 18 \Rightarrow x = 12$. Alternatively, moving $+18$ to the right-hand side (changing the sign from a plus to a minus), we get $x = 30 - 18 \Leftrightarrow x = 12$. Check: when $x = 12$, the left-hand side of the given equation becomes $12 + 18 = 30$. The right-hand side of the given equation is 30. Therefore, the solution is correct.

Equations containing the unknown quantity on both sides: In equations of this kind, we group all the terms containing the unknown quantity on one side of the equation and the remaining terms on the other side.

i. We can solve the equation $4x + 3 = 6x + 11$ as follows: transferring $6x$ to the left-hand side and $+3$ to the right-hand side, we get $4x - 6x = 11 - 3 \Rightarrow -2x = 8 \Rightarrow x = -\frac{8}{2} = -4$. Check: when $x = -4$, the left-hand side becomes $4(-4) + 3 = -13$, and the right-hand side becomes $6(-4) + 11 = -13$. Therefore, the solution is correct.

ii. We can solve the equation $7x - 2 = 5x + 8$ as follows: $7x - 5x = 8 + 2 \Rightarrow 2x = 10 \Rightarrow x = 5$. Check: when $x = 5$, the left-hand side becomes $7 \times 5 - 2 = 33$, and the right-hand side becomes $5 \times 5 + 8 = 33$. Therefore, the solution is correct.

Equations containing brackets: When an equation contains brackets, we remove these first, and then we solve according to the aforementioned methods. For instance, $3(2x - 1) = 9 \Rightarrow 6x - 3 = 9 \Rightarrow 6x = 12 \Rightarrow x = 2$. Check: when $x = 2$, the left-hand side is $3(2 \times 2 - 1) = 9$, and the right-hand side is 9. Therefore, the solution is correct.

Equations containing fractions: When an equation contains fractions, we multiply each term of the equation by the least common multiple of the denominators. For instance, we can solve the equation $\frac{x}{3} + \frac{2}{5} = \frac{5x}{2} - 1$ as follows: The least common multiple of the denominators 3, 5, and 2 is 30. Multiplying each term by 30 gives $\frac{x}{3} \times 30 + \frac{2}{5} \times 30 = \frac{5x}{2} \times 30 - 1 \times 30 \Rightarrow 10x + 12 = 75x - 30 \Rightarrow -65x = -42 \Rightarrow x = \frac{42}{65}$. The solution may be verified by the check method shown in the previous examples.

Simultaneous equations: Consider the two following equations:

$$\begin{cases} ax + by = c \\ px + qy = r \end{cases}.$$

Each equation contains the unknown quantities x and y. The solutions of the equations are the values of x and y that satisfy both equations. Equations such as these are called "simultaneous equations" (or a "system of equations").

i. We can solve the simultaneous equations

$$4x + 5y = 14 \qquad (*)$$

$$x + 2y = 11 \qquad (**)$$

as follows: If we multiply equation $(**)$ by 4, we shall have the same coefficient of x in both equations:

$$4x + 8y = 44 \qquad (***)$$

We can now eliminate x by subtracting equation $(*)$ from equation $(***)$:

$$\begin{array}{r} 4x + 8y = 44 \\ 4x + 5y = 14 \\ \hline 3y = 30 \end{array}$$

Hence, $y = 10$. In order to find x, we substitute $y = 10$ in either of the original equations. Therefore, substituting for $y = 10$ in equation $(*)$, we get $4x + 5 \times 10 = 14 \Rightarrow x =$

-9. In order to check these values, it suffices to substitute them in equation $(**)$.

ii. We can solve the simultaneous equations

$$5x + 7y = 15 \quad (*)$$

$$4x + \frac{8}{5}y = 24 \quad (**)$$

as follows: the same coefficient of x can be obtained in both equations if equation $(*)$ is multiplied by 4 (the coefficient of x in equation $(**)$) and equation $(**)$ is multiplied by 5 (the coefficient of x in equation $(*)$). Multiplying equation $(*)$ by 4, we get

$$20x + 28y = 60 \quad (***)$$

Multiplying equation $(**)$ by 5, we get

$$20x + 8y = 120 \quad (****)$$

Subtracting equation $(***)$ from equation $(****)$, we get

$-20y = 60 \Rightarrow y = -3$.

Substituting for $y = -3$ in equation $(*)$, we get $x = \frac{36}{5}$. In order to check these values, it suffices to substitute them in equation $(**)$.

iii. We can solve the simultaneous equations

$$7x + 4y = 20 \quad (*)$$

$$3x - 2y = 3 \quad (**)$$

as follows: in this system of equations, it is easier to eliminate y, since the same coefficient of y can be obtained in both equations by multiplying equation $(**)$ by 2. In fact, multiplying equation $(**)$ by 2, we get

$$6x - 4y = 6 \quad (***)$$

Adding equations $(*)$ and $(***)$, we get $13x = 26 \Rightarrow x = 2$. Substituting for $x = 2$ in equation $(*)$, we get $y = \frac{3}{2}$. In order to check these values, it suffices to substitute them in equation $(**)$.

iv. We can solve the simultaneous equations

$$\frac{x}{5} - \frac{y}{3} = \frac{1}{10} \quad (*)$$

$$\frac{3x}{4} - \frac{2y}{3} = \frac{2}{3} \quad (**)$$

as follows: first, we shall clear each equation of fractions. In equation $(*)$, the least common multiple of the denominators is 30. Hence, by multiplying equation $(*)$ by 30, we get

$$6x - 10y = 3 \quad (***)$$

In equation ($**$), the least common multiple of the denominators is12. Hence, by multiplying equation $(**)$ by 12, we get

$9x - 8y = 8$ $(****)$

We now proceed in the usual way. Multiplying equation $(**$ $*)$ by 6, we get

$36x - 60y = 18$ (A)

Multiplying equation $(****)$ by 4, we get

$36x - 32y = 32$ (B)

Subtracting equation (B) from equation (A), we get $-28y = -14 \Rightarrow y = \frac{1}{2}$. Substituting for $y = \frac{1}{2}$ in equation $(***)$, we get $x = \frac{8}{6} = \frac{4}{3}$. Therefore, the solutions are $y = \frac{1}{2}$ and $x = \frac{4}{3}$. Since equation $(***)$ came from equation $(*)$, we must do the check in equation $(**)$. Indeed, $\frac{3(4/3)}{4} - \frac{2(1/2)}{3} = \frac{2}{3}$.

Factoring Models

Common factor: $ax + ay = a(x + y)$.
Difference of squares: $x^2 - y^2 = (x + y)(x - y)$.
Trinomial (leading coefficient 1): $x^2 + (a + b)x + ab = (x + a)(x + b)$.
Perfect square trinomial: $x^2 + 2xy + y^2 = (x + y)^2$.
General trinomial: $(ac)x^2 + (ad + bc)x + bd = (ax + b)(cx + d)$.
Sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.
Difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

Real Polynomials

A function of a single variable x is said to be a "polynomial" on its domain if it can be put in the following form:

$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$,

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are constants. Hence, every polynomial can be expressed as a finite sum of monomial terms of the form $a_k x^k$, in which the variable is raised to a non-negative integral power. Notice that $x^0 = 1$, and so $a_0 x^0 = a_0$. For the aforementioned polynomial with $a_n \neq 0$:

the numbers a_i (where $0 \leq i \leq n$) are called "coefficients";
a_n is the "leading coefficient";
$a_n x^n$ is the "leading term";
a_0 is the "constant term" or the "constant coefficient";
a_1 is the "linear coefficient";
$a_1 x$ is the "linear term";

when the leading coefficient, a_n, is equal to 1, the polynomial is said to be "monic";
the non-negative integer n is the "degree" of the polynomial, and we write $\deg(p) = n$.
A "constant polynomial" has only one term, specifically, a_0. A non-zero constant polynomial has degree 0, and, by convention, the "zero polynomial" (with all coefficients vanishing) has degree $-\infty$.
A "zero" of a polynomial $p(x)$ is any number r for which $p(r)$ takes the value 0. Hence, when $p(r) = 0$, we say that r is a "root," or a "solution" of the equation $p(x) = 0$.
Let
$p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ and
$q(x) = b_0 + b_1x + b_2x^2 + \cdots + b_mx^m$
be two arbitrary polynomials. Then we can operate with them as follows:
Sum: $(p+q)(x) = (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + \cdots$
Difference: $(p-q)(x) = (a_0 - b_0) + (a_1 - b_1)x + (a_2 - b_2)x^2 + \cdots$
Product of a constant and a polynomial: $(cp)(x) = ca_0 + ca_1x + ca_2x^2 + \cdots$
Product of two polynomials: $(p \cdot q)(x) = a_0b_0 + (a_0b_1 + a_1b_0)x + (a_0b_2 + a_1b_1 + a_2b_0)x^2 + \cdots + (a_0b_k + a_1b_{k-1} + \cdots + a_ib_{k-i} + \cdots + a_kb_0)x^k + \cdots + (a_nb_m)x^{m+n}$.
Composition of two polynomials: $(p \circ q)(x) = p\big(q(x)\big)$, so that we replace each occurrence of x in the expression for $p(x)$ with $q(x)$.
Notice that we divide one polynomial by another in a manner similar to the division of two integers. Firstly, we arrange the terms of the dividend and the divisor in descending powers of x. If a term is missing, then we write 0 as its coefficient. Then, we divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient. Next, we multiply the entire divisor by the first term of the quotient, and we subtract this product from the dividend. We use the remainder as the new dividend, and we repeat the same process until the remainder is of lower degree than the divisor. As with the division of numbers,
$dividend = (divisor)(quotient) + remainder$.
Remainder Theorem: If a polynomial $p(x)$ is divided by $x - b$, then the remainder is $p(b)$.
Proof: Let $q(x)$ and r be, respectively, the quotient and the remainder when $p(x)$ is divided by $x - b$. Then, given that
$dividend = (divisor)(quotient) + remainder$,
it holds that, for any x,
$p(x) = (x - b)q(x) + r$.
If $x = b$, then $p(b) = r$.■

Factor Theorem: Given an arbitrary polynomial function $y = p(x)$, b is a zero of $y = p(x)$ if and only if $x - b$ is a factor of $p(x)$.
Proof. It can be easily verified using the Remainder Theorem. ■
Remark: The real number zeros of $y = p(x)$ are also the x-intercepts in the graph of $y = p(x)$. If b is a real number zero with multiplicity n of $y = p(x)$, then the graph of $y = p(x)$ crosses the x-axis at $x = b$ if n is odd, whereas the graph turns around and stays on the same side of the x-axis at $x = b$ if n is even. Hence, the x-intercepts can be obtained from the Factor Theorem, and the behavior of the graph at an x-intercept, say $(b, 0)$, is determined by the multiplicity of zero$b$ or, equivalently, by the highest power of $(x - b)$ that is a factor of $p(x)$. For instance, if $p(x) = (x + 1)(x - 2)^2$, then, by setting $x = 0$, we realize that the y-intercept is $(0,4)$. Because $(x + 1)$ is a factor with an odd exponent, it holds that $(-1,0)$ is an x-intercept at which the graph crosses the x-axis. Because $(x - 2)^2$ is a factor with an even exponent, it holds that $(2,0)$ is an x-intercept at which the graph touches the x-axis and then turns around.
In fact, the fundamental problem in algebra consists in finding ways of solving polynomial equations; specifically, we seek formulas for zeros/roots in terms of the coefficients of the corresponding polynomial. A well-known example is the "quadratic formula." If we have the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, then we have the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where the expression $b^2 - 4ac$ is known as the "discriminant," meaning that, if we have a number r such that $r^2 = b^2 - 4ac \Leftrightarrow r = \sqrt{b^2 - 4ac}$, then
$x_1 = \frac{-b+r}{2a}$ and $x_2 = \frac{-b-r}{2a}$
are the solutions of $ax^2 + bx + c = 0$.
If a function $y = f(x)$ satisfies an equation of the form
$p_0(x)y^n + p_1(x)y^{n-1} + \cdots + p_{n-1}(x)y + p_n(x) = 0,$
where $p_0(x), \ldots, p_n(x)$ are polynomials in x, then it is said to be an "algebraic function." In other words, an algebraic function is a function that can be defined as the root of a polynomial equation. If a function can be expressed as the quotient of two polynomials,
$f(x) = \frac{p(x)}{q(x)}$,
then it is called a "rational algebraic function."

Matrices

Matrices are often used in physics, statistics, and economics, and they are particularly useful when they are used in connection with systems of linear equations. For instance, let us considers the following linear simultaneous equations:

$$4x + 5y = 14$$
$$x + 2y = 11$$

By arranging the coefficients of x and y in the way in which they occur in the equations, we obtain the array

$$\begin{pmatrix} 4 & 5 \\ 1 & 2 \end{pmatrix},$$

which is an example of a matrix.

In general, consider the following rectangular array

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix},$$

consisting of m rows (i.e., horizontal n-tuples) and n columns (i.e., vertical n-tuples). This is called an "$m \times n$ matrix," usually denoted by $A = (a_{ij})$. If the number of rows in the matrix is m and the number of columns is n, then the matrix is said to be of order $m \times n$. The term "matrix" was introduced by the nineteenth-century English mathematician James Sylvester, but it was his friend the mathematician Arthur Cayley who developed the algebra of matrices in the 1850s.

Types of matrices:

i. *Row matrix.* This is a matrix having only one row; for instance, the following is a row matrix:
$(4 \quad 5)$.

ii. *Column matrix.* This is a matrix having only one column; for instance, the following is a column matrix:
$\begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

iii. *Null matrix.* this is a matrix with all its elements zero.

iv. *Square matrix.* This is a matrix having the same number of rows and columns.

v. *Diagonal matrix*. This is a square matrix in which all the elements are zero except the main diagonal elements (the main diagonal in a matrix always runs from upper left to lower right, so that the main diagonal of a matrix $A = (a_{ij})$ is the list of entries a_{ij} where $i = j$); for instance, the following is a diagonal matrix:
$\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$.

vi. *Identity matrix.* This is a diagonal matrix in which the main diagonal elements are equal to 1 (an identity matrix is usually denoted by I); for instance, the following is an identity matrix:
$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$.

Addition and Subtraction of Matrixes: Two matrices may be added or subtracted provided that they are of the same order. Addition of matrices is done by adding together the corresponding elements of each of the two matrices. For instance:

$$\begin{pmatrix}4 & 5\\ 1 & 2\end{pmatrix}+\begin{pmatrix}3 & 6\\ 2 & 4\end{pmatrix}=\begin{pmatrix}4+3 & 5+6\\ 1+2 & 2+4\end{pmatrix}=\begin{pmatrix}7 & 11\\ 3 & 6\end{pmatrix}.$$

Subtraction of matrices is done in a similar way except the corresponding elements are subtracted. For instance:

$$\begin{pmatrix}4 & 5\\ 1 & 2\end{pmatrix}-\begin{pmatrix}3 & 6\\ 2 & 4\end{pmatrix}=\begin{pmatrix}4-3 & 5-6\\ 1-2 & 2-4\end{pmatrix}=\begin{pmatrix}1 & -1\\ -1 & -2\end{pmatrix}.$$

Multiplication of Matrices:

i. Scalar multiplication: A matrix may be multiplied by a number as follows:
$$4\begin{pmatrix}5 & -2\\ 1 & 8\end{pmatrix}=\begin{pmatrix}4\times 5 & 4\times(-2)\\ 4\times 1 & 4\times 8\end{pmatrix}=\begin{pmatrix}20 & -8\\ 4 & 32\end{pmatrix}.$$

ii. General Matrix Multiplication: Two matrices can only be multiplied by each other if the number of columns in the one is equal to the number of rows in the other. Multiplication of matrices is done by multiplying a row by a column as follows:
$$\begin{pmatrix}4 & 5\\ 1 & 2\end{pmatrix}\times\begin{pmatrix}3 & 6\\ 2 & 4\end{pmatrix}=\begin{pmatrix}4\times 3+5\times 2 & 4\times 6+5\times 4\\ 1\times 3+2\times 2 & 1\times 6+2\times 4\end{pmatrix}=$$
$$\begin{pmatrix}22 & 44\\ 7 & 14\end{pmatrix}.$$

The product of an $m\times n$ matrix $A=(a_{ij})$ and an $n\times p$ matrix $B=(b_{ij})$ is a matrix $C=AB=(c_{ij})$ whose (i,j) entry is

$$c_{ij}=\sum_{k=1}^{n}a_{ik}\,b_{kj}, \text{ where } 1\le i\le m \text{ and } 1\le j\le p.$$

Inverting a Matrix: An n-square matrix A is said to be "invertible" or "non-singular" if there exists an n-square matrix B with the following property:

$$AB=BA=I_n,$$

where I_n is the n-square identity matrix, namely, the $n\times n$ matrix with ones along the main diagonal and zeros elsewhere. If this is the case, then the matrix B is called the inverse of A, and the notation A^{-1} is used to designate B. If no such B exists, then A is said to be "singular." If

$$A=\begin{pmatrix}a & b\\ c & d\end{pmatrix},$$

then

$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Transposition of Matrices: The "transpose" of a matrix A is denoted by A^T, and it is the matrix obtained by writing the rows of A, in order, as columns; if $A = (a_{ij})$ is an $m \times n$ matrix, then $A^T = (a_{ij}^T)$ is the $n \times m$ matrix where $a_{ij}^T = a_{ji}$, for all i and j. For instance, if

$A = \begin{pmatrix} 1 & 7 \\ 4 & 3 \end{pmatrix}$, then $A^T = \begin{pmatrix} 1 & 4 \\ 7 & 3 \end{pmatrix}$.

Determinants: The determinant of a matrix A is a scalar assigned to A, and it is denoted by $det(A)$. Given a matrix

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$,

its determinant is

$det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$.

Solution of simultaneous equations using matrices: Let us consider a system of two linear equations with two unknowns:

$\begin{cases} a_{11}x_1 + a_{12}x_2 = c_1 \\ a_{21}x_1 + a_{22}x_2 = c_2 \end{cases}$,

which gives rise to the following three matrices:

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $B = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$, and $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

Thus, the original system of linear equations can be reformulated as follows:

$AX = B \Leftrightarrow X = A^{-1}B$,

where A is the matrix of the system's coefficients, X is the matrix of the system's unknowns, and B is the matrix of the system's constant terms. The system has a unique solution if and only if the determinant $\det(A) = a_{11}a_{22} - a_{12}a_{21} \neq 0$, and that solution is:

$x_1 = \frac{B_{x_1}}{\det(A)} = \frac{\begin{vmatrix} c_1 & a_{12} \\ c_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{c_1a_{22} - a_{12}c_2}{a_{11}a_{22} - a_{12}a_{21}}$,

and

$x_2 = \frac{B_{x_2}}{\det(A)} = \frac{\begin{vmatrix} a_{11} & c_1 \\ a_{21} & c_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{a_{11}c_2 - c_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$,

where the numerators B_{x_1} and B_{x_2} are obtained by substituting the column of constant terms in place of the column of coefficients of the corresponding unknown in the matrix of coefficients. If $\det(A) = 0$, then the system has either no solution or an infinite number of solutions.

Consider the 3-square matrix

$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$.

The determinant of A is

$$\det(A) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - a_1c_2b_3 - b_1a_2c_3 - c_1b_2a_3.$$

Moreover, it can be easily shown that

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}.$$

Let us consider a system of 3 linear equations with 3 unknowns:

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}.$$

The aforementioned system has a unique solution if and only if the determinant of the matrix of coefficients is not zero:

$$\det(A) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0.$$

In this case, the unique solution of the given system can be expressed as quotients of determinants as follows:

$x = \frac{B_x}{\det(A)}$,

$y = \frac{B_y}{\det(A)}$,

$z = \frac{B_z}{\det(A)}$,

where the numerators B_x, B_y, and B_z are obtained by substituting the column of constant terms for the column of coefficients of the corresponding unknown in the matrix of coefficients, so that:

$$B_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, B_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \text{ and } B_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}.$$

If $\det(A) = 0$, then the system has either no solution or an infinite number of solutions.

Advances in computing power have contributed significantly to the application of matrix algebra in several scientific disciplines, such as physics and mathematical economics.

The application of matrices in input-output analysis

The major economic tasks that every society must accomplish pertain to decision-making about an economy's inputs and outputs. In economics, the term "input" refers to commodities or services used by firms in their production processes. Thus, by means of its technology, an economy combines inputs to produce outputs. In economics, the term "output" refers to the various useful goods or services that are either employed in further production or consumed.

The acknowledged founder of "input-output analysis" is the Russian-American economist Wassily Leontief, who won the Nobel Prize in Economics in 1973. An input-output matrix is a square matrix, say $A = (a_{ij})$, whose entries a_{ij} represent the amount of input i required per unit of output j. A column of such a matrix depicts the inputs needed for the achievement of a specific output. Therefore, from the perspective of economics, it can be considered as a "production technique." Hence, an input-output matrix is a "constellation" of production techniques. If the list of inputs is complete, including factor inputs, then the input-output matrix contains techniques for the production of the factor services as well. Input-output is an integral part of general equilibrium analysis. As the American economist Campbell R. McConnell has pointed out, the economy is "an interlocking network of prices wherein changes in one market are likely to elicit numerous and significant changes in other markets," so economists need to study "the price system as a whole" and focus on "general equilibrium analysis" (Campbell R. McConnell, *Economics*, fifth edition, New York: McGraw-Hill, 1972, p. 579).

For instance, let us consider a small economic network that consists of two interdependent industries A and B (e.g., A may represent the final goods industry, and B may represent the energy industry). This method can obviously be generalized to any number of industries. We assume that, for each dollar's worth of goods/services produced by A, A needs to consume a quantity of A's output and a quantity of B's output, and, for each dollar's worth of goods/services produced by B, B needs to consume a quantity of B's output and a quantity of A's output. In particular, the production of each dollar's worth of A requires $\$q_{11}$ worth of A and $\$q_{21}$ worth of B; and the production of each dollar's worth of B requires $\$q_{12}$ worth of A and $\$q_{22}$ worth of B. Therefore, both industries sell to each other and buy from each other. In addition, assume that there is an external demand for A and B; specifically, let the final demand from the outside sector of the economy be $\$d_1$ million for A and $\$d_2$ million for B. Let x_1 and x_2 represent the total output from A and B respectively.

Then we formulate the following equation:

$X = QX + D \Rightarrow X - QX = D \Rightarrow IX - QX = D \Rightarrow (I - Q)X = D,$
where: $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $Q = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$, and $D = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$,
and X is the "output matrix" (i.e., X is a column matrix representing the equilibrium output levels in industry A and industry B), Q is the "technology matrix," D is the "final demand matrix," and I is the identity matrix.
If $I - Q$ is invertible, then the solution for X is given by
$X = (I - Q)^{-1}D,$
which is the optimum level of production for the given economic network, meaning that the given economic network must produce x_1 million dollars of A (e.g., final goods) and x_2 million dollars of B (e.g., energy) in order to meet both the internal demand and the external demand for A and B (and thus avoid both oversupplying and undersupplying the market with the corresponding commodities).

The application of matrices in linear programming

By the term "linear programming," we mean a method to achieve the best outcome (e.g., to maximize profit, minimize cost, etc.) in a mathematical model whose requirements are represented by linear functions. The first contributions to linear programming are due to the Soviet mathematician and economist Leonid Vitaliyevich Kantorovich (1912–86), who won the Nobel Prize in Economics in 1975. Moreover, one of the acknowledged founders of linear programming is the American mathematician George Bernard Dantzig (1914–2005), who managed to make significant contributions to industrial engineering, operations research, economics, statistics, and computer science. In fact, input-output analysis is a special and very important case of linear programming.
The "canonical form" of linear programming is the following: given a system of m linear constraints (or linear inequalities) with n variables, we wish to find non-negative values (i.e., ≥ 0) of these variables that will satisfy the constraints and will maximize a function of these variables; symbolically: given m linear inequalities and/or equalities
$\sum_j a_{ij}\, x_j \leq b_i, i = 1,2, \dots, m, and\ j = 1,2, \dots, n,$ $(*)$
we wish to find those values of x_j which satisfy the constraints $(*)$ and the condition that $x_j \geq 0$ (for $j = 1,2, \dots, n$) and simultaneously maximize the linear function
$z = \sum c_j x_j\, , j = 1,2, \dots, n.$ $(**)$
For instance, consider a problem where we wish to maximize the gross profit of an industry (or of a firm offering several product lines) that produces n commodities, and thus has n sectors of production. In this

case, $(*)$ and $(**)$ can be interpreted as follows: z denotes an overall performance measure (specifically, total gross profit); x_j denotes the level of activity j $(j = 1,2, \dots, n)$, specifically, the output of the jth sector of production (i.e., the produced quantity of the jth commodity); c_j denotes the performance measure coefficient for activity j, specifically, the gross profit per unit of output in the jth sector of production (so that the total gross profit in the jth sector of production is $c_j x_j$); b_i denotes the available quantity of resource i $(i = 1,2, \dots, m)$; and a_{ij} denotes the quantity of resource i consumed by each unit of activity j.

In matrix form, the constrained maximization problem $(**)$ can be rewritten as follows:

$$z_{max} = (c_1 \quad c_2 \quad \dots \quad c_n) \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix},$$

under the constraints

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \leq \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix},$$

and

$x_j \geq 0$ for $j = 1,2, \dots, n$. More simply, given the above concepts, we can write:

$$\left.\begin{array}{c} max z = cx \\ under\ the\ constraints \\ Ax \leq b \\ x_j \geq 0 \end{array}\right\}. \qquad (***)$$

Regarding the geometric significance of $(***)$, notice that the constraints $Ax \leq b$ and $x_j \geq 0$ define a convex polyhedron P_n in $\mathbb{R}^n$, and such P_n is called the "feasible region" of the corresponding model, meaning the region of all the feasible solutions of the corresponding problem. In general, a polyhedron P_n in $\mathbb{R}^n$ is the set of all points $x \in \mathbb{R}^n$ that satisfy a finite set of linear inequalities. Moreover, a set Q in $\mathbb{R}^n$ is called "convex" if, for any two points x and y in Q, the line segment joining them is also in Q; symbolically: $\forall x, y \in Q$, the "convex combination" $kx + (1 - k)y \in Q$ for any k such that $0 \leq k \leq 1$. The goal of constrained maximization in the context of linear programming is to choose that feasible combination $(x_1, x_2, \dots, x_n)$ of actions that maximize a given function $z = cx$. This occurs at the maximum (most extreme) point $(x_1^*, x_2^*, \dots, x_n^*)$ of the feasible region.

The constrained maximization problem (∗∗∗) is known as the "primal problem," while the so-called "dual problem" is the corresponding constrained minimization problem where, given a system of m linear constraints (linear inequalities) with n variables, we wish to find non-negative values (i.e., ≥ 0) of these variables that will satisfy the constraints and will minimize a function (e.g., a cost function) of these variables; symbolically:

$$\left.\begin{array}{c} minz = cx \\ under\ the\ constraints \\ Ax \geq b \\ x_j \geq 0 \end{array}\right\}. \qquad (****)$$

For instance, using the "dual problem," we can create models of constrained cost minimization in economics and business management. Firms seek to minimize cost subject to the constraint that they produce at least b units of output, so that the firm's cost minimization problem is given by (∗∗∗∗). In general, linear programming is useful for guiding quantitative decisions in business planning, industrial engineering, and the social and physical sciences.

The Basic Equations of Money

In economics, by the term "interest," we refer to the profit return on investment. The money that is invested is called the "principal." The percentage return per annum is called the "rate per cent." Thus, if P stands for the principal, T stands for the time in years, R stands for the rate per cent per annum, and I stands for the interest, then

$$I = \frac{PRT}{100}$$

where P and I must be in the same monetary units. This formula can be transposed to give P, R, and T in terms of the other letters:

$T = \frac{100I}{PR}$,

$R = \frac{100I}{PT}$, and

$P = \frac{100I}{RT}$.

Compound interest is different from simple interest in that the interest which is added also attracts interest. If a sum of P monetary units is invested at $r\%$ per annum for n years, then the value or amount after n years is

$P\left(1 + \frac{r}{100}\right)^n$.

For instance, the value of \$2,500 invested at 5% compound interest after eight years (i.e., $P = \$2{,}500$, $r = 5$, and $n = 8$) will be

$$P\left(1+\frac{r}{100}\right)^n = \$2{,}500\left(1+\frac{5}{100}\right)^8 = \$3{,}693.$$

The mathematical formula of compound interest is the following: assume that you borrow an amount P of money (the "principal") at an (annual) interest rate of $r > 0$, and that, at the end of each year, you have to pay back a fixed amount (a "deposit") d. Let A_n be the total amount of money owed after n years. The formula for computing A_n in terms of P (the principal of the loan), r (the interest rate of the loan), and d (the loan deposits) is the following (where $0 < r \leq 1$):

$$\begin{aligned} A_n &= A_{n-1}(1+r) - d \\ &= P(1+r)^n - d(1+r)^{n-1} - d(1+r)^{n-2} - \cdots \\ &\quad - d \end{aligned}$$

$$= P(1+r)^n - d\frac{(1+r)^n - 1}{(1+r)-1} \Leftrightarrow A_n = P(1+r)^n - \frac{d}{r}[(1+r)^n - 1],\ r \neq 0;$$

so that the initial condition is $A_0 = P$; at the end of the first year, you owe P (the principal) plus an interest equal to rP minus the deposit you have agreed to pay each year. Therefore, $A_1 = P + rP - d = P(1+r) - d$; by analogy, at the end of the second year, you owe $A_2 = A_1(1+r) - d = P(1+r)^2 - d(1+r) - d$, etc. By allowing the owners of large sums of money to lend (that is, trade) money on interest, we give them power to immunize themselves against loss (in fact, this is the ultimate purpose of charging interest on loans: to immunize the lender of money against loss), while socializing loss and risks. Thus, we create an exceptionally privileged financial oligarchy.

The net present value (NPV) of an investment project consists in calculating the amount by which the value of that investment project exceeds its cost. If i is the interest rate (which, for convenience, is assumed to be fixed for the project under consideration), then the NPV is defined as follows:

$$NPV = \frac{X_1}{1+i} + \frac{X_2}{(1+i)^2} + \cdots + \frac{X_n}{(1+i)^n} - C_0$$

where X_t $(t = 1,2,\ldots,n)$ denotes the cash flow that corresponds to year t, C_0 is the capital cost of the investment project in year 0, and n is the lifetime (in years) of the investment project. Hence, according to the Nobel Prize-winning Italian-American economist Franco Modigliani and the American economist Merton Miller, under certain conditions (in particular, if we assume that there is total information transparency and total rationality), the intrinsic or real value of a company can be considered to be the net present value of all the investment projects of that company.

Probability and Risk Analysis

First of all, it should be clarified that, by the term "quantitative analysis," we mean the study of phenomena by means and on the basis of any type of quantitative information. Such an inquiry takes place by applying suitable methods that determine the nature of the available information and the phenomena under consideration. Quantitative methods mainly include methods that derive from mathematical analysis, mathematical programming, probability theory, and statistics.

In fact, statistics emerged from the constant efforts of humankind to deal with situations of uncertainty in which they lived. In these situations, the element of luck always appeared as a key determining factor which prevented the identification of the existence of systematicness in the manifestations of various phenomena and in the formulation of relations between them. Aristotle was the first philosopher to offer a systematic account of "luck" and to include it as a significant topic in both physics and ethics (Aristotle, *Physics*, 2:4–6, and *Metaphysics*, 7:7–9). A method is called statistical if it relates facts and hypotheses of some kind. Hence, statistics investigates and develops methods for evaluating hypotheses in reference to empirical facts.

In general, luck is involved in all things where actors do not hold full control over the outcome of action. One of the basic attributes of the statistical method is the fact that it refers to properties of populations instead of individual cases. Statistics examines a unit only in its capacity as a member of a population. The statistical method can be applied to any problem related to the definition of overall behavior, based on individual observations expressed numerically. The concept of luck is commonly used in statistics in order to display all the possible outcomes given a very large sample and the probability of each outcome. In science, "probabilities," often called chances or stochastic processes, are relative frequencies in series of events, or tendencies or propensities in the systems that give rise to those events. The "frequency" is the number of times each measurement occurs.

Probability theory is primarily concerned with the issue of uncertainty. In fact, "probability," usually denoted by p, is a quantitative measure of uncertainty. It is a number between 0 and 1, where 0 indicates impossibility and 1 indicates certainty. Assume that we take any very large number, N, out of a series of cases in which an event, A, is in question, and that A happens on pN occasions (where $0 \leq p \leq 1$). The probability of the event A is said to be p (the certainty of the corresponding proposition increases as the number N of specimen cases selected increases).

Furthermore, the following corollaries and extensions may be added to the aforementioned definition of a probability: (i) If the probability of an event is p, then, out of N cases in which it is in question, it will happen pN times, where N is any very large number (where $0 \leq p \leq 1$). (ii) If the probability of an event is p, then the probability of its failing is $1 - p$.

Probability theory is based on set theory. By the term "experiment," we mean a process that leads to one of several possible outcomes. By the term "outcome," we mean an observation or measurement. The "sample space" is the set of all possible outcomes of an experiment. An "event" is a subset of a sample space—or, in other words, a set of basic outcomes. Thus, we say that the event "occurs" if the corresponding experiment gives rise to a basic outcome belonging to the event. Therefore, we obtain the following formula:

$$Probability\ of\ event\ A = \frac{n(A)}{n(S)},$$

where $n(A)$ is the number of elements in the set of the event A, and $n(S)$ is the number of elements in the sample space S. For instance, roulette as it is played in Las Vegas or Atlantic City consists of a wheel that has 36 numbers, numbered 1 through 36, and the numbers 0 and 00 (double zero). Therefore, in this case, the sample space, S, consists of 38 numbers, and the probability of winning a single number that you bet is $P = 1/38$.

When the sets corresponding to two events are disjoint (their intersection is the empty set), then these events are called "mutually exclusive."

The axiomatic definition of probability is the following: Let E be a space of elementary events (i.e., the space of outcomes of experiments, or the space of states of a system, since the state of a system can be construed as the outcome of an experiment). The "probability of an event" $A \subseteq E$ is denoted by $p(A)$, and it is defined as a single number that corresponds to A and has the following properties:

(P1) $p(A) \geq 0$;

(P2) for each pair of mutually exclusive events, $A, B \subseteq E$, it holds that
$p(A \cup B) = p(A) + p(B)$;

(P3) $p(E) = 1$ (i.e., the total probability, after adding all possibilities, is equal to one).

Remark: For each $A, B \subseteq E$, $p(A \cup B) = p(A) + p(B) - p(A \cap B)$; but, in case A and B are mutually exclusive, it holds that $p(A \cap B) = 0$, so we obtain (P2).

By the term "conditional probability," we mean the probability of event A conditional upon the occurrence of event B. Assume that we investigate the probability of an event A given that we know that an event B has

occurred, and that event B influences the probability of event A. The "conditional probability" of event A given the occurrence of event B is defined as the quotient of the probability of the intersection of A and B over the probability of event B; symbolically:
$P(A|B) = \frac{P(A \cap B)}{P(B)}$,
assuming that $P(B) \neq 0$. The aforementioned formula for the computation of conditional probability is known as Bayes's Law, since it was originally formulated by the eighteenth-century English statistician and philosopher Thomas Bayes. Notice that A is independent of B if $P(A|B) = P(A)$; that is, knowing that B occurred does not change the probability that A occurred. Thus, according to Bayes's Law, two events A and B are independent of each other if and only if
$P(A \cap B) = P(A)P(B)$.
Bayes's Law provides a method of revising existing predictions or theories (specifically, updating probabilities) given new additional evidence. In fact, Bayes's Law implies that the interpretation of any risk assessment depends on an estimate of the base rate, and the corresponding base rate, which is never known with complete certainty at the time of the assessment, is a Bayesian "prior probability."
One of the most important methods that is used to discover, describe, and explain "typical" behavior of mass data is the "arithmetic mean." The formula is

$$\bar{X} = \frac{\sum_{i=1}^{N} X_i}{N}$$

where $\bar{X}$ denotes the arithmetic mean, $\sum_{i=1}^{N} X_i$ denotes the summation of the values of the individual observations X_i under consideration ($i = 1,2, \dots, N$), and N is the total number of items in the series that have been summated. It is worth noticing that arithmetic means are often "weighted" averages, in the sense that, when averaging values, it is sometimes logically necessary to assign more importance to some than to others (by multiplying each value with a suitable statistical weight), so that particular values may be more influential in determining the "typical" value than others. Formally, the weighted arithmetic mean of a non-empty finite set of data $\{X_1, X_2, \dots, X_N\}$ with corresponding non-negative weights $\{w_1, w_2, \dots, w_N\}$ is

$$\bar{X} = \frac{\sum_{i=1}^{N} w_i X_i}{\sum_{i=1}^{N} w_i} = \frac{w_1 X_1 + w_2 X_2 + \cdots + w_N X_N}{w_1 + w_2 + \cdots + w_N}$$

(the weights can be in the form of decimals, whole numbers, percentages, etc.). For instance, if $x_1, x_2, x_3, \dots$ are the measured observations and $f_1, f_2, f_3, \dots$ are the corresponding frequencies, then the arithmetic mean is

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + f_3 x_3 + \cdots}{f_1 + f_2 + f_3 + \cdots}$$

(this is the arithmetic mean of a frequency distribution).

One of the most important methods that are used to discover, describe, and explain "risk" or "uncertainty" is the "standard deviation," which is a quantity expressing by how much the members of a database (i.e., the data under consideration) differ from the arithmetic mean of the given database. The formula is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} x_i^2}{N}}$$

where: firstly, we calculate the arithmetic mean $\bar{X}$ of the values X_i $(i = 1,2, \dots, N)$ under consideration; secondly, we record the deviation of each value X_i from the arithmetic mean, namely, $x_i = X_i - \bar{X}$; thirdly, we square these deviations (we compute x_i^2); fourthly, we summate the squared deviations and divide by N (thus finding the "variance" of our data); fifthly, we extract the square root to obtain σ. However, the aforementioned formula for the standard deviation is used when N is the entire population of the species or kind under consideration; if we do not have the entire population, we use the following formula for the standard deviation:

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1}}$$

where n is the size of the sample (i.e., the number of the point data that are contained in the database that we use), X_i is the ith point of the sample $(i = 1,2, \dots, n)$, and $\bar{X}$ is the arithmetic mean of the sample (namely, of the database that we use).

When we have two sets of data and we want to find how strong a relationship is between them, we use Pearson's correlation coefficient (PCC), also known as Pearson's r. In other words, PCC calculates the level of change in one variable due to the change in the other. When applied to a sample of the variables x and y, PCC is commonly represented by r_{xy}.

Given paired data

$\{(x_1, y_1), \dots, (x_n, y_n)\}$, consisting of n pairs, r_{xy} is defined as follows:

$$r_{xy} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2}}$$

where:

n is the sample size,

x_i are the values of the x-variable in the sample,
$\bar{x}$ is the mean of the values of the x-variable,
y_i are the values of the y-variable in the sample, and
$\bar{y}$ is the mean of the values of the y-variable.
PCC returns values between -1 and 1, symbolically,
$-1 \leq r_{xy} \leq 1$,
where:
1 indicates a strong (actually, perfect) positive relationship,
-1 indicates a strong (actually, perfect) negative relationship, and
a result of zero indicates no relationship at all. In general, a positive correlation between two variables means that both the variables move in the same direction, whereas a negative correlation between two variables means that both the variables move in opposite directions.

Classical Euclidean Geometry, Analytic Geometry, and Trigonometry

Geometry is the scientific study of the quantitative and the qualitative properties of spatial forms and relations (the criteria for equality of triangles provide instances of qualitative geometric knowledge, and the computation of lengths, areas, and volumes exemplifies quantitative geometric knowledge).
Around 300 B.C.E., Euclid published the definitive treatment of Greek geometry and number theory in his thirteen-volume *Elements*, building on the experience and the achievements of previous Greek mathematicians: on the Pythagoreans for Books I–IV, VII, and IX, on Archytas for Book VIII, on Eudoxus for Books V, VI, and XII, and on Theaetetus for Books X and XIII. The axiomatic method used by Euclid is the prototype for the entire field of "pure mathematics," which is "pure" in the sense that we need only pure thought, no physical experiments, in order to verify that the statements are correct—that is, we need only to check the reasoning in the demonstrations. All mathematical theorems are conditional statements—namely, statements of the form
If (hypothesis) *then* (conclusion).
Put simply, one condition (hypothesis) implies another (conclusion). In particular, in a given mathematical system, the only statements that are called "theorems" are those statements for which a proof has been supplied. By a "proof," we mean a list of statements that is endowed with a justification for each statement, and it ends up with the conclusion desired. The following are the six types of justifications allowed for statements in proofs: (i) "by hypothesis . . ."; (ii) "by axiom . . ."; (iii) "by

theorem . . .”; (iv) “by definition . . .”; (v) “by step . . .”; (vi) “by rule . . . of logic”; and a justification may involve several of the aforementioned types.

In particular, Euclid articulated:

i. *A set of definitions, such as the following:*
   - A point is that which has no part or magnitude (i.e., it does not have a concrete size).
   - A line is length without breadth.
   - The ends of a line are points. A straight line is a line that lies evenly with the points on itself.
   - A surface is that which has length and breadth only.
   - The edges of a surface are lines.
   - A plane surface is a surface that lies evenly with the straight lines on itself.

ii. *A set of fundamental rules (axioms):*
   - Things that are equal to the same thing are equal to each other.
   - If equals are added to equals, then the wholes are equal.
   - If equals are subtracted from equals, then the remainders are equal.
   - Things that coincide with each other are equal to each other.
   - The whole is greater than the part.
   - Things that are double of the same things are equal to each other.
   - Things that are halves of the same things are equal to each other.

iii. *A set of fundamental propositions (postulates):*
   - Postulate 1: a straight line may be drawn from one point to any other point. Given two distinct points, there is a unique straight line that passes through them.
   - Postulate 2: a terminated straight line can be produced indefinitely.
   - Postulate 3: a circle can be drawn with any center and any radius.
   - Postulate 4: all right angles are equal to each other.
   - Postulate 5 (known as the Parallel Postulate): if a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles.

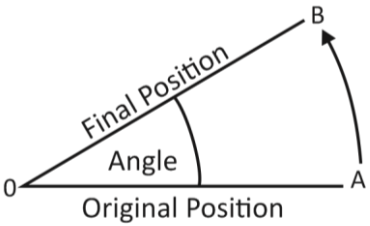
According to Euclidean geometry, space is three-dimensional and isotropic (i.e., it has the same value when measured in different directions). This scientific conception of space clashes with several mythical and folk perceptions of space, according to which space is connected with a form of temporality, and it is unisotropic (for instance, the "upward" and the "forward" directions are evaluated as superior to the "downward" and the "backward" directions). The Euclidean perception of space, combined with the concept of gravity, found its fullest expression in Isaac Newton's calculus and mechanics.

In view of Euclid's geometric treatises and the subsequent development of geometry as a scientific discipline, geometry is "an axiomatic in which we ignore all representation, and in which the word 'space' designates a structure, i.e., a system of axioms and deductions" (Saddo Ag Almouloud, "Demonstration in Geometry: Historical and Philosophical Perspectives," *Quantitative Research Journal*, vol. 8, Special Edition: Philosophy of Mathematics, 2020, p. 562). In other words, in mathematics, by the term "space," we mean a non-empty set endowed with some mathematical structure. In general, in mathematics, the term "structure" refers to a class of mathematical objects described by axioms. Moreover, sometimes mathematicians use the term "structure" in order to refer to the description of the way in which an object could be reconstructed from simpler objects of the same kind.

Euclidean Geometry

The two most basic geometric concepts are those of an angle and of a straight line. An angle may be considered to be an amount of a rotation or turning. In Figure 5-4, the line OA has been rotated about O in an anti-clockwise direction, until it takes up the position OB. The angle through which the line has turned is the amount of opening between the lines OA and OB. If the line OA is rotated until it returns to its original position, then it will have described one revolution. Angles are usually measured in degrees, minutes, and seconds as follows: $60\ seconds = 1\ minute$, $60\ minutes = 1\ degree$, and $360\ degrees = 1\ revolution$. For instance, an angle of 32 degrees 18 minutes and 3 seconds is written as follows: $32^o18'3''$. A "right angle" is the $\frac{1}{4}$th of a revolution, and, therefore, it contains 90^o. An "acute angle" is less than 90^o. An "obtuse angle" lies between 90^o and 180^o. A "reflex angle" is greater than 180^o. "Complementary angles" are angles whose sum is 90^o. "Supplementary angles" are angles whose sum is 180^o.

Figure 5-4: An angle.

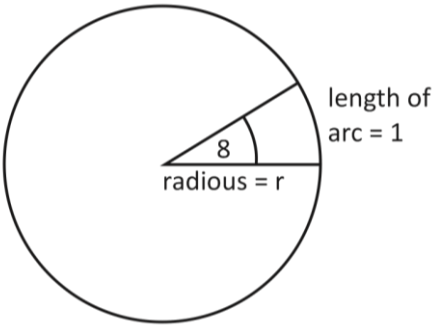


While we usually measure angles in degrees, we can also measure angles in radians. Referring to Figure 5-5,

$$angle\ in\ radians = \frac{length\ of\ arc}{radius\ of\ circle}$$

so that $\theta\ radians = \frac{l}{r} \Leftrightarrow l = r\theta$.

Figure 5-5: Measuring angles in radians; e.g., $l = 1\ unit$ and $\theta = 8\ units$.

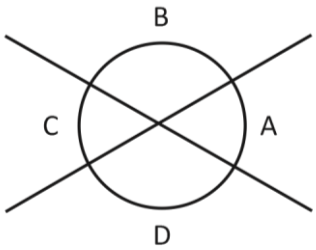


In geometry, the abstraction of a straight line can be attributed to mathematical intuition. According to the ancient Greek mathematician Euclid, an arbitrary straight line can be construed as a "length without breadth" that is perceived as a whole. Furthermore, there are points on every straight line, each point on the straight line corresponds to a real number, and the straight line is complete. For this reason, it is known as the arithmetic or geometric continuum. In fact, the ancient Greek mathematicians' awareness of the existence of real numbers was developed with reference to geometric processes, in the sense that they construed a real number either as a completed process of combining units

or monads (that is, as a rational number) or as an incomplete process of measuring non-commensurable quantities (that is, as an irrational number).
Properties of angles and straight lines:

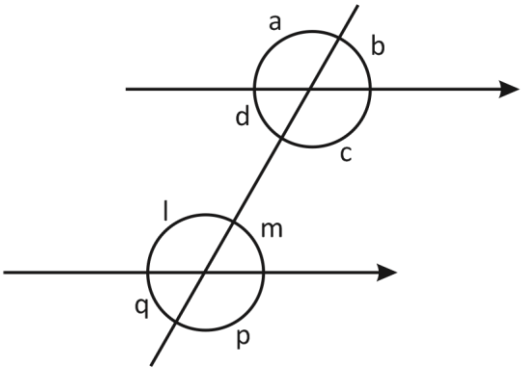
i. The total angle of a straight line is 180^o.
ii. When two straight lines intersect, the opposite angles are equal, as shown in Figure 5-6, where $\angle A = \angle C$ and $\angle B = \angle D$.

Figure 5-6: Opposite angles formed by intersecting straight lines.



iii. If two parallel lines are cut by a transversal, then, as shown in Figure 5-7: the corresponding angles are equal (i.e., $a = l$, $b = m$, $c = p$, and $d = q$); the alternate angles are equal (i.e., $d = m$ and $c = l$); and the interior angles are supplementary (i.e., $d + l = 180^o$ and $c + m = 180^o$). Conversely, if two straight lines are cut by a transversal, the lines are parallel if one of the following is true: two corresponding angles are equal; two alternate angles are equal; two interior angles are supplementary.

Figure 5-7: Angles formed by two parallel lines cut by a transversal.



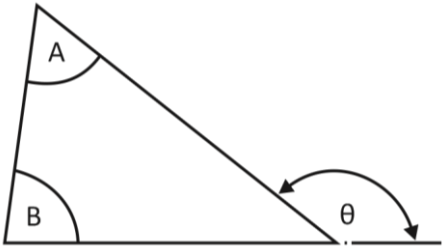
Types of triangles on the basis of their angles and their sides:

i. An "acute-angled" triangle has all its angles less than 90^o.
ii. A "right-angled" triangle has one of its angles equal to 90^o. The side opposite to the right angle is the longest side, and it is called the "hypotenuse."
iii. An "obtuse-angled" triangle has one angle greater than 90^o.
iv. A "scalene" triangle has all three sides of different length.
v. An "isosceles" triangle has two sides and two angles equal. The equal angles lie opposite to the equal sides.
vi. An "equilateral" triangle has all its sides and angles equal. Each angle of an equilateral triangle is equal to 60^o.

Angle properties of triangles:

i. The sum of the angles of a triangle is equal to 180^o
ii. In every triangle, the greatest angle is opposite to the longest side, and the smallest angle is opposite to the shortest side. Moreover, in every triangle, the sum of the lengths of any two sides is always greater than the length of the third side.
iii. When the side of a triangle is produced, the exterior angle so formed is equal to the sum of the opposite interior angles. For instance, in Figure 5-8, $\angle\theta = \angle A + \angle B$.

Figure 5-8: Exterior angle.



iv. In an isosceles triangle, the perpendicular (drawn from the point where the two equal sides meet) to the base bisects the angle between the two equal sides. Moreover, it bisects the base of the triangle.

Two triangles are said to be "congruent" if they are equal in every respect, both with regard to their corresponding angles and to their corresponding sides. If one side and two angles in one triangle are equal to one side and two similarly located angles in another triangle, then these triangles are congruent. Moreover, if two sides and the angle between them in one triangle are, respectively, equal to two sides and the angle between them in another triangle, then these triangles are congruent. Given two right-angled triangles, if their hypotenuses are equal to each other and one other side in each triangle are also equal to each other, then these right-angled triangles are congruent.

Two triangles are said to be "similar" if they are equi-angular. Two triangles are equi-angular if and only if their corresponding sides are proportional. For instance, a triangle $\triangle ABC$ and a triangle $\triangle XYZ$ are equi-angular if and only if

$\frac{AB}{XY} = \frac{AC}{XZ} = \frac{BC}{YZ}$.

Areas of triangles: The area of any triangle is:

$area = \frac{1}{2} \times base \times height$.

Triangles having equal bases and equal heights are equal in area. Moreover, the areas of congruent triangles are equal.

One of the most important geometric theorems is the Pythagorean Theorem, which states that, in every right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. As mentioned earlier, the Pythagorean Theorem led Greek mathematicians to prove the existence of irrational numbers. The Pythagorean Theorem can be proved in an algebraic way, using the concept of a locus, as follows.

Pythagorean Theorem: Consider a right-angled triangle $\triangle ABC$, whose hypotenuse is c, and whose other two sides are a and b, as shown in Figure 5-9. Then
$a^2 + b^2 = c^2$.
Proof: Given the triangle shown in Figure 5-9, we create four triangles identical to it, and we use them in order to form a square with side lengths $a + b$ as shown in Figure 5-10. The area of this square is
$A = (a + b)(a + b)$.

Figure 5-9: A right-angled triangle.

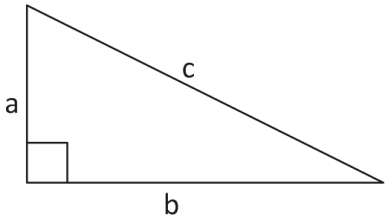
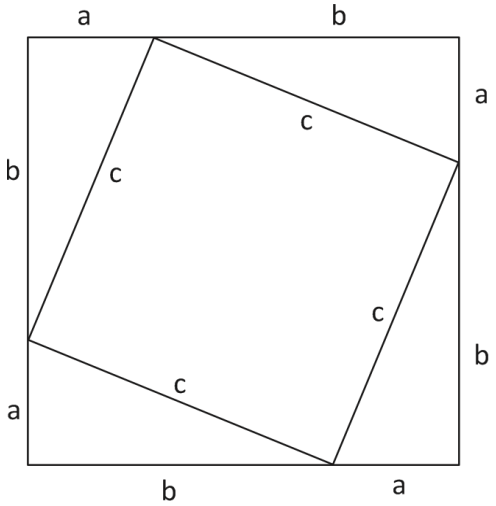


Figure 5-10: Proof of the Pythagorean Theorem.



In Figure 5-10, inside the big square, the hypotenuses of the four identical triangles form another smaller square, whose area is equal to c^2. Each of

the four triangles has an area of $\frac{ab}{2}$. In general, notice that, given an arbitrary rectangle $ABCD$ whose height is h, and whose base is b, its area is equal to hb. Therefore, if we draw a diagonal from one vertex, say diagonal AC, it will break the rectangle into two congruent, or equal, triangles, and the area of each of these triangles is half the area of the rectangle, that is, $\frac{hb}{2}$. The area of all four of the triangles that are shown in Figure 5-10 is equal to $4\frac{ab}{2} = 2ab$. Adding up the areas of the smaller square and of the four triangles, we obtain
$A = c^2 + 2ab$.
Hence, given that, as we have shown, $A = (a + b)(a + b)$, it holds that
$(a + b)(a + b) = c^2 + 2ab \Leftrightarrow a^2 + b^2 = c^2$.■

Quadrilaterals and Polygons

A "quadrilateral" is any four-sided figure. Given that a quadrilateral can be split up into two triangles, the sum of its angles is 360^o.

A "parallelogram" has both pairs of opposite sides parallel. If the base of a parallelogram is equal to b and its height is equal to h, then its area is given by the following formula: $A = bh$. Parallelograms having equal bases and equal heights are equal in area. A parallelogram has the following properties: (i) the sides that are opposite to each other are equal in length; (ii) the angles that are opposite to each other are equal; (iii) the diagonals bisect each other; (iv) the diagonals each bisect the parallelogram.

A "rectangle" is a parallelogram with all its angles equal to 90^o. If the length of a rectangle is equal to l and its width is equal to w, then its area is equal to lw, and its perimeter is equal to $2l + 2w$. A rectangle has all the properties of a parallelogram, but the diagonals are equal in length.

A "rhombus" is a parallelogram with all its sides equal in length. It has all the properties of a parallelogram, but in addition it has the following properties: (i) the diagonals bisect at right angles; (ii) the diagonal bisects the angle through which it passes.

A "square" is a rectangle with all its sides equal in length. If the length of each side of a square is equal to a, then its area is equal to a^2, and its perimeter is equal to $4a$. A square has all the properties of a parallelogram, a rectangle, and a rhombus.

A "trapezoid" is a quadrilateral having only one pair of parallel sides (as opposed to a parallelogram, which has both pairs of opposite sides parallel). The parallel sides are called the "bases" of the trapezoid, while the other two sides are called the "legs" of the trapezoid. If the bases

(parallel sides) of a trapezoid are equal to a and b, respectively, and if its height is equal to h, then its area is equal to $\frac{1}{2}h(a+b)$.

Analytic Geometry and Trigonometric Functions

Analytic geometry signifies the introduction of coordinates into geometry in a systematic way—specifically, by unifying aspects of algebra and aspects of geometry. The development of analytic geometry through the algebraization of geometry set the stage for the development of infinitesimal calculus. The first pioneers of analytic geometry were the second-century B.C.E. Greek astronomer and mathematician Hipparchus of Nicaea, who introduced coordinates for the sphere (in the context of his studies of the night sky), and the third-century B.C.E. Greek geometer Apollonius of Perga, who introduced coordinates for the study of conic sections.

Ancient Greek mathematicians, such as Apollonius of Perga, were the first to observe that circles, ellipses, hyperbolas, and parabolas result from the intersection of a cone by an adequate plane. A cone is defined to be a three-dimensional geometric shape that tapers smoothly from a flat circular base to a point called the vertex (or apex). A circle is produced when the cone is cut by a plane that is parallel to the base of the cone. An ellipse is produced when the cone is cut by a plane that is not parallel to the base of the cone or the side of the cone, and it cuts only one nappe of the cone. A hyperbola is produced when the intersecting plane cuts both nappes of the cone. A parabola is produced when the oblique section of the cone is parallel to the slant height (the height of a cone from the vertex to the periphery, rather than the center, of the base). In the Middle Ages, the use of coordinates in mathematics and analytic geometry was further analyzed and developed by the fourteenth-century French philosopher and mathematician Nicolas d'Oresme.

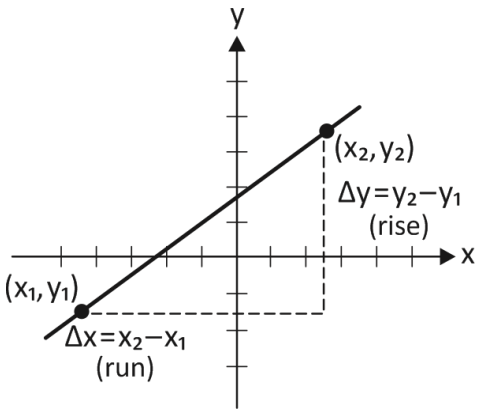
By the term "locus," we mean a set of all the points that satisfy a specific rule. Moreover, the path drawn by a point moving according to a given rule is called the "locus of the point." Thus, using the concept of a locus, we can study geometric problems through algebra. In analytic geometry, we put traditional (Euclidean) geometry on the Cartesian plane. René Descartes has pointed out that "any problem in geometry can easily be reduced to such terms that knowledge of lengths of certain straight lines is sufficient for its construction" (René Descartes, "On Analytic Geometry," translated by David E. Smith and Marcia L. Latham, in *A Source Book in Mathematics*, edited by David E. Smith, New York: Dover, 1959, p. 397). In particular, according to Descartes, "just as arithmetic consists of only four or five operations, namely, addition, subtraction, multiplication,

division, and the extraction of roots, which may be considered a kind of division, so in geometry," we can find required lines by merely adding or subtracting other lines; or else, by working as follows (ibid, pp. 397–98):

> . . . taking one line which I shall call unity in order to relate it as closely as possible to numbers, and which can in general be chosen arbitrarily, and having given two other lines, to find a fourth line which shall be to one of the given lines as the other is to unity (which is the same as multiplication); or, again, to find a fourth line which is to one of the given lines as unity is to the other (which is equivalent to division); or, finally, to find one, two, or several mean proportionals between unity and some other line (which is the same as extracting the square root, cube root, etc., of the given line).

Consider two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ on the xy-plane and connect them with a straight line segment as shown in Figure 5-11.

Figure 5-11: Slope and Distance.



The x-coordinate of point P is x_1, the x-coordinate of point Q is x_2, and the distance between x_1 and x_2 is $x_2 - x_1$; in order to avoid the use of plus and minus signs, we can use the absolute value $|x_2 - x_1|$. The y-coordinate of point P is y_1, the y-coordinate of point Q is y_2, and the distance between y_2 and y_1 is $y_2 - y_1$; in order to avoid the use of plus and minus signs, we can use the absolute value $|y_2 - y_1|$. Therefore, the horizontal distance between points P and Q is $x_2 - x_1$, and the vertical distance between points P and Q is $y_2 - y_1$. Now, consider the right-angled triangle that is defined by the points $P(x_1, y_1)$, $Q(x_2, y_2)$, and the point R (the intersection between the horizontal side and the vertical side):

the three sides of this right-angled triangle are the hypotenuse PQ, the horizontal side, which is $x_2 - x_1$, and the vertical side, which is $y_2 - y_1$. The "slope," or "gradient," of the straight line segment PQ, denoted by m_{PQ}, is the quotient of the "rise" over the "run," comparing how much one travels vertically ("up and down") versus how much one travels horizontally. Thus, it relates the steepness or inclination of the straight line segment PQ to the coordinates; symbolically:

$$slope = m_{PQ} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

(see Figure 5-11; the Greek letter Δ is used to indicate change).

In Figure 5-11, the distance between points P and Q, denoted by d_{PQ}, is given by (and, indeed, is a version of) the Pythagorean Theorem. Therefore, in Figure 5-11,

$$\left(d_{PQ}\right)^2 = (run)^2 + (rise)^2$$

$$\Leftrightarrow d_{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

It can be easily verified that the midpoint of the straight line segment joining points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

All points (x, y) in $\mathbb{R}^2$ satisfying the equation $y = mx + b$ form a straight line, and m is the slope of the straight line. For the slope m of the straight line passing through the points (x_1, y_1) and (x_2, y_2), we have:

i. If $x_1 = x_2$, m is undefined (the line is vertical).
ii. If $x_1 \neq x_2$, then $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

Two non-vertical straight lines y_1 and y_2, with slopes m_1 and m_2 respectively, are parallel if and only if $m_1 = m_2$ (i.e., their slopes are equal), and they are perpendicular if and only if $m_1 m_2 = -1$ (i.e., the product of their slopes is -1).

In order to find the equation of a non-vertical straight line, we work as follows:

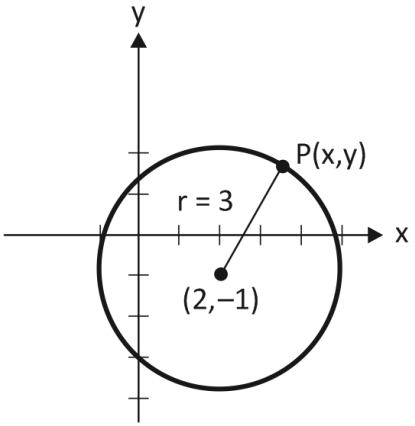
i. we find a point (x_1, y_1) on the line;
ii. we find the slope m of the line;
iii. we write the equation of the line as follows:
$y - y_1 = m(x - x_1)$; this equation is called the "point-slope" form of the equation of a line.

For instance, let us find the equation of the straight line passing through the points $(5, -0.5)$ and $(10, 9.5)$. Firstly, we define the point $(x_1, y_1) = (5, -0.5)$. Secondly, we find the slope of the required line: $m = \frac{9.5-(-0.5)}{10-5} = 2$. Thirdly, we find the equation of the required line: $y - y_1 = m(x - x_1) \Rightarrow y - (-0.5) = 2(x - 5) \Rightarrow y = 2x - 10.5$.

Circle

As we can see in Figure 5-12, a circle with center $O(v,w)$ and radius r is the set of all points in the xy-plane whose distance from O is r (in Figure 5-12, $O(v,w) = O(2,-1)$, and $r = 3$).

Figure 5-12: Circle.



If (x,y) is a point on the circle with center $O(v,w)$ and radius r, then the distance formula implies that

$r = \sqrt{(x-v)^2 + (y-w)^2} \Leftrightarrow r^2 = (x-v)^2 + (y-w)^2$,

which is the standard form of the equation of a circle with center (v,w) and radius r. The circumference of a circle of radius r is $C = 2\pi r$, and the area of a circle of radius r is $A = \pi r^2$, where $\pi \approx 3.14$ is Archimedes's constant (the ratio of the circle's circumference to its diameter). Archimedes approximated π by using the fact that the circumference of a circle is bounded by the perimeter of an inscribed polygon and the perimeter of a circumscribed polygon. In particular, he used a 96-sided inscribed polygon and a 96-sided circumscribed polygon to find the following approximation:

$3 + \frac{10}{71} < \pi < 3 + \frac{10}{70}$.

It is worth mentioning that the degenerate possibilities for a circle are the following: a point or no graph at all.

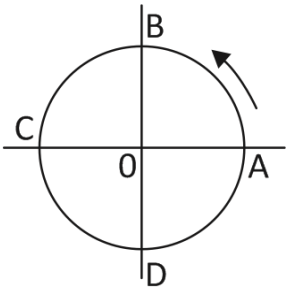
The study of the circle underpins trigonometry. The term "trigonometry" appeared for the first time in the book *Trigonometria* by Bartholomaeus

Pitiscus (1561–1613) in 1595, and it literally means measuring (and, more broadly, studying) "trigons" ("trigon" being the Latin word for "triangle"). The acknowledged founder of trigonometry is the ancient Greek astronomer and mathematician Hipparchus of Nicaea (ca. 190–ca. 120 B.C.E.). Moreover, around 100 C.E., another Greek mathematician, Menelaus of Alexandria, published a series of treatises on chords.

Trigonometric Functions

In the context of analytic geometry, we can also study the basic trigonometric functions on the unit circle (specifically, on a circle whose center is $(0,0)$ and whose radius $r = 1$).

Figure 5-13: The Number Circle.



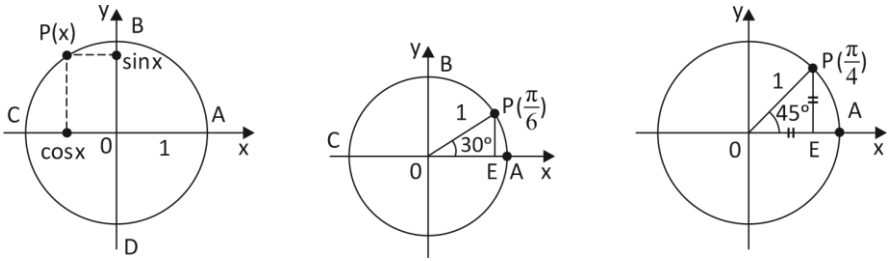
Consider a circle of unit radius, as shown in Figure 5-13, and let point *A* (the right-hand endpoint of the horizontal diameter) be a reference point. Let an anti-clockwise motion round the circle be a positive direction, and a clockwise motion be a negative direction. A circle of unit radius with a reference point and the direction of tracing specified is called the "number circle." Given an arbitrary point *P* of the number circle, there are infinitely many arcs beginning at the point *A* and terminating at the point *P*. One of these arcs is the shortest arc connecting the points *A* and *P*, and all the other arcs are obtained from the shortest arc by adding or subtracting an integral number of complete revolutions. Hence, every point *P* of the number circle is associated with an infinite set of numbers that consists of the values of all the arcs beginning at the point *A* and terminating at the point *P* (the lengths of the arcs are taken with the plus or minus sign according as the motion from the point *A* to the point *P* is anti-clockwise or clockwise, respectively).

The circumference of the circle of unit radius is equal to 2π. Therefore, the lengths of all the arcs terminating at the given point *P* differ from one

another by an integral number multiple of 2π, so that the general form of these quantities is $x + 2\pi a$, where $a \in \mathbb{Z}$, and x is the length of the shortest arc connecting the points A and P. Thus, for every real number x, there is a point $P(x)$ of the number circle such that the length of the arc AP is x, and every point P of the circle corresponds to an infinite set of numbers of the form $x + 2\pi a$, where $a \in \mathbb{Z}$, and x is the length of one of the arcs connecting the points A and P.

Assume that the center of the number circle coincides with the origin $O(0,0)$ of the rectangular coordinate system XOY, as shown in Figure 5-14. Let x be an arbitrary real number. Then, on the number circle, we find the point $P(x)$ that corresponds to x. The ordinate of the point $P(x)$ is called the "sine" of the number x (denoted by $sinx$), the abscissa of the point $P(x)$ is called the "cosine" of the number x (denoted by $cosx$), the ratio $\frac{sinx}{cosx}$ is called the "tangent" of the number x (denoted by $tanx$), and the ratio $\frac{cosx}{sinx}$ is called the "cotangent" of the number x (denoted by $cotx$).

Figure 5-14: Trigonometric Functions.



Notice that the reference point A on the number circle corresponds to the number 0, that is, $A = A(0)$. Since the abscissa and the ordinate of this point are 1 and 0, respectively, we have $cos0 = 1$, $sin0 = 0$, and $tan0 = \frac{sin0}{cos0} = 0$. The point B of intersection of the circle and the positive ray of the axis OY corresponds to the number $\pi/2$. Since the abscissa and the ordinate of the point B are 0 and 1 respectively, we have $\cos(\frac{\pi}{2}) = 0$ and $\sin(\frac{\pi}{2}) = 1$, whereas $\tan(\frac{\pi}{2})$ is not defined. Similarly, as shown in Figure 5-14, given the coordinates of the points C and D, we realize that $cos\pi = -1$, $sin\pi = 0$, $tan\pi = 0$, $\cos(\frac{3\pi}{2}) = 0$, $sin(\frac{3\pi}{2}) = -1$, and $tan(\frac{3\pi}{2})$ is not defined.

We can summarize the basic definitions and the basic formulas of trigonometry as follows:

$Sine: \sin\theta = \frac{opposite\ side}{hypotenuse}$,

$Cosine: cos\theta = \frac{adjacent\ side}{hypotenuse}$,

$Tangent: tan\theta = \frac{opposite\ side}{adjacent\ side}$,

$Cosecant: csc\theta = \frac{hypotenuse}{opposite\ side} = \frac{1}{sin\theta}$,

$Secant: sec\theta = \frac{hypotenuse}{adjacent\ side} = \frac{1}{cos\theta}$,

$Cotangent: cot\theta = \frac{adjacent\ side}{opposite\ side} = \frac{1}{tan\theta}$,

and the basic trigonometric identities:

$sin^2a + cos^2a = 1$,

$sin(-a) = -sina$,

$cos(-a) = cosa$,

$\sin(a \pm b) = sina \cdot cosb \pm cosa \cdot sinb$,

$\cos(a \pm b) = cosa \cdot cosb \mp sina \cdot sinb$,

$sina + sinb = 2sin\frac{1}{2}(a+b) \cdot cos\frac{1}{2}(a-b)$,

$cosa + cosb = 2cos\frac{1}{2}(a+b) \cdot cos\frac{1}{2}(a-b)$,

$sin2a = 2sina \cdot cosa$,

$cos2a = cos^2a - sin^2a$,

$sin\frac{1}{2}a = \sqrt{\frac{1-cosa}{2}}$,

$cos\frac{1}{2}a = \sqrt{\frac{1+cosa}{2}}$,

$\sin(a \pm \pi/2) = \pm cosa$,

$\cos(a \pm \pi/2) = \mp sina$.

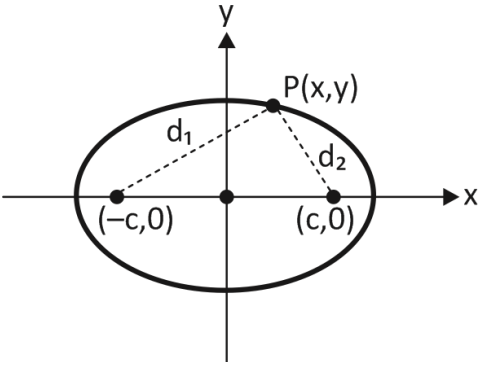
The inverse trigonometric functions are denoted as follows: $arcsinx \equiv sin^{-1}x$ $(y = arcsinx \Leftrightarrow x = siny)$, $arccosx \equiv cos^{-1}x$ $(y = arccosx \Leftrightarrow x = cosy)$, and $arctanx \equiv tan^{-1}x$ $(y = arctanx \Leftrightarrow x = tany)$.

Ellipse

As we can see in Figure 5-15, an "ellipse" is the set of all points in a plane the sum of whose distances from two fixed points ("foci") is constant. Foci: $(-c, 0)$ and $(c, 0)$. Notice that, if the two foci coincide, then we receive a circle. The Greek word ellipse, literally meaning "omission," was first applied by Apollonius of Perga, because, in the case of an ellipse,

the conic section of the cutting plane makes a smaller angle with the base than does the side of the cone.

Figure 5-15: Ellipse.



The standard form of the equation of an ellipse with center at the origin and foci on the x-axis is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

By setting $y = 0$, we find that the x-intercepts are $(-a, 0)$ and $(a, 0)$. By the setting $x = 0$, we find that the y-intercepts are $(0, -b)$ and $(0, b)$. The larger segment from $(-a, 0)$ to $(a, 0)$ is called the "major axis," while the "minor axis" is the segment from $(0, -b)$ to $(0, b)$. The endpoints of the major axis are called the "vertices of the ellipse"; vertices: $(-a, 0)$ and $(a, 0)$.

If the foci are placed on the y-axis at $(0, -c)$ and $(0, c)$, then the standard form of the equation of an ellipse is

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$

In this case, the major axis is along the y-axis, the foci are $(0, c)$ and $(0, -c)$, and the vertices are $(0, a)$ and $(0, -a)$.

Given the definition of an ellipse, the degenerate possibilities for an ellipse are the following: a point or no graph at all.

In our solar system, many bodies revolve in elliptical orbits around a larger body that is located at one focus. In the seventeenth century, Johannes Kepler, based on Apollonius's mathematical study of the ellipse, articulated a rigorous explanation of planetary motions.

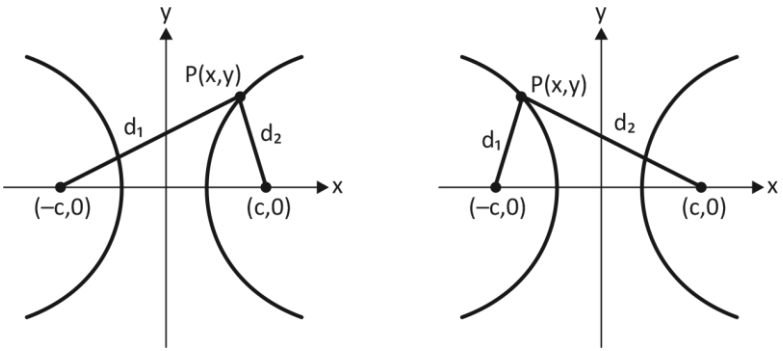
Moreover, regarding the ellipse, it should be mentioned that it has a reflection property that causes any ray or wave that originates at one focus to strike the ellipse and pass through the other focus. In terms of acoustics, the aforementioned property implies that, in a room with an elliptical ceiling, even a slight noise made at one focus can be heard at the other focus, but, if people are standing between the foci, then they hear nothing. Such rooms are known as whispering galleries.

As regards architecture, it should be mentioned that ornamental arches are often elliptical in shape; in other words, arches whose main purpose is beauty and not strength are often elliptical in shape.

Hyperbola

As we can see in Figure 5-16, a "hyperbola" is the set of all points in a plane the difference of whose distances from two fixed points ("foci") is a positive constant (the Greek word hyperbola literally means "extravagance"). Hence, the distances between the foci and a point on the figure maintain a *constant difference* for a hyperbola and a *constant sum* for an ellipse.

Figure 5-16: Hyperbola.



Given the definition of a hyperbola, the degenerate possibilities for a hyperbola are two intersecting straight lines.

The standard form of a hyperbola with center at the origin and foci on the x-axis is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

By setting $y = 0$, we find that the x-intercepts are $(-a, 0)$ and $(a, 0)$. The line segment joining these two points is called the "transverse axis." The endpoints of the transverse axis are called the "vertices of the hyperbola."

By setting $x = 0$, we find that there are no y-intercepts. The line segment from $(0, b)$ to $(0, -b)$ is called the "conjugate axis." In order to determine the significance of b, we write

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ as $y = \frac{\pm bx}{a}\sqrt{1 - \frac{a^2}{x^2}}$.

As $|x|$ tends to infinity, $1 - \frac{a^2}{x^2}$ tends to 1, and, therefore, the graph of the hyperbola approaches the lines

$y = \pm\frac{b}{a}x$.

These lines are called the "asymptotes of the hyperbola" (they are the diagonals of a rectangle of dimensions $2a$ by $2b$).

If the foci are placed on the y-axis at $(0, -c)$ and $(0, c)$, then the standard form of the equation of a hyperbola is

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$,

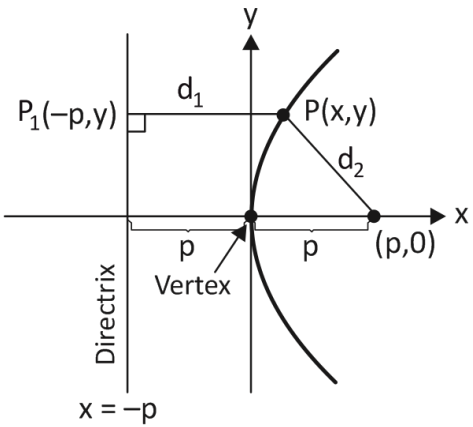
and, in this case, the asymptotes are given by

$y = \pm\frac{a}{b}x$.

Parabola

As we can see in Figure 5-17, a "parabola" is the set of all points in a plane that are equidistant from a fixed line ("directrix") and a fixed point ("focus") not on the line (the word "parabola" derives from the Greek terms "parā," meaning "beside," and "bolē," meaning "a throw," and, therefore, "parabola" literally means "para-beside"—that is, placing side by side).

Figure 5-17: Parabola.



The standard form of the equation of a parabola with directrix $x = -p$ and focus at $(p, 0)$ is
$4px = y^2$.
The line through the focus that is perpendicular to the directrix is called the "axis of symmetry." In this case, the axis of symmetry is the x-axis, and the parabola opens to the right. The point on the axis of symmetry that is midway between the focus and the directrix is called the "vertex," and the vertex is the turning point of the parabola. The standard form of the equation of a parabola with directrix $x = p$ and focus at $(-p, 0)$ is
$-4px = y^2$,
and, in this case, the parabola opens to the left.
Obviously, the axis of symmetry of a parabola may be the y-axis. If the directrix is $y = -p$ and the focus is at $(0, p)$, then the standard form of the equation of a parabola is
$x^2 = 4py$,
and the parabola opens upward. If the directrix is $y = p$ and the focus is at $(0, -p)$, then the standard form of the equation of a parabola is
$x^2 = -4py$,
and the parabola opens downward.
As regards the parabola in general, it should be mentioned that it has a reflection property that causes any ray or wave that originates at the focus and strikes the parabola to be reflected parallel to the axis of symmetry. Thus, for instance, flashlights and searchlights use a parabolic reflector with the bulb located at the focus. Additionally, due to the reflection property of a parabola, any ray or wave that comes into a parabolic reflector parallel to the axis of symmetry is directed to the focus point. For this reason, radars, radio antennas, and reflecting telescopes operate according to this principle. In astronomy, the parabola features in both the construction of telescopes and in the motion of comets around the Sun. Finally, due to their great strength, parabolic arches are used extensively in bridges, cathedrals, and elsewhere in architecture and engineering, especially in case we have equally spaced load.

Volumes and Surface Areas

By the term "volume," we mean the amount of three-dimensional space enclosed by a closed surface. The volume of any solid having a uniform cross-section is equal to:
cross-sectional area×length of solid.
The surface area of any solid having a uniform cross-section is equal to:
curved surface+ends; namely:

perimeter of cross-sections×length of solid+total area of ends.
The volume of a sphere with radius r is equal to
$\frac{4}{3}\pi r^3$,
and its surface area is equal to
$4\pi r^2$.
The volume of a cylinder whose height is h and whose base is a circle with radius r is equal to
$\pi r^2 h$,
and its surface area is equal to
$2\pi rh + 2\pi r^2 = 2\pi r(h + r)$.
The volume of a cone whose vertical height is h and whose base is a circle with radius r is equal to
$\frac{1}{3}\pi r^2 h$,
and, if l is its slant height, then its surface area is equal to
$\pi rl + \pi r^2$.
The volume of a pyramid whose height is h and whose base's area is equal to A is given by the following formula:
$V = \frac{1}{3}Ah$.
The surface area of a pyramid is equal to the sum of the areas of the corresponding triangles plus the area of the base.

Vectors, Vector Spaces, and Normed Vector Spaces

A "scalar" is a quantity that can be specified by determining only its magnitude. However, the quantities that are specified by determining both magnitude and direction are called "vectors." In other words, a "vector" is a quantity that has both a direction and a magnitude of length; therefore, it is graphically denoted by an oriented line segment ("arrow"). In physics, vectors are very useful, because they can visually represent position, displacement, velocity, and acceleration. Moreover, vector graphics are used in computers, since they can be scaled to a larger size without losing any image quality.
If the coordinates of a point P in the coordinate plane are (x, y), and if we denote the origin of the coordinate system by $O(0,0)$, then a vector OP is denoted by $\overrightarrow{OP}$, since the length OP represents the magnitude, and the arrow represents the direction. The column vector (matrix) corresponding to $\overrightarrow{0P}$ is
$\begin{pmatrix} x \\ y \end{pmatrix}$.

Since the coordinates of point P are (x, y), the length from $O(0,0)$ to P is $\sqrt{x^2 + y^2}$, according to the Pythagorean Theorem. Notice that, frequently, we do not need to use arrows in order to indicate that letters represent vectors (in particular where there is no likelihood of confusion).

The operations between vectors are based on matrix algebra. For instance, given two vectors $\overrightarrow{OA} = \begin{pmatrix} p \\ q \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} r \\ s \end{pmatrix}$,

their sum is a vector $\overrightarrow{OC}$ such that

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB} = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p + r \\ q + s \end{pmatrix}$.

In general, we can define the following vector operations:

Vector addition: $\vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n)$;

Scalar multiplication: $k\vec{u} = (ku_1, ku_2, \dots, ku_n)$;

Negation: $-\vec{u} = (-1)\vec{u} = (-u_1, -u_2, \dots, -u_n)$;

Dot Product (or Scalar Product or Inner Product):

$\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + \cdots + u_nv_n = \sum_{i=1}^{n} u_iv_i$;

Norm (Length): $\|\vec{u}\| = \sqrt{\vec{u} \cdot \vec{u}} = \sqrt{u_1^2 + u_2^2 + \cdots + u_n^2}$

(specifically, the norm of a vector is the distance of the vector from the origin);

where: $\vec{u} = (u_1, u_2, \dots, u_n)$ and $\vec{v} = (v_1, v_2, \dots, v_n)$ are vectors in $\mathbb{R}^n$, and k is a real number (scalar).

Cross Product of two vectors in a 3-dimensional space: Consider two vectors $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$, and let $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$ be the unit vectors of the three coordinate axes respectively. Then the cross product of $\vec{u}$ and $\vec{v}$ is a vector given by the following determinant:

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \vec{\imath} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \vec{\jmath} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \vec{k} =$$

$(u_2v_3 - u_3v_2)\vec{\imath} - (u_1v_3 - u_3v_1)\vec{\jmath} + (u_1v_2 - u_2v_1)\vec{k}$.

The geometric significance of this operation is that, if θ is the angle between $\vec{u}$ and $\vec{v}$ with $0 \leq \theta \leq \pi$, then

$\vec{u} \times \vec{v} = \|\vec{u}\|\|\vec{v}\|(sin\theta)\vec{n}$,

where $\vec{n}$ is a unit vector perpendicular to the plane containing $\vec{u}$ and $\vec{v}$ (with your right hand, point your index finger along vector $\vec{u}$, and point your middle finger along vector $\vec{v}$; then $\vec{n}$ goes in the direction of your extended thumb). The magnitude of the cross product ($|\vec{u} \times \vec{v}|$) can be interpreted as the positive area of the parallelogram having $\vec{u}$ and $\vec{v}$ as its sides. Whilst the resultant of the dot product of two vectors $\vec{u}$ and $\vec{v}$ is a scalar quantity, the cross product of two vectors $\vec{u}$ and $\vec{v}$ is a third vector whose direction is perpendicular to both $\vec{u}$ and $\vec{v}$ (the direction is given by

the aforementioned right-hand rule). Two vectors are parallel to each other if and only if they are scalar multiples of each other.

The most abstract definition of a vector is that a vector is an element of a "vector (or linear) space," which, in turn, can be defined as follows: let U be a set endowed with two operations: addition and scalar multiplication, defined in the following way:

$+: U \times U \to U$ defined by $(u, v) \in U \times U \to u + v \in U$ for all $u, v \in U$, that is, U is "closed under addition";

$\cdot: k \times U \to U$ defined by $(k, u) \in K \times U \to k \cdot u \in U$ for every $k \in K$ (where K is a field, such as $\mathbb{R}$) and for every $u \in U$, that is, U is "closed under scalar multiplication." Of course, $0 \in U$, since, for every $u \in U$, $(-1)u \in U$, and, therefore, $u - u \in U \Rightarrow 0 \in U$. As a result of the aforementioned definition, we say that U under the operations of $+$ (addition) and $\cdot$ (scalar multiplication) forms a "vector space" (or "linear space") over the field K; and, therefore, a "vector" can be defined as an element of such a U.

For instance, we can prove that, if

$V = \{ax^2 + bx + c | a, b, c \in \mathbb{R}\}$,

then V is a vector space over $\mathbb{R}$ as follows:

Step 1: $0 = 0x^2 + 0x + 0 \in V$.

In other words, $0 \in V$.

Step 2: Let

$$\begin{cases} v_1 = a_1x^2 + b_1x + c_1 \\ v_2 = a_2x^2 + b_2x + c_2 \end{cases}.$$

Then $v_1 + v_2 = (a_1 + a_2)x^2 + (b_1 + b_2)x + (c_1 + c_2) \in V$.

In other words, V is closed under addition.

Step 3: Let $v = ax^2 + bx + c$ with $a, b, c \in \mathbb{R}$.

Then $kv = (ka)x^2 + (kb)x + (kc) \in V$.

In other words, V is closed under scalar multiplication.

Therefore, $V = \{ax^2 + bx + c | a, b, c \in \mathbb{R}\}$ is a vector space over $\mathbb{R}$. In other words, the set of all real quadratic polynomials forms a vector space over $\mathbb{R}$.

On the other hand, we can prove that a sphere S is not a vector space as follows: let v be a vector belonging to the sphere S. If we multiply v by an adequate number k, then kv does not belong to S any more (it "pierces" the sphere). Hence, a sphere is not a vector space (it is not closed under scalar multiplication). This example helps us to understand why no bounded set, in general, is a vector space.

Linearly Independent Vectors: Let V be a vector space over K. The vectors $v_1, v_2, \ldots, v_n$ of V are "linearly independent" if and only if every time

$k_1v_1 + k_2v_2 + \cdots + k_nv_n = 0 \Rightarrow k_1 = k_2 = \cdots = k_n = 0$.

For instance, the vectors $v_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $v_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, and $v_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ are linearly independent, since

$$k_1 v_1 + k_2 v_2 + \cdots + k_n v_n = 0$$
$$\Rightarrow \begin{pmatrix} k_1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & k_2 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ k_3 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & k_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$
$$\Rightarrow \begin{pmatrix} k_1 & k_2 \\ k_3 & k_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow k_1 = k_2 = k_3 = k_4 = 0.$$

Linearly Dependent Vectors: Let V be a vector space over K. The vectors $v_1, v_2, \dots, v_n$ of V are "linearly dependent" if and only if $k_1 v_1 + k_2 v_2 + \cdots + k_n v_n = 0$ for some $k_i \neq 0$, where $i = 1,2, \dots, n$.

For instance, the vectors $v_1 = (0,1)$, $v_2 = (1,0)$, and $v_3 = (1,1)$ are linearly dependent.

Basis: Let V be a vector space over K. The vectors $v_1, v_2, \dots, v_n$ form a "basis" of V if and only if these vectors are linearly independent and generate (or span) V; that is, every vector of V must be expressed in terms of $v_1, v_2, \dots, v_n$. For instance, if $V = \{a + bx + cx^2 | a, b, c \in \mathbb{R}\}$, then $v_1 = 1$, $v_2 = x$, and $v_3 = x^2$ form a basis of V, because: (i) v_1, v_2, and v_3 are linearly independent, since no vector from $\{1, x, x^2\}$ can be written in terms of the other vectors; and (ii) $\{1, x, x^2\}$ generate V, since, for any $v \in V$, it holds that $v = k + lx + mx^2 = k \cdot 1 + lx + mx^2$. Every (non-zero) vector space over a field K has at least one basis (actually, it has many different bases). However, every vector space V has an invariant property: the number of vectors in every basis of V remains the same, and the "dimension" of a vector space V is the number of elements of any of its bases.

When we study vector spaces, and abstract mathematical spaces in general, we must keep in mind that the term "space" signifies a collection of vectors that interact in a certain way, which is determined by the corresponding structure (e.g., by a set of operations, by a norm, etc.). We can define a norm in an abstract way as follows: given a vector (or linear) space X over $\mathbb{R}$, a "norm" $\|\cdot\|$ for X is a function on X that assigns to each element a real number (symbolically: $\|\cdot\|: X \to \mathbb{R}$) satisfying the following properties:

for every $x \in X$:

i. $\|x\| \geq 0$,
ii. $\|x\| = 0$ if and only if $x = 0$,
iii. $\|kx\| = |k|\|x\|$ for any scalar k, and,
for every $x, y \in X$,
iv. $\|x + y\| \leq \|x\| + \|y\|$ (the triangle inequality).

A vector (or linear) space that is equipped with a norm $\|\cdot\|$ is denoted by $(X, \|\cdot\|)$ and is called a "normed vector space" (or "normed linear space"). Different norms can be defined on the same vector space, thus giving rise to different normed vector spaces.

Example 1: $(\mathbb{R}, |\cdot|)$. The set of real numbers ($\mathbb{R}$) is a normed vector space with norm given by the absolute value (or modulus), that is,

$\|x\| = |x|$,

and we call this the "usual norm" for $\mathbb{R}$.

Example 2: $(\mathbb{R}^n, \|\cdot\|_2)$. The set of ordered n-tuples of real numbers ($\mathbb{R}^n$) is a normed vector space with norm $\|\cdot\|_2$ defined as follows:

for any real vector $x = (k_1, k_2, \dots, k_n)$,

$\|x\|_2 = \sqrt{|k_1|^2 + |k_2|^2 + \cdots + |k_n|^n}$,

and we call this the "Euclidean norm."

Example 3: $(\mathbb{R}^n, \|\cdot\|_1)$. The set of ordered n-tuples of real numbers ($\mathbb{R}^n$) is a normed vector space with norm $\|\cdot\|_1$ defined as follows:

for any real vector $x = (k_1, k_2, \dots, k_n)$,

$\|x\|_1 = |k_1| + |k_2| + \cdots + |k_n|$.

Example 4: $(\mathbb{R}^n, \|\cdot\|_\infty)$. The set of ordered n-tuples of real numbers ($\mathbb{R}^n$) is a normed vector space with norm $\|\cdot\|_\infty$ defined as follows:

for any real vector $x = (k_1, k_2, \dots, k_n)$,

$\|x\|_\infty = max\{|k_i|, where\ i = 1,2, \dots, n\}$,

and we call this the "supremum (or uniform) norm" for $\mathbb{R}^n$.

Example 5: $(\mathcal{B}(X), \|\cdot\|_\infty)$. For any non-empty set X, we denote by $\mathcal{B}(X)$ the set of bounded real functions on X. Notice that a function f on some set X with real values is said to be "bounded" if the set of its values is bounded—that is, if there exists a real number M such that, for every $x \in X$, it holds that $|f(x)| \leq M$.

$\mathcal{B}(X)$ is a real vector space under the pointwise definitions of addition and scalar multiplication. Moreover, $\mathcal{B}(X)$ is a normed vector space with norm $\|\cdot\|_\infty$ defined by

$\|f\|_\infty = sup\{|f(x)|, where\ x \in X\}$,

and we call this the "supremum (or uniform) norm" for $\mathcal{B}(X)$. Notice that Example 4 is the special case when $X = \{1,2, \dots, n\}$.

Example 6: l_2-space, also known as the "Hilbert (sequence) space." This is a generalization of the Euclidean n-space. The set l_2 whose elements are sequences of scalars (real numbers) $x = \{k_1, k_2, \dots, k_n, \dots\}$ such that $\sum|k_n|^2$ is convergent, is a real vector space under the pointwise definitions of addition and scalar multiplication, and it is a normed vector space with norm $\|\cdot\|_2$ defined by

$\|x\|_2 = \sqrt{\sum_{i=1}^{\infty}|k_i|^2}$.

In an arbitrary normed vector space $(X, \|\cdot\|)$, the set

$$S(0;1) = \{x \in X \; such \; that \; \|x\| = 1\}$$

is called the "unit sphere"; the set

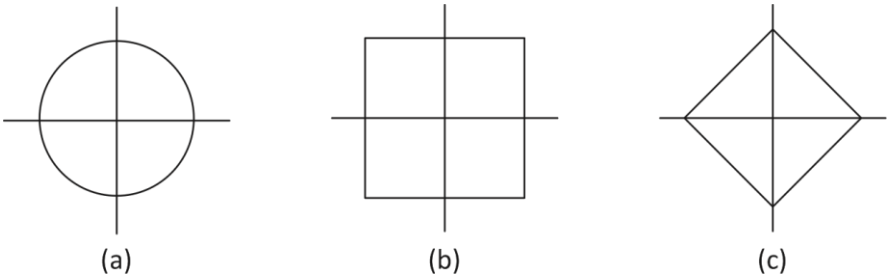
$$B[0;1] = \{x \in X \; such \; that \; \|x\| \leq 1\}$$

is called the "closed unit ball"; and the set

$$B(0;1) = \{x \in X \; such \; that \; \|x\| < 1\}$$

is called the "open unit ball." In Figure 5-18, we consider the shape of the unit sphere in several coordinate space examples: (a) in $(\mathbb{R}^2, \|\cdot\|_2)$, where $S\big((0,0);1\big) = \{(k,l) \; such \; that \; k^2 + l^2 = 1\}$; (b) in $(\mathbb{R}^2, \|\cdot\|_\infty)$, where $S\big((0,0);1\big) = \{(k,l) \; such \; that \; max\{|k|,|l|\} = 1\}$; and (c) in $(\mathbb{R}^2, \|\cdot\|_1)$, where $S\big((0,0);1\big) = \{(k,l) \; such \; that \; |k| + |l| = 1\}$.

Figure 5-18: The shape of the unit sphere in several coordinate space examples.



Non-Euclidean Geometries

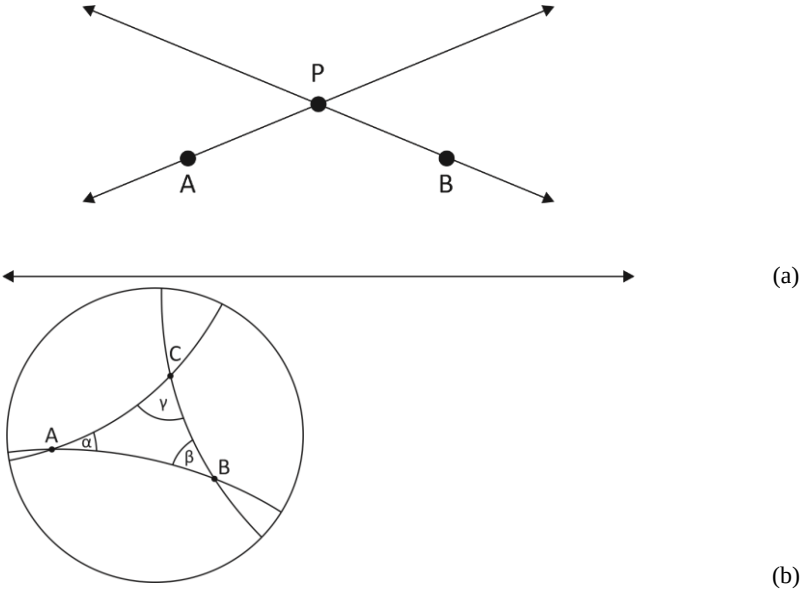
Ancient geometry culminated in Euclid's *Elements*. However, in the fifth century C.E., the Greek philosopher Proclus criticized Euclid's parallel postulate ("if a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles") by arguing that it should be struck out of the axioms of geometry altogether, because it is actually a theorem involving many difficulties. Proclus offered the example of a hyperbola that approaches its asymptotes as closely as one likes without ever meeting them, thus indicating that the opposite of Euclid's conclusion is at least conceivable. Consequently, according to Proclus, Euclid's parallel postulate should be treated as a theorem, which should be proved from the other axioms.

Euclid's parallel postulate was so obscure and so intimately related to the other axioms that, for many centuries, mathematicians vainly attempted to

prove it by proceeding from the others. Leo Gersonides (1288–1344) was arguably the first mathematician in Western Europe who tried to prove the parallel postulate. Euclid's parallel postulate became the focus of keen attention by famous mathematicians, such as: Ch. Clavius (1574), P. Cataldi (1603), G. A. Borelli (1658), G. Vitale (1680), J. Wallis (1663), G. G. Saccheri (1733), J. H. Lambert (1766), A.-M. Legendre (1800), F. K. Schweikart (1818), F. Taurinus (1825), and C. F. Gauss. Although their attempts were unsuccessful, they played a very important role in the development of modern mathematics, because they laid the foundations for a new, non-Euclidean geometry. This geometry was based on the rejection of Euclid's parallel postulate, and was invented by the Russian mathematician Nikolai Ivanovich Lobachevski (1792–1856), who initially called this geometry "imaginary" and, later, "pangeometry." In 1826, Lobachevski delivered his first communication on non-Euclidean geometry. Lobachevski's work opened up a new era in the development of geometry. In 1832, a similar research paper of the Hungarian mathematician J. Bolyai was published. In the 1830s, Lobachevski argued that, in order to establish the validity of his non-Euclidean geometry, he needed the aid of experiments, such as astronomical observations, as in the case of other natural laws (see: Eric Temple Bell, *The Search for Truth*, New York: Reynal and Hitchcock, 1934). Similar ideas were put forward by C. F. Gauss, who did not, however, publish his work.

In Gaussian–Lobachevskian geometry, known as hyperbolic geometry, Euclid's parallel postulate is replaced by the so-called "hyperbolic axiom": for any given line L and point P not on L, in the plane containing both line L and point P, there exist at least two distinct lines through P that do not intersect L, as shown in Figure 5-19(a). In Euclidean geometry, the sum of the three interior angles of a triangle is always equal to π radians (i.e., 180^o, a straight line). In hyperbolic geometry, however, the sum of the three interior angles of a triangle is always strictly less than π radians, as shown in Figure 5-19(b). The difference is referred to as the "defect."

Figure 5-19: Hyperbolic Axiom and Hyperbolic Triangle.

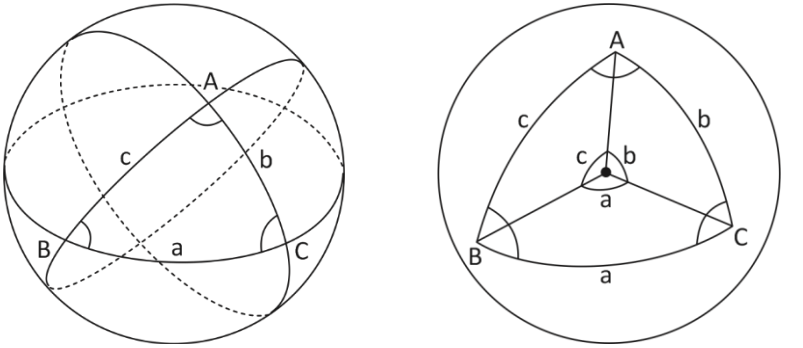


The renowned German mathematician Bernhard Riemann (1826–66), who was a student of Gauss, had the most profound insight in non-Euclidean geometry. In his investigations of the function theory, Riemann developed new methods based on geometric representation. Remember that $\mathbb{R}^n$ is the space of all ordered sets (called points or vectors) $x = (x_1, x_2, \dots, x_n)$ of n real numbers; the numbers $x_1, x_2, \dots, x_n$ are the coordinates of a point (or vector) x. We say that $\mathbb{R}^n$ is "standardly embedded" in $\mathbb{R}^{n+k}$ if a point $(x_1, x_2, \dots, x_n)$ from $\mathbb{R}^n$ is identified with the point $(x_1, \dots, x_n, 0, \dots, 0)$ from $\mathbb{R}^{n+k}$. In the 1850s, Riemann invented the concept of an abstract geometric surface that need not be embeddable in Euclidean three-dimensional space. On this surface, the "lines" can be interpreted as geodesics, and the intrinsic curvature of the surface can be precisely defined, as shown in Figure 5-20(a): a "geodesic" is the shortest path between two points on a curved surface (i.e., the non-Euclidean equivalent of a Euclidean straight line); like, for instance, on the surface of the Earth (e.g., airplanes, wishing to minimize the time that they spend on the air, do not follow Euclidean straight lines, but they follow shortest curves known as geodesics). In spherical geometry, "great circles," or

"geodesics," are intersections with planes through the center of the sphere. Thus, it is not quite true that, given any two points, there is a unique line through them, because, if one chooses two points on the sphere that are opposite, or "antipodal," then there is a whole family of great circles that go through them.

In general, Riemannian geometry is geometry on the ellipsoid or on the sphere; thus, it exists on surfaces that have constant positive curvature (Fig. 5-20). Gaussian–Lobachevskian geometry exists on surfaces that have constant negative curvature (Fig. 5-19). Euclidean geometry exists on surfaces that have constant zero curvature. This is the way in which modern geometers construe the reality of non-Euclidean planes. Therefore, whereas hyperbolic triangles are "thin" triangles (i.e., their angle sum is strictly less than 180°, as shown in Figure 5-19(b)), Riemannian triangles (i.e., triangles on the ellipsoid or on the sphere) are "fat" triangles (i.e., their angle sum is strictly greater than 180°, as shown in Figure 5-20(b)).

Figure 5-20: Riemannian Geometry on the Sphere (where "Lines" are Geodesics) and a Spherical Triangle.



(a) (b)

When the great French mathematician and philosopher Henri Poincaré (1854–1912) was asked which geometry is true, he answered as follows:

> If geometry were an experimental science, it would not be an exact science. It would be subjected to continual revision . . . The geometric axioms are therefore neither synthetic *a priori* intuitions [as Kant has contended] nor experimental facts [as Newton has assumed]. They are conventions. Our choice among all possible conventions is guided by experimental facts; but it remains free, and is only limited by the necessity of avoiding every contradiction, and thus it is that postulates may remain

> rigorously true even when the experimental laws which have determined their adoption are only approximate . . . One geometry cannot be more true than another: it can only be more convenient (Henri Poincaré, *Science and Hypothesis*, translated by Mélanie Frappier, Andrea Smith, and David J. Stump, London: Bloomsbury, 2017, p. 50).

Guided by Max Planck's research work in quantum physics, by Bernhard Riemann's research work in non-Euclidean geometry, and by Constantin Carathéodory's research work in mathematical analysis and the axiomatization of thermodynamics, Albert Einstein concluded that space and time are functions of each other, so that, by referring to space, we actually refer to a temporal correspondence of space, and vice versa. Einstein's theory of relativity implies that, in contrast to Newton's perception of a three-dimensional space, we should perceive a four-dimensional space whose fourth dimension is time. Thus, time is part of the substance of space; conversely, time is underpinned by the three classical dimensions of space. The experiments on which Einstein was based in order to articulate his general theory of relativity—according to which mass and energy are, in essence, mutually transformable forms of the same reality—presupposed the existence of a four-dimensional continuum (space-time) whose curvature is determined by gravity.

Astronomy, Celestial Mechanics, and Infinitesimal Calculus

"Infinitesimal calculus" is a branch of mathematical analysis that concerns itself with the systematic study of the concept of an "infinitely small function," a function of a variable x whose absolute value, $|f(x)|$, becomes and remains smaller than any given number as a result of variation of x. The method of the "infinitesimals" ("infinitely small" quantities) was originally used by ancient Greek mathematicians, who determined areas and volumes by the so-called "method of exhaustion," in which infinitesimal quantities are used in order to prove that two given magnitudes (or two ratios between given magnitudes) are equal.

The method of exhaustion was originally developed in the fifth century B.C.E. by the Athenian scholar Antiphon, and it was put in a rigorous scientific setting shortly afterwards by the Greek mathematician and astronomer Eudoxus of Cnidus, who used it in order to calculate areas and volumes. The Greek mathematician and acknowledged father of "Euclidean geometry" Euclid, and the Greek mathematician, physicist, and engineer Archimedes, made extensive use of the method of exhaustion in order to prove several mathematical propositions. For instance, as already mentioned, Archimedes used the method of exhaustion in order to

compute the area of a circle by approximating the area of a circle from above and below, by circumscribing and inscribing regular polygons of an increasingly larger number of sides (so that sides become "infinitesimals," namely, infinitely small). Moreover, Archimedes was able to calculate the length of various tangents to the spiral (i.e., to a curve emanating from a point moving farther away as it revolves around the point).

In few words, infinitesimal calculus, or simply calculus, is concerned with two kinds of problems: problems of tangents to curves, and problems of areas or volumes of regions. Thus, having studied both of these kinds of problem in a rigorous and systematic way, Archimedes can be considered to be the first pioneer of calculus. Some other great pioneers of calculus are the Flemish Jesuit and mathematician Gregory of Saint Vincent (1584–1667), the Dutch-French philosopher and mathematician René Descartes (1596–1650), the Italian mathematician and Jesuate Bonaventura Francesco Cavalieri (1598–1647), the French lawyer and amateur mathematician Pierre de Fermat (ca. 1607–65), the English clergyman and mathematician John Wallis (1616–1703), the English Christian theologian and mathematician Isaac Barrow (1630–77), and the Scottish mathematician and astronomer James Gregory (1638–75).

Infinitesimal calculus is primarily aimed at solving problems concerning "change." Thus, infinitesimal calculus is used in many fields, including physics, engineering, biology, economics, statistics, and the mathematical modelling of social, political, military, and psychological problems. In the seventeenth century, infinitesimal calculus was erected as a rigorous framework of science as a result of, and in the context of, the revolutionary achievements that took place in the scientific discipline of celestial mechanics, whose protagonists were Nicolaus Copernicus, Galileo Galilei, Tycho Brahe, Johannes Kepler, and Isaac Newton. In its contemporary rigorous form, calculus was formulated independently in England by Sir Isaac Newton and in Germany by Gottfried Wilhelm Leibniz in the last quarter of the seventeenth century, using the algebraic set-up and, especially, the Cartesian set-up, which had been introduced and developed by their predecessors. Calculus consists of "differential calculus" (which is concerned with problems of tangents to curves) and "integral calculus" (which is concerned with problems of areas or volumes of regions).

Limit and Continuity of a Function

Whenever, by a known value of one quantity, we can find the value of another quantity, we say that there is a "functional dependence" between these quantities. For instance, if the length x of the side of a square is

known, then its area can be found by the formula $A = x^2$. In this way, we specify the functional dependence between the length of the side of a square and its area.

As already explained, the specification of a "numerical function" requires a set of numbers X and a rule f, according to which every number x that belongs to the set X is associated with a certain number (the value of the function). An independent variable taking on values from the set X is said to be the "argument" of the function. Given a member a of the set X, the value of the function f for the argument a is denoted by $f(a)$.

If a function f is specified on a set X, then the set X is said to be the "domain" of this function, and the set of all the values of the function is said to be its "range." As already mentioned, a function $f: X \to Y$ assigns to each element $x \in X$ exactly one element $y \in Y$.

We can read the expression $y = f(x)$ as follows: "y is a function of x," meaning that, as the variable x varies, the variable y also varies according to some rule f; in this case, y is the dependent variable, and x is the independent variable.

Analytic representation of a function: Assume that we are given a collection of operations that must be performed with the argument x in order to obtain a function value. Then the function is said to be represented by an "analytic expression." For instance, consider the following functions: $y = x^2 + x + 1,\ x \in [0,1]$; $y = x^2 + x + 1,\ x \in [-2,3]$; and $y = x^2 + x + 1, x \in (-\infty, +\infty)$. Even though the analytic expressions of these functions are the same in form, we have three different functions, because they are defined on three different sets (their domains are different).

Graphical representation of a function: Assume that a function f is given by an analytic expression $f(x)$, that is, $y = f(x)$ with $x \in X$, where X is the corresponding real interval, on which f is defined. The "graph" of the function f is a set of points of the coordinate plane that have coordinates $\big(x, f(x)\big)$, where $x \in X$. If a function is even, then its graph is symmetric with respect to the axis of ordinates. If a function is odd, then its graph is symmetric about the origin.

A function $y = f(x)$ is defined to be "increasing" on its domain if, for any two of its points x_1 and x_2 such that $x_1 < x_2$, the inequality $f(x_1) < f(x_2)$ is satisfied; in other words, if to a greater value of the argument there corresponds a greater value of the function. A function $y = f(x)$ is defined to be "decreasing" on its domain if, for any two of its points x_1 and x_2 such that $x_1 < x_2$, the inequality $f(x_1) > f(x_2)$ is satisfied; in other words, if a smaller value of the function corresponds to a greater value of the argument.

A function f is said to have a "period" T if, for any value of x for which f is defined, the following equalities hold:
$f(x-T) = f(x) = f(x+T)$.
The aforementioned definition implies that, if a function f with period T is defined at the point x, it is also defined at the points $x+T$ and $x-T$. If a function f has a non-zero period T, then it is said to be "periodic." For instance, if time is measured in years, then the distance from the Earth to the Sun is given by a periodic function whose period is equal to 1.
One of the simplest functions is the "linear function" (or "linear equation"), where $y = mx + c$. In this, y and x are "variables" (that is, they can take on many values), while m and c are "constants" (that is, they have fixed values). As already explained, if we plot y against x on a diagram, the result will be a straight line, hence the name. A "non-linear function" ("non-linear equation") is any other sort of function (equation). For instance, $y = x^2$ is a quadratic equation that is downward-sloping for negative values of x and upward-sloping for positive values of x. Functions come in many forms, and they are very useful as models of the real world when they are simple or can be satisfactorily approximated by, or manipulated into simple forms.
The concept of a limit, or a limiting process, is central to all mathematical analysis. In fact, one can argue that, from the perspective of mathematical analysis, "analysis" means taking limits. In his book entitled *Cours d'analyse*, the French mathematician Augustin-Louis Cauchy (1789–1857), one of the founders of modern mathematical analysis, explained the concept of a limit of a function in a clear, formal, and arithmetic, rather than geometric, way by arguing as follows: "when the successive values attributed to a variable approach indefinitely a fixed value so as to end by differing from it by as little as one wishes, this last is called the limit of all the others" (quoted in: Carl B. Boyer, *The History of Calculus and Its Conceptual Development*, New York: Dover, 1959, p. 272).
Consider an arbitrary function $f(x)$ defined at all values in an open interval of the number line $\mathbb{R}$ containing a point x_0, with the possible exception of x_0 itself, and let L be a real number. The "limit of a function" $f(x)$ at a point x_0 is L if and only if the values of x (where $x \neq x_0$) approach the number x_0 (notice that $f(x_0)$ may not be defined, since, according to the definition of a limit, x tends to x_0, but x never becomes equal to x_0). In other words, as x gets closer to x_0, $f(x)$ gets closer and stays close to L; symbolically:
$lim_{x \to x_0} f(x) = L$.
Remark: Let a be a real number and c a constant. Then
$lim_{x \to a} x = a$, and

$lim_{x\to a} c = c$.

Let us recall that the distance between any two points a and b on the number line $\mathbb{R}$ is $|a - b|$. Therefore, the statement

$$|f(x) - L| < \varepsilon$$

means that the distance between $f(x)$ and L is less than ε, and, by the definition of an absolute value, the statement

$$0 < |x - a| < \delta$$

is equivalent to the statement

$$a - \delta < x < a + \delta, \text{ so that } x \neq a.$$

Thus, the *Cauchy epsilon-delta definition of a limit* is the following: assume that, for all $x \neq a$, an arbitrary function $f(x)$ is defined over an open interval containing a. Then

$$lim_{x\to a} f(x) = L$$

if and only if, for every $\varepsilon > 0$, there exists a $\delta > 0$ such that, if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$. The statement (with the universal quantifier) "for every $\varepsilon > 0$" means "for every positive distance ε from L"; the statement (with the existential quantifier) "there exists a $\delta > 0$" means that there is a positive distance δ from a; and the conditional statement "if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$" means that, if x is closer than δ to a, and $x \neq a$, then the value of $f(x)$ is closer than ε to L.

Consider a function f whose domain is D_f. Let a be an interior point of D_f. Then f is said to be "continuous at the point" a if

$lim_{x\to a} f(x)$ exists finitely and

$lim_{x\to a} f(x) = f(a)$,

meaning: if the limit of $f(x)$ as x tends to a is equal to the value of $f(x)$ at a. If a is a boundary point of D_f (i.e., in this case, an endpoint of a closed interval), then we distinguish the following two cases:

i. if $D_f = (x_1, a]$, then $f(x)$ is said to be "continuous from the left" at a if $lim_{x\to a^-} f(x) = f(a)$;
ii. if $D_f = [a, x_2)$, then $f(x)$ is said to be "continuous from the right" at a if $lim_{x\to a^+} f(x) = f(a)$.

The aforementioned definition of continuity can also be given in the following equivalent forms:

(i) A function f is continuous at $a \in D_f$ if and only if, for every sequence (x_n) with $lim_{n\to\infty} x_n = a$, where $x_n \in D_f$, it holds that $lim_{n\to\infty} f(x_n) = f(a)$. An infinite sequence (x_n) of real numbers $x_1, x_2, \ldots, x_n$ has a limit a if and only if the distance $|x_n - a|$ tends to zero as the indices of the terms of this sequence become greater than some value n_0. This means that, after a finite set of n_0 terms of this sequence, the remaining infinitely many terms of

the given sequence, namely, $x_{n_0+1}, x_{n_0+2}, x_{n_0+3}, \ldots$, converge indefinitely to the value a. The sequential definition of continuity was originally developed by the German mathematician Heinrich Eduard Heine (1821–81).

(ii) A function f is continuous at $x = a \in D_f$ if and only if:
$\forall \varepsilon > 0, \exists \delta > 0 | |x - a| < \delta \Rightarrow |f(x) - f(a)| < \varepsilon$.

A function f is said to be "continuous over (or on, or in) an open interval" (x_1, x_2) if f is continuous at every point in that interval (x_1 may be $-\infty$, and/or x_2 may be $+\infty$). A function f is said to be "continuous over (or on, or in) the closed interval" $[x_1, x_2]$ if the following conditions hold: firstly, f is continuous at every x in the open interval (x_1, x_2); secondly, $f(x_1)$ and $f(x_2)$ both exist; and, thirdly, $lim_{x \to x_1^+} f(x) = f(x_1)$, and $lim_{x \to x_2^-} f(x) = f(x_2)$.

If we compare the definition of the limit of a function with the definition of the continuity of a function, we realize that they have the same structure, but they also have the following differences:

i. In the case of the limit of a function (Cauchy epsilon-delta definition), we have $0 < |x - a| < \delta$, or $x \neq a$, whereas, in the case of continuity, we have only $|x - a| < \delta$, meaning that the definition of continuity holds also when $x = a$.

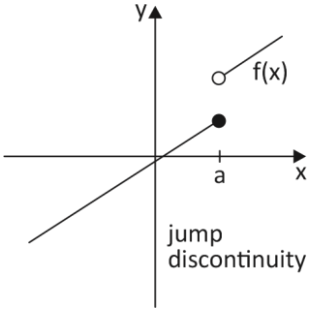
ii. Instead of the value L that is used in the definition of the limit of a function, the definition of the continuity of a function uses the value $f(a)$, meaning that, in the case of the continuity of a function, the function must be defined at the point a. Indeed, it is meaningless to talk about the continuity (or the discontinuity) of a function at a point that does not belong to its domain of definition.

iii. In the definition of the limit of a function (Cauchy epsilon-delta definition), the point a must be an accumulation point of the domain of definition D_f of the corresponding function. Therefore, it may not belong to D_f. In the definition of the continuity of a function, meanwhile, the point a must belong to the domain of definition D_f of the corresponding function.

In intuitive terms, a function is said to be continuous if it varies with no abrupt breaks or jumps. Hence, points of continuity are characterized by the fact that, for small changes in the argument, the value of the function changes but little, whereas points of discontinuity are characterized by the fact that, for small changes in the argument, the function can change considerably. For instance, consider a load that is suspended on a thread above a table. Due to this load (supposed to be a material particle), the thread extends, and the distance l from the load to the point of thread

suspension is a function of the mass m of the load, symbolically, $l = f(m)$, where $m \geq 0$. For small changes in the mass of the load, the distance l will change but little. But, if the mass of the load approaches the tensile strength m_0 of the thread, then a small increase in the mass of the load may cause a break in the thread. Thus, the distance l will increase jump-wise and become equal to the distance L from the suspension point to the surface of the table. On the half-closed interval $[0, m_0)$, the graph of the function $l = f(m)$ is a continuous line, and, at the point m_0, it suffers a discontinuity. Consequently, we get a graph consisting of two branches: at all points except m_0, the function $l = f(m)$ is continuous, in the sense that it exhibits a smooth change. At the point m_0, however, it has a discontinuity, in the sense that it exhibits a jump-wise change. In Figure 5-21, we see an example of a "jump discontinuity."

Figure 5-21: Jump Discontinuity.



Differential Calculus

Assume that a function $y = f(x)$ is defined at the points x and x_1. The difference $x_1 - x$ is called the "increment of the argument," and it is denoted by Δx. The difference $f(x_1) - f(x)$ is called the "increment of the function," and it is denoted by Δf or Δy. Therefore, $\Delta x = x_1 - x \Leftrightarrow x_1 = x + \Delta x$, and $\Delta f = f(x_1) - f(x) = f(x + \Delta x) - f(x)$. Using this formula, we can compute the value of Δf for any given x and Δx. Moreover, notice that a function $y = f(x)$ is continuous at a point $x = a$ if and only if $lim_{\Delta x \to 0} \Delta f = 0$, where $\Delta x = x - a$ and $\Delta f = f(x) - f(a)$. Assume that, for function $y = f(x)$, at a given point x, there exists the limit of the ratio of the increment of the function, Δf, to the increment of the argument, Δx, provided that $\Delta x \to 0$. Then the function $y = f(x)$ is said to be "differentiable at the point x," and this limit is called the

"derivative of the function $y = f(x)$" at the point x, and it is denoted by $\frac{df(x)}{dx}$, or $f'(x)$, or y'. Symbolically:

$\frac{df(x)}{dx} \equiv f'(x) \equiv y' = lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$.

Notice that $f'(x)$ is a new function defined at every such point x at which the indicated limit exists; this function is called the "derivative of the function $y = f(x)$," and it measures the rate of change of y with regard to x.

Let $s = s(t)$ denote the distance travelled by a point moving in a straight line on which a reference point, the unit of measurement, and the direction are chosen (notice that $s(t)$ is the position of the point on the straight line at instant t). In physics, the "average velocity" of motion during a time interval is defined as the ratio of the net displacement to the elapsed time—that is, the average velocity during the time interval from t_1 to t_2 is expressed by the quantity

$v_{av} = \frac{s(t_2)-s(t_1)}{t_2-t_1}$.

If we set $t_1 = t$, then we obtain

$v_{av} = \frac{s(t+\Delta t)-s(t)}{\Delta t} = \frac{\Delta s}{\Delta t}$.

Suppose that the average velocity of a particle is measured for a number of different time intervals, and that it is not constant. In other words, the particle under consideration is moving with varying velocity. We then have to compute the velocity of the particle at any given instant of time. This is called the instantaneous velocity. The (numerical value of the) "instantaneous velocity," or the (numerical value of the) velocity at instant t, is defined as the limit of the average velocity of motion during the time interval $[t, t + \Delta t]$ provided that $\Delta t \to 0$, symbolically:

$v_{inst} = lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$,

which is the derivative of displacement $s = s(t)$ with respect to time,

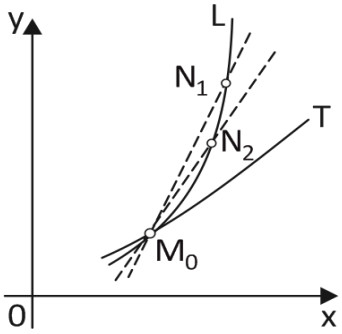
$v_{inst} = \frac{ds(t)}{dt}$.

Similarly, we can compute the instantaneous rate of change of any other physical or (quantifiable) socio-economic phenomenon with respect to its independent variable. For instance, in economics, inflation is defined as the derivative of price (as a function of time) with respect to time; the rate of change of demand with respect to price is defined as the derivative of the quantity demanded (as a function of price) with respect to price (dQ/dP, where $Q = f(P)$); and the point price-elasticity of demand, which measures the degree to which the desire for something changes as its price changes within the same demand curve, is equal to the absolute

value of the derivative of the quantity demanded with respect to price multiplied by the point's price divided by its quantity ($\left|\frac{dQ}{dP}\right| \frac{P}{Q}$).

Given a function $y = f(x)$, we realize that, in order to find the rate of change of y with regard to x at a particular point, we need to find the slope of the tangent line to the curve at that point. In differential calculus, a main objective is to try to understand tangents to curves, as illustrated in Figure5-22. Hence, it is important to define a tangent line to an arbitrary plane curve in a rigorous way. A tangent line cannot be rigorously defined as a straight line having only one common point with the corresponding curve. In order to define a tangent line to an arbitrary plane curve in a rigorous way, we must use the concept of a limit. Let L be an arc of some curve, and M_0 be a point of this curve. We draw a secant M_0N through the point M_0. If the point N, moving in the curve, approaches the point M_0, then the secant M_0N turns about the point M_0. Thus, it may so happen that, as the point N approaches M_0, the secant tends to a certain limit position M_0T, so that M_0T is referred to as the "secant" to the curve L at the point M_0, as illustrated in Figure 5-22. Then the "tangent line" to the curve L at the point M_0 is defined as the limit position of the secant M_0N as $N \to M_0$.

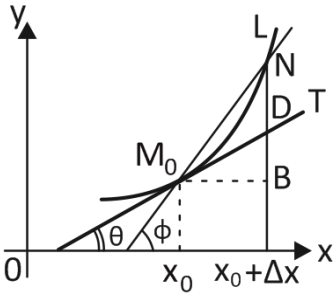
Figure 5-22: A tangent line to a curve.



Let us try to compute the slope of the tangent line for the case when the curve L is the graph of a certain function $y = f(x)$. Let M_0 be a point of the graph with abscissa x_0 and ordinate $y_0 = f(x_0)$. Assuming that the tangent line to the curve L at the point M_0 does exist, we take one more point $N(x_0 + \Delta x, y_0 + \Delta y)$ on the curve, as illustrated in Figure 5-23, and we draw a straight line through the points M_0 and N. If φ is the slope of this secant to the positive direction of the x-axis, then

$|BN| = \Delta y$, $|M_0B| = \Delta x$, and $tan\varphi = \frac{|BN|}{|M_0B|} = \frac{\Delta y}{\Delta x}$,
so that the slope of this secant is $k_{tan} = lim_{N \to M_0} tan\varphi = lim_{\Delta x \to 0} tan\varphi$.
If we denote the slope of the tangent line to the axis of abscissas with θ, as shown in Figure 5-23, then the slope of the tangent line is
$k_{tan} = tan\theta = lim_{\Delta x \to 0} tan\varphi = lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$.

Figure 5-23: The slope of a tangent line.



Consequently, in order to draw a non-vertical tangent line to the graph of the function $y = f(x)$ at a point with abscissa x_0, it is necessary and sufficient that, at this point, the limit $lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ exists finitely; in fact, this limit is equal to the slope of the tangent line. In other words, we create an infinite sequence of slopes, and then we say that the slope of the given tangent line is the infinite limit of this sequence. Hence, infinitesimal calculus provides us with abstract objects (such as a tangent to a curve) at which only infinite tasks can arrive through the concept of a limit. The concept of a limit has a deep philosophical significance, because it secures the theoretical convenience of being able to do an infinite number of tasks through a theoretical concept—namely, that of a limit—without actually doing each one of them, which would be practically impossible. This abstraction underpins the foundations of calculus as it was articulated by Newton and Leibniz in the seventeenth century. In view of the foregoing, the slope of the tangent line to the graph of a function $y = f(x)$ at the point x_0 is equal to the value of the derivative at the point of tangency; symbolically: $k_{tan} = f'(x)$. *This is the geometric significance of the derivative.*

Basic rules of differentiation: Let $X \subseteq \mathbb{R}$ be an interval, $a \in X$, and $f: X \to \mathbb{R}$ and $g: X \to \mathbb{R}$ be functions that are differentiable at a. Then the following relations hold:

If $k \in \mathbb{R}$, then the function kf is differentiable at a, and
$(kf)'(a) = kf'(a)$.
The function $f + g$ is differentiable at a, and
$(f + g)'(a) = f'(a) + g'(a)$.
The function $f \cdot g$ is differentiable at a, and
$(f \cdot g)'(a) = f'(a)g(a) + f(a)g'(a)$.
If $g(a) \neq 0$, then the function $\frac{f}{g}$ is differentiable at a, and
$\left(\frac{f}{g}\right)'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{g(a)^2}$.

Power rule: $\frac{d}{dx}x^n = nx^{n-1}$, where n is an arbitrary real number.

Obviously, $\frac{d}{dx}(c) = 0$ for any constant c; and $\frac{d}{dx}(x) = 1$ for any $x \in \mathbb{R}$.

Differentiation of a composite function.: $\Big(f\big(g(x)\big)\Big)' = f'\big(g(x)\big) \cdot g'(x)$.

In other words, if $y = y\big(u(x)\big)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

Higher order derivatives: It is evident that the first derivative $\frac{dy}{dx}$ expresses the rate of change of y with respect to x (e.g., velocity). Then $\frac{d}{dx}\left(\frac{dy}{dx}\right) \equiv \frac{d^2y}{dx^2} \equiv y''$ expresses the rate of change of the first derivative of y with respect to x (e.g., acceleration), and $\frac{d^3y}{dx^3} \equiv y''' \equiv y^{(3)}$ expresses the rate of change of the second derivative of y with respect to x (e.g., jerk). Of course, we can compute the nth derivative of $y = f(x)$, denoted by $\frac{d^ny}{dx^n} \equiv y^{(n)}$, where n is called the order of the derivative.

Basic differentiation formulas (following directly from the definition of the derivative of a function):

i. $\frac{d}{dx}(a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0) = a_n \cdot nx^{n-1} + a_{n-1} \cdot (n-1)x^{n-2} + \cdots + a_1$.
ii. $\frac{d}{dx}(e^x) = e^x$.
iii. $\frac{d}{dx}(lnx) = \frac{1}{x}$.
iv. $\frac{d}{dx}(a^x) = a^x lna$.
v. $\frac{d}{dx}(x^x) = x^x(1 + lnx)$; notice that $y = x^x \Leftrightarrow lny = lnx^x = xlnx$ (and we apply the product rule for derivatives).
vi. $\frac{d}{dx}(log_ax) = \frac{1}{xlna}$.
vii. $\frac{d}{dx}(sinx) = cosx$.
viii. $\frac{d}{dx}(cosx) = -sinx$.

ix. $\frac{d}{dx}(tanx) = \frac{1}{cos^2x} = sec^2x.$
x. $\frac{d}{dx}(cotx) = -\frac{1}{sin^2x} = -csc^2x.$

Investigation of the behavior of a function using differential calculus: If a function $y = f(x)$ is differentiable on an interval (a, b), then:

i. f is increasing on the interval (a, b) if and only if its derivative is non-negative in this interval; symbolically: $f'(x) \geq 0 \; \forall \; x \in (a, b)$;
ii. f is decreasing on the interval (a, b) if and only if its derivative is non-positive in this interval; symbolically: $f'(x) \leq 0 \; \forall \; x \in (a, b)$.

Geometric significance: A differentiable function increases where its graph has positive slopes, and decreases where its graph has negative slopes. If $f'(x) = 0$, then $f(x)$ is constant (in a sense, it increases and decreases *simultaneously*).

We often have to solve optimization problems—that is, to choose from various variants the best one for some reasons. For instance, builders must know how to select the dimensions of a square beam in order to ensure its best tensile strength, aircraft builders must know what orbit ensures minimum fuel consumption, agronomists must know what seeding rate will guarantee the richest harvest, logistics managers must know how to minimize the transportation cost, production managers must know how to minimize costs and maximize utility, artillery officers must know what inclination of a gun tube will result in the greatest range of fire, and so on. Most optimization problems reduce to finding the extreme values, meaning the greatest and the lowest values, of a function.

Assume that a function $y = f(x)$ is continuous at a point $x = c$, and that there exists a neighborhood $(c - \delta, c + \delta)$ of this point such that the inequality $f'(x) > 0$ holds in the interval $(c - \delta, c)$, and the inequality $f'(x) < 0$ holds in the interval $(c, c + \delta)$. Then $x = c$ is a "point of maximum" for $f(x)$. In other words, if $f(x)$ increases in the interval $(c - \delta, c)$ to the left of c, and decreases in the interval $(c, c + \delta)$ to the right of c, then $x = c$ is a "point of maximum" for $f(x)$.

On the other hand, assume that a function $y = f(x)$ is continuous at a point $x = c$, and that, for some $\delta > 0$, it holds that $f'(x) < 0$ in the interval $(c - \delta, c)$, and $f'(x) > 0$ in the interval $(c, c + \delta)$. Then $x = c$ is a "point of minimum" for $f(x)$. In other words, if $f(x)$ decreases in the interval $(c - \delta, c)$ to the left of c, and increases in the interval $(c, c + \delta)$ to the right of c, then $x = c$ is a "point of minimum" for $f(x)$.

Consequently, we obtain the following algorithm for investigating a function $y = f(x)$ for an extremum (maximum or minimum):

i. Find the derivative $f'(x)$.
ii. Find the critical points, that is, the points at which the function is continuous and the derivative $f'(x)$ is either equal to zero or does not exist.
iii. Consider the neighborhood of each critical point found that does not contain another critical point and investigate the sign of the derivative to the left and to the right of the critical point under consideration.
iv. Using the aforementioned sufficient conditions for a maximum and a minimum, draw relevant conclusions (when passing through a maximum, the derivative changes sign from plus to minus, whereas, when passing through a minimum, the derivative changes sign from minus to plus).

For instance, let us investigate the function $f(x) = x^3 - 9x^2 + 24x$ for an extremum. We work as follows:

i. We have $f'(x) = 3x^2 - 18x + 24$.
ii. Equating the derivative to zero, we find the two roots (solutions) of the equation $3x^2 - 18x + 24 = 0$, namely: $x_1 = 2$ and $x_2 = 4$ (the curve has horizontal tangents at these values). In this case, the derivative is defined everywhere, and, therefore, there are no other critical points.
iii. We study the behavior of the function in a neighborhood of the point $x_1 = 2$ and in a neighborhood of the point $x_2 = 4$. We see the following: when passing through the point $x_1 = 2$, the derivative changes sign from plus to minus, whereas, when passing through the point $x_2 = 4$, the derivative changes sign from minus to plus.
iv. At $x_1 = 2$, the function has a maximum $y_{max} = 20$. At $x_2 = 4$, the function has a minimum $y_{min} = 16$.

Note that, if x is a critical point of $f(x)$ and the second derivative of $f(x)$ is positive (resp. negative), then x is "local minimum" (resp. "local maximum") of $f(x)$.

A function $f(x)$ is said to be "concave up" on an interval X if all the tangents to $f(x)$ on X are below the graph of $f(x)$, as shown, for instance, in Figure 5-24 (i.e., it "opens" up). A function $f(x)$ is said to be "concave down" on an interval X if all the tangents to $f(x)$ on X are above the graph of $f(x)$, as shown, for instance, in Figure 5-25 (i.e., it "opens" down).

Figure 5-24: A Concave-Up Function.

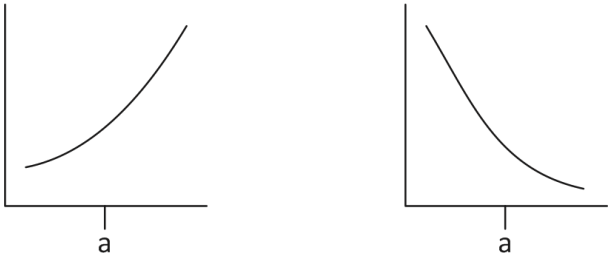
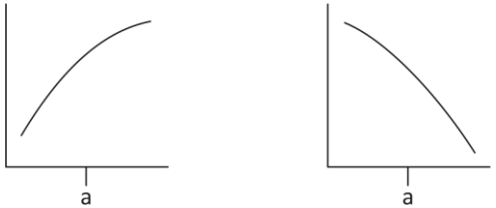


Figure 5-25: A Concave-Down Function.



Let f be a function differentiable on (a, b). (i) If f' is increasing (namely, if $f''(x) > 0$ on (a, b)), then f is concave up on (a, b). (ii) If f' is decreasing (namely, if $f''(x) < 0$ on (a, b)), then f is concave down on (a, b). (iii) If f' is constant, then the graph of f has no concavity.

If $f: (a, b) \to \mathbb{R}$ changes its direction of concavity at x_0, then the point $(x_0, f(x_0))$ is said to be a "point of inflection." In other words, x_0 is a point of inflection if $x_0 \in (a, b)$ so that either f is concave down in (a, x_0) and concave up in (x_0, b), or f is concave up in (a, x_0) and concave down in (x_0, b).

Rolle's Theorem: Let $f: [a, b] \to \mathbb{R}$ be a function satisfying the following conditions:

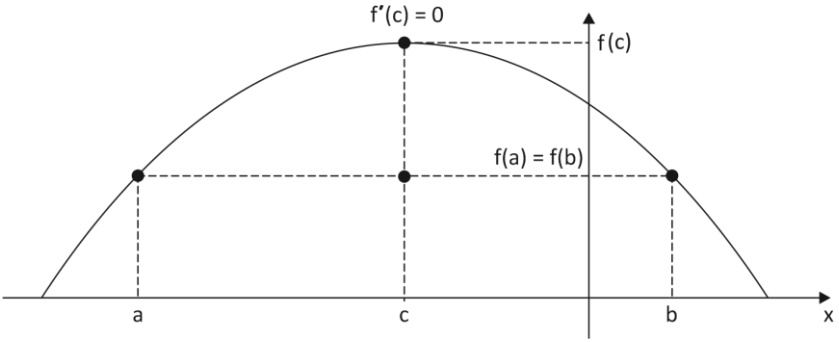
i. f is continuous on the closed interval $[a, b]$,
ii. f is differentiable on the open interval (a, b), and
iii. $f(a) = f(b)$.

Then there exists at least one point $c \in (a, b)$ such that $f'(c) = 0$.

Geometric interpretation of Rolle's Theorem: Under the above conditions, there exists a point c at which the tangent line to the graph of $y = f(x)$ is parallel to the x-axis, as shown in Figure 5-26. In particular, conditions (i) and (ii) imply that the curve $y = f(x)$ is continuous from $x = a$ to $x = b$, and it has a definite tangent at each point between $x = a$ and $x = b$; and condition (iii) implies that the ordinates at the endpoints a and b are equal.

Algebraic interpretation of Rolle's Theorem: Since, according to condition (iii), $f(a) = f(b)$, let $f(a) = f(b) = 0$. Then Rolle's Theorem means that, if $f(x)$ is a polynomial in x, and if a and b are two roots of the equation $f(x) = 0$, then the equation $f'(x) = 0$ has at least one root between a and b. In fact, the French mathematician Michel Rolle, after whom the above theorem is named, proved the given theorem in 1691 only in the case of polynomial functions, and a general proof of this theorem was achieved and published by Augustin-Louis Cauchy in 1823. The name "Rolle's Theorem" was first used by the German mathematician, logician, psychologist, and philosopher Moritz Wilhelm Drobisch in the 1830s.

Figure 5-26: Rolle's Theorem.



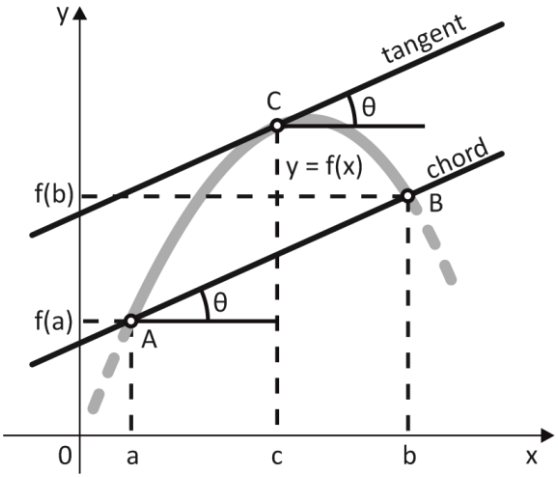
In mathematical analysis, the mean value theorems play a very important role, because they examine the relationship between the values of a function and the values of the derivative of the given function. The Italian-French mathematician and astronomer Joseph-Louis Lagrange (1736–1813) has proved the following mean value theorem, which allows us to express the increment of a function on an interval through the value of the derivative at an intermediate point of the corresponding segment:

Lagrange's Mean Value Theorem: If $f: [a, b] \to \mathbb{R}$ is a function continuous on $[a, b]$ and differentiable on (a, b), then there exists a $c \in (a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Geometric interpretation of Lagrange's Mean Value Theorem: As shown in Figure 5-27, Lagrange's Mean Value Theorem implies that the slope of the chord passing through the points of the graph corresponding to the ends of the segment a and b is equal to $k = tan\theta = \frac{f(b)-f(a)}{b-a}$, and then there exists a point $x = c$ inside the closed interval $[a, b]$ such that the

tangent to the graph at $x = c$ is parallel to the chord. In other words, if a function f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a point c in the interval (a, b) such that $f'(c)$ is equal to the function's average rate of change over $[a, b]$.

Figure 5-27: Lagrange's Mean Value Theorem.



For instance, given $f(x) = x^2 + x + 1$, if we are asked to find the point c at which $f'(x)$ gets its mean value over $[0,2]$, then we work as follows: we confirm that the hypotheses of Lagrange's Mean Value Theorem are satisfied, and, therefore, $\exists c \in (a, b) | \frac{f(b)-f(a)}{b-a} = f'(c) \Rightarrow \frac{f(2)-f(0)}{2-0} = 3 = f'(c) = 2c + 1 \Rightarrow c = 1$.

Optimization: (i) If we are enclosing a rectangular field (whose length is x and whose width is y) with $100\ ft$ of fence material, and one side (x) of the field is a building, then we can determine the dimensions that will maximize the enclosed area as follows: we have to maximize the function $A = xy$ subject to the constraint of $x + 2y = 100$. Hence, $x = 100 - 2y$, and then $A = (100 - 2y)y = 100y - 2y^2$. We differentiate and find the critical point(s). In fact, $A' = 100 - 4y$; thus, the critical point is $y = 25$, and this is a maximum, since $A''(25) < 0$. Finally, we find x as follows: $x = 100 - 2(25) = 50$. Consequently, the required dimensions are 50×25. (ii) We can determine the points on $y = x^2 + 1$ that are closest to the point $(0,2)$ as follows: we have to minimize the distance function

$f = d^2 = (x-0)^2 + (y-2)^2$ subject to the constraint of $y = x^2 + 1$. Hence, $x^2 = y - 1$, and then the distance function becomes $f = y^2 - 3y + 3$. We differentiate and find the critical point(s). In fact, $f' = 2y - 3$; thus, the critical point is $y = \frac{3}{2}$, and this is a minimum, since $f''\left(\frac{3}{2}\right) > 0$. Finally, we find x as follows: $x^2 = \frac{3}{2} - 1 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$. Consequently, the points on $y = x^2 + 1$ that are closest to the point $(0,2)$ are $\left(\frac{1}{\sqrt{2}}, \frac{3}{2}\right)$ and $\left(-\frac{1}{\sqrt{2}}, \frac{3}{2}\right)$.

The curvature of a curve: By the term "curvature," we refer to the measure of how sharply a curve bends. If $y = f(x)$ is a plane curve, then the curvature at any point $P(x, y)$ is expressed in terms of the first and the second derivatives of the function $f(x)$ by the formula

$$K = \frac{|f''(x)|}{[1 + (f'(x))^2]^{\frac{3}{2}}}$$

where *K* characterizes the speed of rotation of the tangent to the curve at the given point. Curvature is one of the key concepts of differential geometry. Differential geometry is a combination of calculus and analytic geometry applied to curves and surfaces. The pioneers of differential geometry are C. Huygens, A. C. Clairaut, L. Euler, A.-L. Cauchy, and G. Monge. In the twentieth century, curvature played a very important role in the development of modern physics due to the theory of relativity.

Integral Calculus

As already mentioned, in calculus, we start with two general questions about functions. Firstly, how steep is a function at a point? Secondly, what is the area underneath a graph over some region? The first question is answered using a tool called the "derivative." In other words, the derivative measures the rate of change of a function at a point. The second question is answered using a tool called the "integral."

Integration can be construed as the inverse of differentiation. Let $f: I \to \mathbb{R}$ be a function, where I is an interval; in fact, I may have one of the following forms:

$[a, b], [a, b), (a, b], (a, b), [a, +\infty), (a, +\infty), (-\infty, b], (-\infty, b), (-\infty, +\infty)$

where $a, b \in \mathbb{R}$.

If $F: I \to \mathbb{R}$ is a function such that $F'(x) = f(x)\ \forall x \in I$, then F is called the "antiderivative" of f in I, and it is denoted by

$F(x) = \int f(x)dx$, where $x \in I$,

according to Leibniz's notation. In other words, $\int f(x)dx = F(x) + c$ if and only if $[F(x) + c]' = f(x)$. The aforementioned definition implies that the "indefinite integral" of a given function with respect to x is a new

function plus a constant if and only if the derivative of the new function and of the constant equals the given function. Thus, differentiation can be used in order to verify the result of an integral.

Examples:

i. $\int adx = ax + c$, because $(ax + c)' = a$;

ii. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ over the following intervals: (i) $n \neq -1, x > 0$; (ii) $n \neq -1, x < 0$; and (iii) $n \geq 0, x \in \mathbb{R}$. For instance, $\int \sqrt{x}dx = \int x^{1/2}dx = \frac{x^{3/2}}{\frac{3}{2}} + c = \frac{2}{3}x^{3/2} + c$, and $\int xdx = \frac{x^2}{2} + c$. If $n = -1$, then $\int x^n dx = ln|x| + c$; i.e., $\int \frac{dx}{x} = ln|x| + c$;

iii. $\int a^x dx = \frac{a^x}{lna} + c$;

iv. $\int lnaxdx = x(lnax - 1) + c$;

v. $\int log_a xdx = xlog_a x - \frac{x}{lna} = \frac{xlnx - x}{lna}$;

vi. $\int sinxdx = -cosx + c$;

vii. $\int cosxdx = sinx + c$;

viii. $\int tanxdx = -ln|cosx| + c$ (notice that $\int tanxdx = \int \frac{sinx}{cosx}dx$ and set $u = cosx$ and $du = -sinxdx$, so that $\int \frac{sinx}{cosx}dx = -\int \frac{du}{u} = -ln|u| + c$, where $u = cosx$).

ix. $\int cotxdx = ln|sinx| + c$.

Let $f: I \to \mathbb{R}$ and $g: I \to \mathbb{R}$ be two functions. If their indefinite integrals exist over I, then there exists the indefinite integral of $af + bg$, where a and b are constants, and

$\int [af(x) + bg(x)]dx = a\int f(x)dx + b\int g(x)dx$.

If the integral includes the expression $\sqrt{a^2 - x^2}$, then we set $x = |a|sin\theta$ or $x = |a|cos\theta$, so that: (i) if $x = |a|sin\theta$, then $dx = |a|cos\theta d\theta$ and $\sqrt{a^2 - x^2} = |a|cos\theta$; (ii) if $x = |a|cos\theta$, then $dx = -|a|sin\theta d\theta$ and $\sqrt{a^2 - x^2} = |a|sin\theta$.

If the integral includes the expression $\sqrt{a^2 + x^2}$, then we set $x = |a|tan\theta$ or $x = |a|cot\theta$. If $x = |a|tan\theta$, then $dx = \frac{|a|}{cos^2\theta}d\theta$ and $\sqrt{a^2 + x^2} = \frac{|a|}{cos\theta}$.

If the integral includes the expression $\sqrt{x^2 - a^2}$, then we set $x = |a|\frac{1}{cos\theta}$, so that $dx = |a|\frac{sin\theta}{cos^2\theta}d\theta$ and $\sqrt{x^2 - a^2} = |a|\frac{sin\theta}{cos\theta} = |a|tan\theta$.

If the integral includes the expression $\sqrt{ax + b}$, then we set $\sqrt{ax + b} = t$.

Integration by parts: $\int udv = uv - \int vdu$,

where: $u = u(x)$ and $du = u'(x)dx$ while $v = v(x)$ and $dv = v'(x)dx$.

For instance, given the integral $\int xe^x dx$, we set $u = x$ and $dv = e^x dx$. Then $du = dx$ and $v = \int e^x dx = e^x$. Hence, $\int xe^x dx = \int x de^x = xe^x - \int e^x dx = xe^x - e^x + c$.

The "definite integral" is written as

$$\int_a^b f(x)dx$$

and represents the area bounded by the curve $y = f(x)$, the x-axis, and the ordinates $x = a$ and $x = b$ if $f(x) \geq 0$. If $f(x)$ is sometimes positive and sometimes negative, then the definite integral represents the algebraic sum of the areas above and below the x-axis. In particular, the areas that are above the x-axis are considered to be positive, whereas the areas that are below the x-axis are considered to be negative.

As shown in Figure 5-28, the definite integral $\int_a^b f(x)dx$ can be defined as follows:

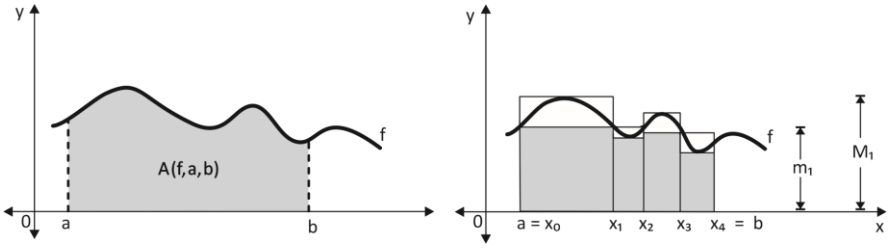
We subdivide the closed interval $[a, b]$ into n subintervals

$[a, x_1], [x_1, x_2], \dots, [x_{k-1}, x_k], \dots, [x_{n-1}, b]$

by means of the points $x_1, x_2, \dots, x_{n-1}$, which have been chosen arbitrarily. Hence, the set of points $P = \{a = x_0, x_1, x_2, \dots, x_{k-1}, x_k, \dots, x_{n-1}, x_n = b\}$ is a "partition" of

$[a, b]$. Let Δx_k be the length of the kth subinterval, that is, $\Delta x_k = x_k - x_{k-1}$. Then the "norm" of partition P is denoted by $\|P\|$, and it is equal to $max\{\Delta x_k | k = 1,2, \dots, n\}$.

Figure 5-28: The Integral as the Limit of a Sum.



In each of the n subintervals mentioned in the aforementioned partition, we choose points $c_1, c_2, \dots, c_n$ in an arbitrary way, and we form the sum

$$S(P, f, c_k) = f(c_1)\Delta x_1 + f(c_2)\Delta x_2 + \cdots + f(c_k)\Delta x_k + \cdots + f(c_n)\Delta x_n = \sum_{k=1}^{n} f(c_k)\Delta x_k.$$

Notice that, as the number of subdivisions n increases, $\|P\|$ vanishes—that is, $\|P\| \to 0$ as $n \to \infty$. Hence, if $lim_{\|P\| \to 0} S(P, f, c_k)$ exists and is

independent of the mode of subdivision of $[a, b]$, then this limit is said to be the integral of f on $[a, b]$; symbolically:

$$lim_{\|P\|\to 0} S(P, f, c_k) = \int_a^b f(x)dx$$

where $f(x)dx$ is called the "integrand," $[a, b]$ is called the "range of integration," and a and b are called the lower and the upper "limit of integration" respectively. Notice that the aforementioned limit exists if $f(x)$ is continuous (or sectionally continuous) on $[a, b]$. Leibniz symbolized the definite integral of a function $f(x)$ on $[a, b]$ as $\int_a^b f(x)dx$, because the sign $\int$ is an elongated S standing for the word "sum," since Leibniz defined $\int_a^b f(x)dx$ as the sum of infinitely many rectangles of height $f(x)$ and infinitesimally small width dx.

Example: We calculate the value of $\int_2^3 x^2 dx$ as follows:

$$\int_2^3 x^2 dx = \frac{x^3}{3}|_2^3 =$$

$$\left(value\ of \frac{x^3}{3} when\ x = 3\right) - \left(value\ of \frac{x^3}{3} when\ x = 2\right) = \frac{3^3}{3} - \frac{2^3}{3} = \frac{19}{3}.$$

The average value of a function $f(x)$ on the closed interval $[a, b]$ is

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

(where$b - a$ is the length of the interval over which we are averaging, and $\int_a^b f(x)\,dx$ is the area underneath the curve that we are averaging).

When the integrand $f(x)$ is known only at certain points (e.g., those obtained by sampling), or when a formula for the integrand is known but it is difficult or impossible to find an antiderivative that is an elementary function, we may use numerical methods of integration—that is, approximate formulas for definite integrals. The simplest approximate formula for definite integrals is

$\int_a^b f(x)dx \approx \frac{1}{2}(b-a)[f(a) + f(b)]$,

which is exact when $f(x)$ is linear. However, a much better approximate formula for definite integrals is

$\int_a^b f(x)dx \approx \frac{1}{6}(b-a)\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$,

which is known as "Simpson's Rule" (named after the eighteenth-century British mathematician Thomas Simpson, who formulated it. Before him, however, Johannes Kepler had already used similar formulas. For this reason, "Simpson's Rule" is sometimes called "Kepler's Rule"). Simpson's Rule derives from the observation that, if $p(x) = Ax^2 + Bx +$

C, then $\int_a^b p(x)\,dx = \frac{b-a}{6}\left[p(a) + 4p\left(\frac{a+b}{2}\right) + p(b)\right]$, and it is used in order to approximate any integral $\int_a^b f(x)dx$, where f is an arbitrary function, and not necessarily a quadratic polynomial (i.e., parabola).

The calculation of the area between two arbitrary curves: In the first case, we want to determine the area A between the equations $y = f(x)$ and $y = g(x)$ over the interval $[a, b]$ under the assumption that $f(x) \geq g(x)$, meaning that the graph of $f(x)$ is above the graph of $g(x)$. Then

$A = \int_a^b [(upperfunction) - (lowerfunction)]dx = \int_a^b [f(x) - g(x)]\,dx,$

where $a \leq x \leq b$.

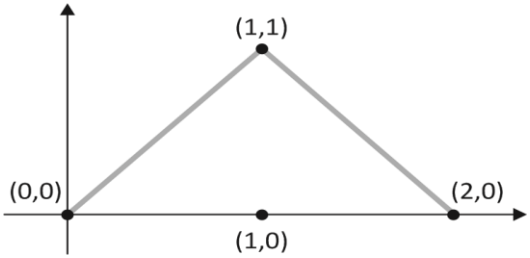
In the second case, we want to determine the area A between the equations $x = f(y)$ and $x = g(y)$ over the interval $[c, d]$ under the assumption that $f(y) \geq g(y)$, namely, $x = f(y)$ is on the right-hand side of $x = g(y)$. Then

$A = \int_c^d [(rightfunction) - (leftfunction)]dy = \int_c^d [f(y) - g(y)]\,dy,$

where $c \leq y \leq d$.

The area of a triangle can be calculated as follows: A triangle consists of tree lines connecting the three vertices. In order to find the area bounded by these three lines, we must find the equations of these three lines and integrate their differences. For instance, in order to find the area of the triangle with vertices $(0,0)$, $(1,1)$, and $(2,0)$, we notice that it consists of the following three lines: $y = 0$, $y = x$, and $y = 2 - x$, as shown in Figure 5-29.

Figure 5-29: Calculating the area of a triangle.



For the left half of the triangle (i.e., between the points $x = 0$ and $x = 1$), we need to find the area between $y = x$ and $y = 0$. For the right half of the triangle (i.e., between the points $x = 1$ and $x = 2$), we need to find the area between $y = 2 - x$ and $y = 0$. Hence, finally, we calculate

$A(triangle) = A(left\ half) + A(right\ half)$,
symbolically:
$\int_0^1 (x - 0)\, dx + \int_1^2 [(2 - x) - 0]\, dx = 1\ square\ unit.$
The area of a square can be calculated as follows: If a is the length of the side of the square, then the area of the square is given by $A(square) = \int_0^a adx = ax|_0^a = a^2$.
The area of a circle whose radius is r can be calculated as follows: If we chop up the circle into triangular pie wedges with base dx and height r, then the area of each triangle is one-half the base dx times the height r. Adding them up, we obtain:

$$\int_0^{2\pi r} \frac{1}{2} r dx = \frac{1}{2} rx|_0^{2\pi r} = \pi r^2$$

given that the circumference of a circle of radius r is $2\pi r$. Similarly, we can work as follows: if we chop up the circle into circular rings with radius x and thickness dx, then the area of each ring is its circumference $2\pi x$ times the thickness dx. Adding them up, we obtain:
$\int_0^r 2\pi x dx = \pi x^2\,|_0^r = \pi r^2$.
The calculation of the volume of a solid of revolution: In order to obtain a solid of revolution, we start out with a curve $y = f(x)$ on an interval $[a, b]$, as shown, for instance, in Figure 5-30, and then we rotate this curve (360º) about a given axis, so that a volume is generated, as shown, for instance, in Figure 5-31.
In order to determine the volume of a solid of revolution on the interval $[a, b]$, we work as follows: we divide the interval $[a, b]$ into n subintervals, each of which has width $\Delta x = \frac{b-a}{n}$, and then we choose a point x_k^* (where $k = 1,2, \ldots, n$) from each subinterval. When we want to determine the area between two curves, we approximate the area by using rectangles on each subinterval. Understandably, when we want to compute the volume of a solid of revolution, we use disks on each subinterval to approximate the area. The area of the face of each disk is given by $A(x_k^*)$, and the volume of each disk is given by $V_k = A(x_k^*)\Delta x$. Hence, the volume of the corresponding solid of revolution on the interval $[a, b]$ can be approximated by $V \approx \sum_{k=1}^n A(x_k^*)\Delta x$. Then, its exact volume is
$V = lim_{n\to\infty} \sum_{k=1}^n A(x_k^*)\Delta x = \int_a^b A(x)dx$,
where $a \le x \le b$.
In other words, in this case, the volume is the integral of the cross-sectional area $A(x)$ at any x, and $x \in [a, b]$. Given that $A = \pi r^2$, $r = f(x)$, and $f(x)$ is a non-negative continuous function from $[a, b]$ to $\mathbb{R}$, the volume of the solid generated by a region under $y = f(x)$ bounded by the

x-axis and the vertical lines $x = a$ and $x = b$ via revolution about the x-axis is

$V = \pi \int_a^b [f(x)]^2 \, dx$;

we take disks with respect to x, and $r = y = f(x)$; dx indicates that the area is rotated about the x-axis.

Figure 5-30: A Curve.

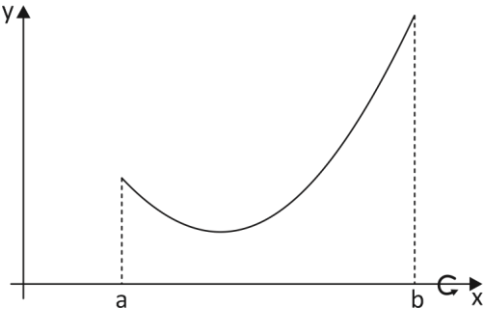
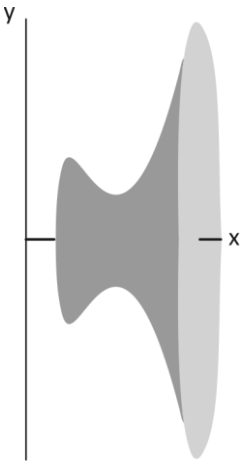


Figure 5-31:A Solid of Revolution.



If we rotate a curve about the y-axis, thus obtaining a cross-sectional area that is a function of y instead of x, then the aforementioned formula becomes

$V = \int_c^d A(y) dy$,

where $c \le y \le d$. Given that, in this case, $A = \pi r^2$, and $r = f(y)$, the volume of the solid generated by a region under $x = f(y)$ bounded by the y-axis and the horizontal lines $y = c$ and $y = d$ via revolution about the y-axis is

$V = \pi \int_c^d [f(y)]^2 \, dy$;

we take disks with respect to y, and $r = x = f(y)$; dy indicates that the area is rotated about the y-axis.

If we have two curves y_1 and y_2 that enclose some area, and we rotate that area about the x-axis, then the volume of the solid formed is given by

$V = \pi \int_a^b [(y_2)^2 - (y_1)^2] \, dx$.

The volume of a sphere can be calculated as follows: A sphere of radius r centered at the origin $(0,0,0)$ can be generated by revolving the upper semicircular disk enclosed between the x-axis and $x^2 + y^2 = r^2$ about the x-axis. If we revolve the semi-circle given by

$$y = f(x) = \sqrt{r^2 - x^2}$$

about the x-axis, we obtain a sphere of radius r. A cross-section of the sphere is a circle with radius $f(x)$ and area $\pi[f(x)]^2$. If we slice the sphere vertically into disks, then each disk has infinitesimal thickness dx, and the volume of each disk is approximately $\pi[f(x)]^2 dx$. If we add up the volumes of the disks, then we obtain the volume of the sphere—namely:

$V = \pi \int_a^b [f(x)]^2 \, dx = \pi \int_{-r}^{r} (r^2 - x^2) dx = \pi \left(r^2 x - \frac{x^3}{3} \right) |_{-r}^{r} =$

$\pi \left(\frac{2}{3} r^3 \right) - \pi \left(-\frac{2}{3} r^3 \right) = \frac{4}{3} \pi r^3$.

The volume of a cone can be calculated as follows: A cone with base radius r and height h can be formed by rotating a straight line through the origin $(0,0,0)$ about the x-axis. The slope of the straight line is $tan\theta = \frac{r}{h}$, so that the equation of the line is $y = \frac{r}{h} x$, and the limits of integration are $x = 0$ and $x = h$. Therefore, the volume of the corresponding cone is

$V = \pi \int_0^h \left(\frac{r}{h} x \right)^2 dx = \frac{\pi r^2}{h^2} \left(\frac{x^3}{3} \right) |_0^h = \frac{1}{3} \pi r^2 h$.

The volume of a cylinder with base radius r and height h (assuming that the plane xOy is the cylinder's base plane) is $V = \pi \int_0^h r^2 dx = \pi r^2 h$.

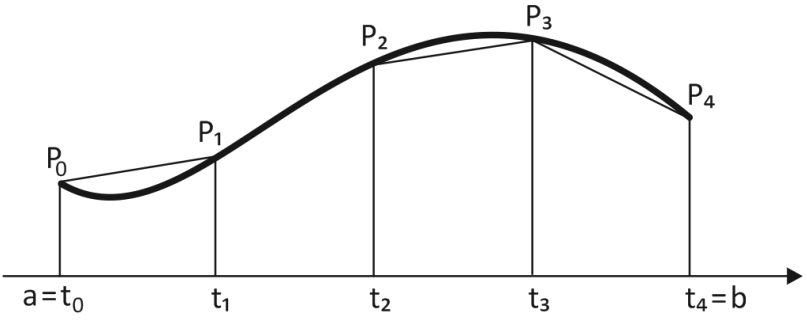
The calculation of the arc length of a curve: Let us consider a curve γ defined by the parametric equations

$x = g(t)$ and $y = f(t)$, where $t \in [a, b]$,

as shown, for instance, in Figure 5-32, and let $P = \{t_0, t_1, \ldots, t_n\}$ be a partition of $[a, b]$. Intuitively, if we regard parameter t as the time

variable, then the curve may be thought of as the path of a moving point whose position vector at time t is $\gamma(t) = \big(g(t), f(t)\big)$.

Figure 5-32: The Arc Length of a Curve.



Let $A_k = [g(t_k), f(t_k)]$, where $k = 1,2, \dots, n$, be the corresponding points of γ, as shown in Figure 5-32. Then these points define a polygonal line. The sum

$$L_P = \sum_{i=1}^{n} \sqrt{[g(t_i) - g(t_{i-1})]^2 + [f(t_i) - f(t_{i-1})]^2}$$

is the length of the polygonal line that is defined by the points A_k (corresponding to a partition P); and the finer the partition P, the more the corresponding polygonal line tends to be identified with the curve γ. Now, let us consider the set L of all numbers L_P, which correspond to all possible partitions P of $[a, b]$, symbolically:

$L = \{L_P | P \; is \; a \; partition \; of \; [a, b]\}$.

If this set L is bounded, then the curve is said to be "alignable," and the supremum $S = L(\gamma)$ of this set is said to be the length of the curve γ. Moreover, we write $S = L_a^b(\gamma)$ in order to denote the length of the arc of the curve that is defined on the interval $[a, b]$.

Notice that, if γ is an alignable curve on $[a, b]$, and if $a < c < b$, then $L_a^b(\gamma) = L_a^c(\gamma) + L_c^b(\gamma)$.

If the derivatives g' and f' are continuous on $[a, b]$, then the curve γ is alignable on $[a, b]$, and its length is given by

$S = L(\gamma) = \int_a^b \sqrt{[g'(t)]^2 + [f'(t)]^2}\, dt$,

where $t \in [a, b]$. If γ is defined by $y = f(x)$, where $x \in [a, b]$, and if the derivative $f'(x)$ exists and is continuous on $[a, b]$, then, setting $x = t$ and $y = f(t)$ in the aforementioned equation, we obtain the following formula:

$S = \int_a^b \sqrt{1 + [f'(x)]^2 dx}$,
where $x \in [a, b]$.

The physical significance of the integral, the method of infinitesimal calculus, and differential equations

The development of infinitesimal calculus by Newton and Leibniz is intimately related to the study of celestial mechanics, and physics in general, by them. Infinitesimal calculus, known also as the differentiation–integration method, is concerned with the limits of applicability of physical laws. Physical laws are not absolute, and the validity of a law is restricted to the framework of the applicability limits (i.e., certain conditions). However, a physical law can be expanded by changing its form beyond the limits of applicability by means of infinitesimal calculus. This method is based on the following two principles: (i) the principle that a law can be represented in differential form, and (ii) the superposition principle, according to which the quantities that enter into the law are additive.

Suppose that a physical law has the form

$X = YZ$, $(*)$

where X, Y, and Z are physical quantities, and, in particular, Y is a constant representing the given law's limits of applicability. We can generalize the given law to the case where Y is not a constant but a function of Z, that is, $Y = Y(Z)$, as follows: As shown in Figure 5-33, we isolate an interval dZ so small that the variation of Z over this interval can be ignored. Hence, in the interval ("infinitesimal") dZ, we can approximately assume that Y is constant, and that the law $(*)$ is valid in this interval. Therefore,

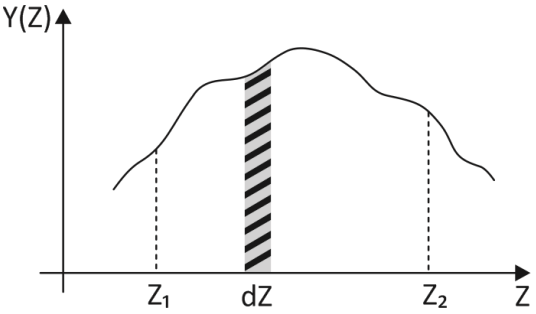
$dX = Y(Z)dZ$, $(**)$

where dX is the variation of X over dZ. Due to the superposition principle, that is, by summing the quantities $(**)$ over all the intervals of variation of Z, we obtain an expression for X in the form

$X = \int_m^M Y(Z)dZ$,

where m and M are the initial and the final values of Z, respectively.

Figure 5-33: The method of infinitesimal calculus.



As a conclusion, the method of infinitesimal calculus consists of two parts: in the first part of the method, we find the differential $(**)$ of the quantity under investigation; in the second part of the method, we sum, or "integrate," having adequately determined the integration variable and the limits of integration (in order to determine the integration variable, we must analyze the quantities on which the differential of the investigated quantity depends and choose the most important variable; and the limits of integration are the lower and the upper values of the integration variable).

The method of infinitesimal calculus underpins the theory of differential equations, which is used to describe numerous phenomena, including the expansion of the universe, pandemic models, arms races, climate models, financial markets, quantum mechanics, etcetera. A differential equation is an equation where the rate of change of some variable is related to the absolute size of the variable. For instance, the rate of growth of a tree of a certain species might be related to its height. If y denotes the size of the tree, then we read the expression $\frac{dy}{dt}$ as "the rate of growth of y (namely, the size) with respect to time." We can write this as follows: $\frac{dy}{dt} = f(y)$, which is a differential equation.

The fundamental problem of the theory of differential equations is to find all the functions $y = f(x)$ that satisfy some differential equation. Every function $y = f(x)$ that satisfies some differential equation is said to be a "solution" of the given differential equation.

A family of functions

$$y = f(x, c) \qquad (A)$$

where c is a constant belonging to $A \subseteq \mathbb{R}$, is said to be a "general solution" of a differential equation

$$y' = F(x, y) \qquad (B)$$

if, for every $c \in A$, (A) is a solution of (B). The solution that we obtain for each particular value of c is said to be a "partial solution" of the differential equation (B).

Differentiation and Integration of Multivariable Functions

So far, we have studied exclusively functions of a single (independent) variable x, but we can also apply the concept of differentiation to functions of several variables $x, y, ...$ Suppose that $f(x, y)$ is a function of two variables x and y, and that the limits

$$lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

and

$$lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$$

exist for all values of x and y in question—that is, $f(x, y)$ possesses a derivative $\frac{df}{dx}$ with respect to x and a derivative $\frac{df}{dy}$ with respect to y. These derivatives are called the "partial derivatives" of f, and they are respectively denoted by

$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ or

$f_x^{'}(x, y), f_y^{'}(x, y)$.

Similarly, we can differentiate functions of three or more variables.

In general, when calculating partial derivatives, we treat all independent variables other than the variable with respect to which we differentiate as constants. For instance, if $f(x, y) = x^2 - 3xy - 5$, then

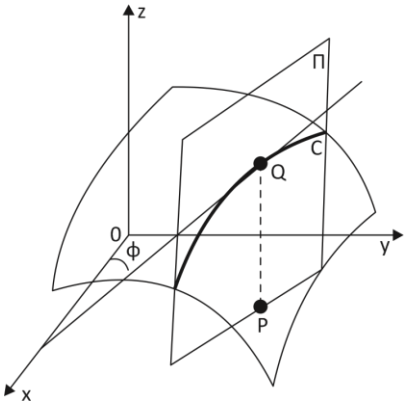
$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 - 3xy - 5) = \frac{\partial}{\partial x}(x^2) - \frac{\partial}{\partial x}(3xy) - \frac{\partial}{\partial x}(5) = 2x - 3y$, and

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 - 3xy - 5) = \frac{\partial}{\partial y}(x^2) - \frac{\partial}{\partial y}(3xy) - \frac{\partial}{\partial y}(5) = -3x.$

The geometric significance of $\frac{\partial f}{\partial x}|_{(x_0, y_0)}$ and $\frac{\partial f}{\partial y}|_{(x_0, y_0)}$ is illustrated in Figure 5-34. Let us consider a function $z = f(x, y)$, whose graph in $\mathbb{R}^3$ is a surface. We suppose that $P(x_0, y_0)$ is an arbitrary point of the domain of f. Notice that, in $\mathbb{R}^3$, the equation $y = y_0$ represents a plane Π that is perpendicular to the y-axis. This plane intersects the surface $z = f(x, y)$ by a curve C whose equation is $z = f(x, y_0)$. If $Q(x_0, y_0, z_0)$ is a point belonging to C, so that its orthogonal projection to the plane xOy is the point P, then the slope of the tangent to the curve C at Q is equal to

$\frac{\partial f}{\partial x}|_{(x_0,y_0)} = tan\varphi$, where φ is the angle formed by the x-axis and the tangent to the curve C at Q, as shown in Figure 5-34. In the same way, we can show that the slope of the tangent to the curve C at Q is equal to $\frac{\partial f}{\partial y}|_{(x_0,y_0)} = tan\theta$, where θ is the angle formed by the y-axis and the tangent to the curve C at Q.

Figure 5-34: The geometric significance of a partial derivative



Generalization: If $f: \mathbb{R}^n \to \mathbb{R}$ is a function,
$\mathbb{R}^n \ni (x_1, x_2, \dots, x_n) \to f(x_1, x_2, \dots, x_n) \in \mathbb{R}$,
then

$$\frac{\partial f(x_1, x_2, \dots, x_i, \dots, x_n)}{\partial x_i} = lim_{\Delta x_i \to 0} \frac{f(x_1, x_2, \dots, x_i + \Delta x_i, \dots, x_n) - f(x_1, x_2, \dots, x_i, \dots, x_n)}{\Delta x_i}$$

is the partial derivative of $f(x_1, x_2, \dots, x_n)$ with respect to x_i, where $i = 1,2, \dots, n$.

We take for granted the obvious generalizations of the theorems of differentiation to two or more variables.

We can integrate functions of several variables as follows: suppose that the domain of a bivariate function is the Cartesian product of two closed intervals—that is, a rectangle—say
$R = [a, b] \times [c, d] = \{(x, y) \in \mathbb{R}^2 | a \le x \le b, c \le y \le d\}$.
If $R = [a, b] \times [c, d]$, whenever the integrand is $f(x, y)$, we have to integrate over two variables, x and y, so that, for each variable, we have

an integration sign. In order to indicate the variables involved, we have dx and dy, symbolically:

$\iint_R f(x,y)\,dxdy \equiv \int_c^d \int_a^b f(x,y)dxdy,$

where $f(x,y)$ is an integrable function of two real variables. In this case, we compute the innermost integral first, and then we work our way outward. In particular, we compute the dx integral inside first, while treating y as a constant, and then we integrate the result over y as we would do with any variable. One interpretation of the double integral of $f(x,y)$ over the rectangle R is the volume under the function (surface) $f(x,y)$ and above the xy-plane. For instance, $\int_0^2 \int_0^1 x^2y^2dxdy$ can be calculated as follows: We focus on the inner integral first: $\int_0^2 \left[\int_0^1 x^2y^2dx\right] dy$; and, treating y as a constant, we integrate normally for x^2dx, thus obtaining $\int_0^2 \left[\frac{x^3y^2}{3}|_0^1\right] dy = \int_0^2 \left[\frac{1^3y^2}{3} - \frac{0^3y^2}{3}\right] dy = \int_0^2 \left[\frac{y^2}{3}\right] dy$. Now, we are left with a normal definite integral: $\int_0^2 \frac{y^2}{3} dy = \frac{y^3}{3\cdot 3}|_0^2 = \frac{y^3}{9}|_0^2 = \frac{2^3}{9} - \frac{0^3}{9} = \frac{8}{9}$. Therefore, $\int_0^2 \int_0^1 x^2y^2dxdy = \frac{8}{9}$.

Furthermore, double integrals can be used in order to compute areas as follows: the area A of a plane region R is given by the formula

$A = \iint_R dxdy,$

in Cartesian coordinates. Note that, if

$R = \{(x,y) \in \mathbb{R}^2 | a \le x \le b, h_1(x) \le y \le h_2(x)\},$

then the area of R is

$A = \iint_R dxdy = \int_a^b \int_{h_1(x)}^{h_2(x)} dy\,dx = \int_a^b [h_2(x) - h_1(x)]\,dx,$

where $h_1(x)$ and $h_2(x)$ are two curves.

The order in which we do the integrations does not matter, provided that we keep track of the limits of integration of each variable. For instance, in the double integral $\int_c^d \int_a^b f(x,y)dxdy$, dx is associated with the x integrand, which runs from a to b, while dy is associated with the y integrand, which runs from c to d, and, therefore,

$$\int_c^d \int_a^b f(x,y)dxdy = \int_a^b \int_c^d f(x,y)dydx$$

meaning that the limits of integration of each integrand remain the same (this result is known as Fubini's Theorem).

Increasing the number of integrals in the context of multiple integration is the same as increasing the number of dimensions: a single integral gives a two-dimensional area, a double integral gives a three-dimensional volume, a triple-integral gives a four-dimensional hypervolume, etc. In general, the

multiple integral of a function $f(x_1, \dots, x_n)$ in n variables over a domain U is represented by n nested integral signs in the reverse order of computation (in the sense that the leftmost integral is computed last), followed by the function and the integrand arguments in such an order that indicates that the integral with respect to the rightmost argument is computed last; and the domain of integration is either represented symbolically for every argument over each integral sign or it is indicated by a characteristic letter (variable) at the rightmost integral sign:

$$\int \dots \int_U f(x_1, \dots, x_n) d\, x_1 \dots dx_n$$

$(x_1, \dots, x_n \in U)$. We take for granted the obvious generalizations of the theorems of integration to two or more variables.

Complex Numbers and Complex Analysis

As we have already mentioned, the concept of a number has been extended from natural to real numbers, both because of human practice and because of the needs of mathematics itself. In particular, the concept of a number grew out of the counting of objects. Counting gave rise to the numbers 1, 2, 3, and so on, which are called natural numbers. Then the necessity of performing the operation of division led to the concept of positive fractional numbers; furthermore, the necessity of performing the operation of subtraction led to the concepts of zero and negative numbers; finally, the necessity of taking roots of positive numbers led to the concept of irrational numbers. The aforementioned operations are feasible in the set of real numbers. However, there are still impracticable operations—for instance, taking a square root of a negative number. Hence, there is a need to extend the concept of a number even further, specifically, to invent new numbers different from the real numbers.

Indeed, if we adjoin to the real system $\mathbb{R}$ a root i of the polynomial $x^2 + 1 = 0$, which is irreducible to $\mathbb{R}$, we obtain the system of complex number $\mathbb{C} \equiv \mathbb{R}(i)$. In other words, a (two-dimensional) number of the form $z = a + bi$, where $a, b \in \mathbb{R}$ and $i = \sqrt{-1}$, is called a "complex number"; the number a is called the "real part" of $z = a + bi$, and it is denoted by $Re(z)$; the number b is called the "imaginary part" of $z = a + bi$, and it is denoted by $Im(z)$; and $i = \sqrt{-1}$ is called the "imaginary unit." Hence, any polynomial equation with coefficients can be solved in the system of complex numbers; indeed, the system of complex numbers is the fundamental connection between geometry and algebra.

We picture the complex number $z = a + bi$ by putting a on the x-axis and b (or rather bi) on the y-axis.

The "modulus" or "absolute value" of $a + bi$ is $\sqrt{a^2 + b^2}$, and it is denoted by $mod(a + bi)$ or $|a + bi|$. The square of the modulus of a complex number $z = a + bi$ is called its "norm," and it is denoted by $Nm(z)$; so that, if $z = a + bi$, then $Nm(z) = a^2 + b^2$.

The "argument" of $z = a + bi$, denoted by $arg(z)$, is a quantity θ such that $cos\theta = \frac{a}{|z|}$ and $sin\theta = \frac{b}{|z|}$. It is many-valued and determined only up to multiples of 2π.

If we define an origin O and rectangular axes $X'OX$ and $Y'OY$, and mark the point P whose coordinates referred to these axes are x and y respectively, then the complex number $z = x + yi$ may be considered to be represented either by the point P or by the vector $\overrightarrow{OP}$, drawn to P from the origin. The numbers 1 and $i = \sqrt{-1}$ will be, respectively, represented by points A and B on OX and OY (respectively) at unit distance from O. Therefore, $z = x + yi$ denotes the operation by which the vector $\overrightarrow{OA} = 1$ is converted into the vector $\overrightarrow{OP} = x + yi$. The modulus and the argument of $x + yi$ are, respectively, equal to the radius vector and the vectorial angle of P, assuming that OX is the initial line, and that the radian is the unit angle; so that, calling these r and θ, we obtain $x + yi = r(cos\theta + isin\theta) = re^{\theta i}$.

In particular, the number $i = \sqrt{-1}$ signifies a 90^o rotation about the real axis, turning 1 into -1. Hence, $i = \sqrt{-1}$ done twice, or squared, is equal to -1. From the perspective of real numbers, we are used to thinking of numbers on a line, but, with the introduction of imaginary numbers, we come up with two-dimensional numbers. Indeed, complex numbers are two-dimensional, since they are made up of the real axis and the imaginary axis, and we can transit from one to the other by rotating.

In 1833 at the Royal Irish Academy, the Irish mathematician and astronomer Sir William Rowan Hamilton presented the complex numbers as ordered pairs of real numbers, thus denoting a complex number by an ordered pair (a, b), and denoting the imaginary unit by $i = \sqrt{-1}$, so that $i^2 = (0,1) \cdot (0,1) = (-1,0) = -1$.

The zero of $\mathbb{C}$ is $(0,0)$, and the unit of $\mathbb{C}$ is $(1,0)$.

In fact, as Hamilton has originally shown, the complex number system $\mathbb{C}$ is the set $\mathbb{R} \times \mathbb{R}$ with operations defined as follows:

$$(a + bi) + (c + di) = (a + c) + (b + d)i,$$
$$(a + bi) - (c + di) = (a - c) + (b - d)i,$$
$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i, \text{ and}$$

$$\frac{(a+bi)}{(c+di)} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

where $a, b \in \mathbb{R}$, and $i = \sqrt{-1}$.

The (complex) "conjugate" of $a+bi$ is $a-bi$, and the conjugate of a complex number z is denoted by $\bar{z}$ or by z^*; so that, if $z = a+bi$, then: $z+\bar{z} = 2a$, $z-\bar{z} = 2ib$, $z\bar{z} = a^2+b^2$.

If a function g takes real inputs and gives complex outputs, then the derivative with respect to its real input is computed by taking the derivatives of the real and the imaginary parts separately, namely:

$$\frac{dg}{dx} = \frac{dRe(g)}{dx} + i\frac{dIm(g)}{dx}$$

where $i = \sqrt{-1}$, $Re(g)$ is the real part of g, and $Im(g)$ is the imaginary part of g.

The integral of a complex function $f(x) = g(x) + ih(x)$ between the limits a and b is defined by

$$\int_a^b f(x)dx = \int_a^b [g(x)+ih(x)]dx = \int_a^b g(x)dx + i\int_a^b h(x)dx$$

where $i = \sqrt{-1}$, and x is a real variable. Obviously, the properties of such integrals may be deduced from the properties of the real integrals.

Vector Calculus

When a function takes a real number and sends it to a vector, then it is said to be a vector-valued function. In the real plane, or in the xy-plane, the general form of a vector-valued function is the following:

$\vec{r}(t) = f(t)\hat{\imath} + g(t)\hat{\jmath}$; (1)

and, in the real 3-dimensional space, or in the xyz-space, the general form of a vector-valued function is the following:

$\vec{r}(t) = f(t)\hat{\imath} + g(t)\hat{\jmath} + h(t)\hat{k}$; (2)

where the component functions f, g, and h are real-valued functions of the parameter t, and $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are the corresponding unit vectors on the x-axis, the y-axis, and the z-axis respectively. The standard unit vectors in the direction of the x, y, and z axes of a 3-dimensional Cartesian coordinate system are

$\hat{\imath} = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $\hat{\jmath} = \begin{pmatrix}0\\1\\0\end{pmatrix}$, and $\hat{k} = \begin{pmatrix}0\\0\\1\end{pmatrix}$.

The "limit" of a vector-valued function $\vec{r}(t)$ is $\vec{L}$ as t tends to a, symbolically:

$$lim_{t\to a}\vec{r}(t) = \vec{L}$$

if and only if

$$lim_{t\to a}\left\|\vec{r}(t)-\vec{L}\right\|=0.$$

Therefore, (1) implies that

$lim_{t\to a}\vec{r}(t)=[lim_{t\to a}f(t)]\hat{\imath}+[lim_{t\to a}g(t)]\hat{\jmath}$,

and (2) implies that

$lim_{t\to a}\vec{r}(t)=[lim_{t\to a}f(t)]\hat{\imath}+[lim_{t\to a}g(t)]\hat{\jmath}+[lim_{t\to a}h(t)]\hat{k}$,

provided that the limits of the component functions f, g, and h as $t\to a$ exist. Similarly, we can define the limit of a vector-valued function of n component functions for $n>3$.

A vector-valued function $\vec{r}(t)$, where $t\in[a,b]$, is said to be "continuous" at a point $t_0\in[a,b]$ if $lim_{t\to t_0}\vec{r}(t)=\vec{r}(t_0)$; and $\vec{r}(t)$ is said to be continuous on $[a,b]$ if it is continuous at every point of $[a,b]$.

The derivative of a vector-valued function $\vec{r}(t)$, where $t\in[a,b]$, is defined as follows:

$$\overrightarrow{r'}(t)=lim_{\Delta t\to 0}\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t}$$

provided that the limit exists. If $\overrightarrow{r'}(t)$ exists, then $\vec{r}(t)$ is said to be differentiable at t. If $\overrightarrow{r'}(t)$ exists $\forall t\in(a,b)$, then $\vec{r}(t)$ is said to be differentiable on the interval (a,b). In order for $\vec{r}(t)$ to be differentiable on $[a,b]$, $\vec{r}(t)$ must be differentiable on the interval (a,b), and the following two limits must exist as well:

$\overrightarrow{r'}(a)=lim_{\Delta t\to 0^+}\frac{\vec{r}(a+\Delta t)-\vec{r}(a)}{\Delta t}$ and

$\overrightarrow{r'}(b)=lim_{\Delta t\to 0^-}\frac{\vec{r}(b+\Delta t)-\vec{r}(b)}{\Delta t}$.

Consequently, (1) implies that

$\overrightarrow{r'}(t)=f'(t)\hat{\imath}+g'(t)\hat{\jmath}$,

and (2) implies that

$\overrightarrow{r'}(t)=f'(t)\hat{\imath}+g'(t)\hat{\jmath}+h'(t)\hat{k}$.

The properties of the derivative of a vector-valued function are analogous to those of the derivative of a scalar-valued function.

Let f, g, and h be integrable real-valued functions on $[a,b]$. Then (1) implies that the indefinite integral of a vector-valued function $\vec{r}(t)=f(t)\hat{\imath}+g(t)\hat{\jmath}$ is

$\int[f(t)\hat{\imath}+g(t)\hat{\jmath}]\,dt=[\int f(t)dt]\hat{\imath}+[\int g(t)dt]\hat{\jmath}$,

and the definite integral of a vector-valued function $\vec{r}(t)=f(t)\hat{\imath}+g(t)\hat{\jmath}$ is

$\int_a^b[f(t)\hat{\imath}+g(t)\hat{\jmath}]dt=\left[\int_a^b f(t)dt\right]\hat{\imath}+\left[\int_a^b g(t)dt\right]\hat{\jmath}$.

By analogy, (2) implies that

$\int\left[f(t)\hat{\imath}+g(t)\hat{\jmath}+h(t)\hat{k}\right]dt=[\int f(t)dt]\hat{\imath}+[\int g(t)dt]\hat{\jmath}+[\int h(t)dt]\hat{k}$,

and

$\int_a^b [f(t)\hat{\imath} + g(t)\hat{\jmath} + h(t)\hat{k}]dt = \left[\int_a^b f(t)dt\right]\hat{\imath} + \left[\int_a^b g(t)dt\right]\hat{\jmath} + \left[\int_a^b h(t)dt\right]\hat{k}.$

The properties of the integral of a vector-valued function are analogous to those of the integral of a scalar-valued function.

Let us consider a function $f(x, y)$; f depends on both x and y, and its graph is a surface in space. Then, in order to interpret and compute the rate of change of $f(x, y)$, we find the rate of change of $f(x, y)$ in a specific direction independently. If we want the rate of change in the x-direction, we differentiate $f(x, y)$ with respect to x while treating y as a constant. In other words, we compute the partial derivative $\frac{\partial f(x,y)}{\partial x}$. Similarly, if we want the rate of change in the y-direction, we differentiate $f(x, y)$ with respect to y while treating x as a constant. In other words, we compute the partial derivative $\frac{\partial f(x,y)}{\partial y}$. The "gradient" of $f(x, y)$ is denoted by ∇f, and it is a concept that combines the two aforementioned partial derivatives; specifically, the gradient of $f(x, y)$ is a vector consisting of both partial derivatives of f in their associated positions, symbolically:

$grad f \equiv \nabla f = \frac{\partial f(x,y)}{\partial x}\hat{\imath} + \frac{\partial f(x,y)}{\partial y}\hat{\jmath},$

where $\hat{\imath}$ is the unit vector in the x-direction, and $\hat{\jmath}$ is the unit vector in the y-direction. By analogy, we can define the gradient of a function $f(x, y, z)$, etc. The gradient of a function is a vector field. A vector field V in $\mathbb{R}^3$ is a rule that assigns a tangent vector V_p to every point p of $\mathbb{R}^3$; and tangent vector V_p is said to be the value of the vector field V at the point p.

Let us consider a vector-valued function $\vec{r}(x, y, z) = f(x, y, z)\hat{\imath} + g(x, y, z)\hat{\jmath} + h(x, y, z)\hat{k}$ such that the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial g}{\partial y}$, and $\frac{\partial h}{\partial z}$ exist and are continuous on $U \subseteq \mathbb{R}^3$. Then the "divergence" of $\vec{r}(x, y, z)$ is a vector operator that operates on a vector field, producing a scalar field that gives the quantity of the vector field's source at each point; and it is defined as follows:

$div\vec{r} \equiv \vec{\nabla}\vec{r} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}.$

In other words, the divergence of a function tells us how the corresponding vector field behaves towards or away from a point.

From Mathematics to Physics

The subject of physics very much depends on mathematics. Whereas mathematicians are concerned only with the structure of reasoning,

physicists endow mathematical expressions with physical meanings. Therefore, they create mathematical models of the physical world.

Energy, Force, and Work

By the term "energy," we mean the impetus that underpins all motion and all activity—more specifically, the capacity for doing work. In physics, we typically look at the work that a constant force, F, does when moving an object over a distance of s. In these cases, the work is

$W = Fs$;

the force is parallel to the displacement.

But most forces are not constant and depend upon where exactly the force is acting. Therefore, assuming that the force at any x is $F(x)$, the work done by the force in moving an object from $x = a$ to $x = b$ is given by

$W = \int_a^b F(x)dx$,

where the force is parallel to the displacement.

Mechanics is the branch of physics that studies the relationships between the following three physical concepts:

i. *Force:* an agent that changes or tends to change the state of motion (i.e., the state of rest or of uniform motion) of an object. The "velocity" of an object is the rate of change of its position with respect to a frame of reference, and it is a function of time (i.e., velocity is the first derivative of displacement with respect to time).
ii. *Mass:* the quantity of matter that is concentrated in an object. The product of the mass times the velocity of an object is the "momentum" of that object.
iii. *Motion:* a change in the position of an object with respect to time.

The part of mechanics that is concerned with the study of motion is called kinematics. Due to the rigorous study of classical mechanics by Isaac Newton, the SI (Système International) unit of force, newton (denoted by N), has been named in his honor. One newton is defined as the force needed in order to accelerate one kilogram (kg) of mass at the rate of one meter (m) per second (sec) squared in the direction of the applied force.

First Law of Motion: An object will remain at rest or in a uniform state of motion unless that state is changed by an external force.

Second Law of Motion: The vector sum of the forces on an object is equal to the mass of that object multiplied by the acceleration of that object ("acceleration" is the rate of change of the velocity of an object with respect to time. Thus, acceleration is the first derivative of velocity with respect to time or, equivalently, the second derivative of displacement with respect to time); symbolically:

$F = ma$,
where F denotes force, m denotes the mass of an object, and a denotes the acceleration of the given object (thus, for any force you put on an object, an object of small mass will accelerate a lot, and an object of large mass will accelerate just a little). In case of circular motion (i.e., a movement of an object along the circumference $C = 2\pi r$ of a circle of radius r), if the period for one rotation is T, then:
the angular velocity (i.e., the angular rate of rotation) is
$\omega = \frac{2\pi}{T} = \frac{d\varphi(t)}{dt}$,
where $\varphi(t)$ denotes the angular displacement from the x-axis and is measured in radians, and t denotes time (measured in seconds);
the speed of the object travelling the circle is
$v = \frac{2\pi r}{T} = \omega r$;
the angular acceleration of the particle is
$\alpha = \frac{d\omega}{dt}$,
and, in case of uniform circular motion, $\alpha = 0$;
the acceleration due to change in the direction is
$\alpha_c = \frac{v^2}{r} = \omega^2 r$;
and the centripetal and centrifugal force can be computed using acceleration as follows (the centripetal force and the centrifugal force are actually the same force, depending upon the frame of reference):
$F_c = m\alpha_c = \frac{mv^2}{r}$.
Third Law of Motion: For every action in nature, there is an equal and opposite reaction.
Newton's Law of Universal Gravitation: An object attracts another object with a force that is directly proportional to the product of the masses of the objects and inversely proportional to the square of the distance between them, symbolically:
$F_g = G\frac{m_1 m_2}{r^2}$,
where F_g is the magnitude of the gravitational force on either object, m_1 and m_2 are their masses, r is the distance between them, and G is the gravitational constant, whose value is found to be (in SI units) $6.673 \times 10^{-11} N \cdot m^2 \cdot kg^{-2}$ (thus, the "weight" of a body is the total gravitational force exerted on the body by all other bodies in the universe).
Coulomb's Law: The magnitude of the force of interaction between two point charges (i.e., electric charges) is directly proportional to the product of the charges and inversely proportional to the square of the distance between them, symbolically:

$F = k\frac{|q_1 q_2|}{r^2}$,

where F denotes the magnitude of the force that each of two point charges q_1 and q_2 a distance r apart exerts on the other, and k is a proportionality constant, whose value is (in SI units) approximately $8.988 \times 10^9 N \cdot m^2 \cdot C^{-2}$. Due to the rigorous description of the electrostatic force of attraction and repulsion by the French military engineer and physicist Charles-Augustin de Coulomb (1736–1806), the SI unit of electric charge, the coulomb (denoted by C), has been named in his honor; it is approximately equivalent to 6.24×10^{18} electrons. "Charge" is a property of matter (just like mass, volume, or density), and it can come in two types: positive $(+)$ or negative $(-)$. In particular, a positive charge occurs when the number of protons exceeds the number of electrons, and a negative charge occurs when the number of electrons exceeds the number of protons.

By the term "field," we mean an area in which forces are exerted on things in its midst. The modern concept of a physical field was originally formulated in the nineteenth century by the English physicist Michael Faraday. An electric charge creates an "electric field" in the region of space surrounding it, in the sense that the properties of space are modified by the presence of an electric charge. "Electric field" (sometimes called "electric intensity") is defined as the electric force per unit charge. Therefore (in SI unites), the unit of electric field magnitude is one newton per coulomb (i.e., $1N \cdot C^{-1}$).

By the term "flux," we mean the quantity of a substance passing through a given area. Thus, "electricity" is the flow of electric charge along a path provided by a conductor (conductors are materials with high electron mobility). The "electric flux" through a surface is proportional to the number of field lines crossing that surface. In other words, its magnitude is proportional to the portion of the field perpendicular to the area:

$Electric\ Flux = Electric\ Field \times Area \times cos\theta$,

where $cos\theta$ denotes the cosine of the angle θ between the electric field and the vector that is perpendicular to the area. A "field line" is an imaginary line drawn through a region of space in such a way that, at every point, it is tangent to the direction of the electric-field vector at that point. In particular, in an "electrostatic field," every field line is a continuous curve with a positive charge at one end and a negative charge at the other. The amount of work needed in order to move a unit of electric charge from a reference point to a specific point in an electric field without producing acceleration is called an "electric potential." In terms of SI units, it is represented by

$V = \frac{joule}{coulomb}$,

where joule is the unit for work done, and $1\,joule = (1\,newton)(1\,meter)$; coulomb is the unit for the charge; and V denotes "volt," the derived unit for electric potential (electromotive force), and is named after the Italian physicist Alessandro Volta (1745–1827). The key to the flow of electricity is making a continuous electric circuit: connecting a wire between a source of electrons and an attractor of electrons (for which reason, for instance, a battery has two poles: a source (a negative), and an attractor (a positive); similarly, an electric plug has at least two tongs, one for incoming electrons and one for outgoing electrons). Electrons do not cease to exist. Rather, being carriers of charge, they move from the negative (source) to the positive (attractor), and they are useful as they follow the path to their destination in the context of a continuous electric circuit. By contrast, connecting two poles of a power source directly can actually be very dangerous: this is what is called a "short circuit," because there is no electric device between the source and the destination of electrons to power, such as a PC or a TV set. In case of a short circuit, the electron flow does not encounter any resistance, therefore the release of energy is instant, often paired with the involved wire heating dangerously.

Total Mechanical Energy of a System: $E_m = K + U$,

where E_m denotes mechanical energy, K denotes kinetic energy, and U denotes potential energy.

By the term "potential energy," we mean the energy possessed by a body by virtue of its position relative to others, stresses within itself, its electric charge, or other factors. For instance, gravitational potential energy (e.g., in the case of a ball whose mass is m and is dropped from height h) can be computed using the following formula:

$U = mgh$,

where m denotes the mass of the object, g denotes the acceleration constant due to gravity ($\approx 9.8\,m/sec^2$), and h denotes the height (displacement) of the object as a function of time.

By the term "kinetic energy," we mean the energy possessed by a body by virtue of its motion. Let us consider a body of mass m moving along the x-axis under the action of a constant resultant force of magnitude F directed along the axis. The body's acceleration is constant, and, according to Newton's Second Law of Motion, it is given by $F = ma$. The kinetic energy of this body can be computed using the following formula:

$K = \frac{1}{2}mv^2$,

where v denotes the body's velocity (which is, by definition, a function of time), and m denotes the mass of the object. Thus, the work done by the

resultant external force on a body is equal to the change in kinetic energy of the body.

Relativity

Simply put, the central meaning of "relativity" is that different people may experience the same situation differently. The theory of relativity explains how to convert from the point of view of one observer to the point of view of another observer. For instance, Galileo theorized that, when two frames of reference (e.g., two drivers) are moving in uniform motion together, it is impossible to determine if one of the frames is moving or stationary from the perspective of the other frame in this system; it is only possible to understand an object's movement through a different frame of reference. The idea works for any two frames of reference moving with respect to each other (by the term "frame of reference," we mean any place where some physical process occurs or could occur).

Observers moving at different speeds will disagree about the distance and the time between two events. The German mathematician Hermann Minkowski depicted time as a length by proposing the following definition:

$distance = speed\ of\ light \times time = ct.$

Hence, if the speed of light in vacuum, commonly denoted by the letter c, is approximately $300{,}000{,}000\ meters/second$ (according to Rosa and Dorsey), then we say that $1/300{,}000{,}000$ of a second is one meter. In other words, one meter is the distance travelled by light in vacuum during a time interval of $1/300{,}000{,}000$ of a second.

Einstein's special theory of relativity implies that our perception of "now," of presence, is subjective; it does not exist in the fundamental laws of nature. According to Einstein's special theory of relativity, the speed of light is finite, it is the same for all observers, and nothing can go faster than light (in vacuum). Hence, by the time an observer A sees an object B standing in front of A, the object B has already moved into the future. In fact, A sees B as B was in the past (how much in the past depends on the distance between them and on their relative motion). This delay (which may be very tiny, but still non-zero) exists in everything we see. The main reason that we do not perceive relativistic effects (such as the visual delay and the relativity of simultaneity) in our everyday life is because life around us is moving very slowly compared to the speed of light.

Conservation of Energy

The eighteenth-century French mathematician and natural philosopher Émilie du Châtelet proposed and tested the law of "conservation of

energy," according to which the total energy of an "isolated system" (i.e., one that does not interact with other systems) remains constant.

In order to clarify the meaning of the principle of the conservation of energy, let us consider the following example: setting fire to coal. The chemical bonds of the coal molecules store great amounts of energy. If we set fire to coal, then fire causes a chain reaction between the coal and oxygen in the air. In this reaction, energy from the chemical bonds is converted into kinetic energy of air molecules. Hence, the air becomes warm, and, for this reason, it will rise. This rising air can be used in order to drive a turbine and, for instance, move a vehicle, or in order to create electricity (by feeding it into the grid).

Alternatively, we can just burn coal without doing anything with the produced energy. This does not change the total energy in the system, because the total energy in the system is conserved. The chemical energy of the coal is converted into kinetic energy of air molecules, which are distributed in the atmosphere. Even though, in this case, the energy is useless, the total energy in the system remains the same. The difference between the aforementioned cases is entropy, or the measure of the molecular disorder, or randomness, of the system under consideration. Initially, the energy was packed into the coal, and the level of entropy was low. By setting fire to coal, the energy was distributed in the motion of air molecules, and the level of entropy became high.

When a system has energy in a state of low entropy, its energy can be used in order to create macroscopic change (e.g., drive a turbine), and this useful energy is called "free energy." Free energy is a type of energy that does "work." But, if the energy in the system is in a state of high entropy, then the energy is useless, and it is called "heat." Heat is a type of energy that does not do "work." Even though *total* energy is conserved, *free* energy is not conserved.

The conversion of mechanical energy into heat (thermal energy) is called "dry friction." Similarly, the conversion of electrical energy in a conductor into heat is called "resistance." In general, by the term "friction," we refer to a force that resists the relative motion between two objects; and its causes are molecular adhesion, surface roughness, and deformations. Friction converts its work into heat.

Electromagnetism and Light

The electric field at a point can be calculated by using Coulomb's law in order to find the total force F on a test charge q' placed at the point, and then we divide F by q' to obtain the electric field E. If q' is positive, then

the direction of E is the direction of F. The force on a negative charge, such as an electron, is opposite to the direction of E.

In order to analyze the motion of a particle with charge q in an electric field, we need to use Newton's Second Law of Motion, $F = ma$, with F caused by the electric field E, so that the magnitude of the electric force F is given by

$$F = qE.$$

If the field is uniform, then the acceleration is constant.

In simple terms, electrical interactions can be described as follows: a charge distribution sets up an electric field E, and the field exerts a force $F = qE$ on any charge q that is present. The same pattern can be followed in order to describe magnetic interactions (phenomena of attraction or repulsion that arise between electrically charged particles because of their motion). A moving charge, or a current, sets up a magnetic field in the space around it, and this field exerts a force F on a moving charge. Like electric field, magnetic field is a vector field (a vector quantity associated with each point in space). The symbol for magnetic field is B.

Whereas the electric-field force is the same whether the charge is moving or not, the magnetic force is proportional to the particle's speed. Thus, a particle at rest experiences no magnetic force at all. Furthermore, the magnetic force F acting on a charge q moving with velocity v does not have the same direction as the corresponding magnetic field B, but it is perpendicular to both the magnetic field B and v. Hence, the magnitude of the magnetic force F is given by

$F = |q|vBsin\varphi,$

where $|q|$ is the magnitude of the charge, and φ is the angle measured from the direction of v to the direction of B. The SI unit of B is $1N \cdot s \cdot C^{-1} \cdot m^{-1}$, where N stands for newton, s stands for second, C stands for coulomb, and m stands for meter. This unit is called 1 tesla ($1T$), in honor of the prominent Serbian-American scientist and inventor Nikola Tesla (1857–1943).

Using vector notation, the force that a magnetic field $\vec{B}$ exerts on a charge q with velocity $\vec{v}$ is given by

$\vec{F} = q\vec{v} \times \vec{B},$

where $\vec{v} \times \vec{B}$ denotes the cross product of the velocity and the magnetic field.

In 1831, the English scientist Michael Faraday discovered electromagnetic induction: he placed a stationary magnet inside or outside a coil, and he observed no deflection in the galvanometer. However, at the moment that he moved the magnet towards (into/above/below) the coil, he saw the pointer deflecting in one direction, and, at the moment that he moved the

magnet way from the coil, he saw the pointer deflecting in the opposite direction. Using the aforementioned notation, the entire electromagnetic force F on the charged particle is called the Lorentz force (after the Dutch physicist H. A. Lorentz), and its magnitude is given by

$$F = F_{electric} + F_{magnetic}.$$

Faraday's discovery was really amazing, because one could make something move without ever touching it, only by using the field. Indeed, we can affect things far away and develop telecommunications using electromagnetic fields. Moreover, Faraday was the first to understand that waves of the electromagnetic field are what we call light. In simple terms, electromagnetic radiation consists of electric and magnetic fields oscillating around each other, creating a freely propagating wave that can travel from one place to another. This event explains light, the operation of radio stations, the operation of microwave ovens, etc. These are electromagnetic phenomena, and they differ from each other only with respect to the wavelength of the corresponding oscillation, so that we use different names for electromagnetic radiation depending on the corresponding wavelength; for instance, if we can see electromagnetic radiation, then we call it light, light with large wavelengths is red, light with larger wavelengths that is invisible is called infrared, while, at even larger wavelengths, electromagnetic radiations are called microwaves, and, if the wavelengths are even larger, then electromagnetic radiations are called radio-waves.

By the term "radiation," we generally mean energy transferred by waves or particles. For instance, radiation may take the form of electromagnetic waves—which, however, are made of particles, photons specifically. A photon is a type of elementary particle that serves as the quantum of the electromagnetic field and the force carrier for the electromagnetic force. In particular, quantum electrodynamics describes the manner in which electrically charged particles interact by shooting photons back and forth between each other. Electrons, being zero-dimensional, lack spatial extension (that is, they have practically zero volume). Therefore, they interact with each other by exchanging photons. As two electrons move towards each other, a photon is passed from one to another, and it changes the momentum of both of them, thus pushing them off.

Gravitational radiation is transferred in gravitational waves, which are actually periodic deformations ("wiggles") of space-time. According to rigorous physical hypotheses, gravitational waves are made of a peculiar kind of particles called gravitons. A graviton is assumed to be a quantum of gravity (an elementary particle mediating the force of gravity). The term

"graviton" was coined in the 1930s by the Soviet physicists Dmitrii Blokhintsev and F. M. Galperin.

The four fundamental forces

The laws of nature can be distilled into the following four fundamental forces:

i. *gravity:* a natural phenomenon by which all things with mass or energy are brought towards each other (it helps us to calculate the motions of celestial bodies);
ii. *electromagnetism:* a type of physical interaction that occurs between electrically charged particles (it has given us the wonders of the electric age);
iii. *weak nuclear force:* the mechanism of interaction between subatomic particles (it is responsible for the radioactive decay of the subatomic particles, and thus plays an essential role in nuclear fission, which is a form of nuclear transmutation);
iv. *strong nuclear force:* the mechanism that binds the component particles of an atom's nucleus (in 1911, the New Zealand physicist Ernest Rutherford discovered the basic structure of the atom: it consists of a small and dense core of positive electric charge called the nucleus, surrounded by a cloud of negatively charged electrons). An energy field that permeates the entire universe is known as the "Higgs field" (the smallest bit of which is called the "Higgs boson"), and it explains why some subatomic particles have a great deal of mass, while others have little, and others have none at all: the Higgs field interacts with the subatomic particles and determines their mass (very massive particles interact a lot with the Higgs field, while massless particles do not interact at all).

Quantum Physics

In quantum physics, everything is described in terms of wave functions, a wave function is a vector in a complex Hilbert space, and the vector coefficients are complex numbers. According to Paul Dirac's notation, in quantum physics, vectors are symbolized in the following way, known as the bra-ket notation:

$$|\Psi\rangle = a_1\begin{pmatrix}1\\0\\0\end{pmatrix} + a_2\begin{pmatrix}0\\1\\0\end{pmatrix} + a_3\begin{pmatrix}0\\0\\1\end{pmatrix}, \text{ where } a_1, a_2, a_3 \in \mathbb{C}.$$

The aforementioned type of brackets helps us to keep track of whether a vector is a row vector or a column vector: $|\Psi\rangle$ is a column vector, whereas $\langle\Psi|$ is a row vector. In quantum mechanics, if we convert a row vector to a

column vector, then we have to take the complex conjugate of each coefficient. In other words, for instance,

$|\Psi\rangle = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\langle\Psi| = (a_1^*, a_2^*, a_3^*)$, where a_1^*, a_2^*, a_3^* are, respectively, the complex conjugates of a_1, a_2, a_3.

In quantum mechanics, all vectors describe probabilities. Usually, we choose the basis of the space under consideration in such a way that the basis vectors correspond to possible measurement outcomes; for instance:

$|\Psi\rangle = a_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + a_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + a_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ corresponds to

$|\Psi\rangle = a_1|X\rangle + a_2|Y\rangle + a_3|Z\rangle$.

Hence, the probability of a particular measurement outcome is the absolute square of the scalar product with the basis vector that corresponds to the outcome; so that, for instance, the probability of measuring X is

$|\langle X|\Psi\rangle|^2 = a_1 a_1^*$,

and this is known as Born's Rule. In other words, the probability density of finding a particle at a given point, when measured, is proportional to the square of the amplitude of the particle's wave function at that point. In quantum physics, the gradient of a wave function is denoted as follows:

$\nabla|\Psi\rangle = \frac{\partial}{\partial x}|\Psi\rangle\hat{\imath} + \frac{\partial}{\partial y}|\Psi\rangle\hat{\jmath} + \frac{\partial}{\partial z}|\Psi\rangle\hat{k}$.

In order to understand quantum physics, we must understand the difference between the potential mode of being and the actual mode of being. Hence, we must never confuse the realm of potentiality with the realm of actuality—that is, we must never attribute actuality to probability. As the famous quantum physicists Alain Aspect, John Clauser, and Anton Zeilinger have experimentally shown, particles do not have definite values before they are measured, but they have definite values after they are measured.

As already mentioned, in quantum physics, every system is described by a wave function, usually denoted by the Greek letter Ψ, from which physicists calculate the probability of obtaining a specific measurement outcome. In other words, this wave function is a way of studying the realm of potentiality in a scientifically rigorous way. For instance, from this wave function, one can calculate that a particle that enters a beam-splitter has a 50% chance of going left and a 50% chance of going right. This is a way of analyzing that particle's *potential* mode of being. On the other hand, we can analyze that particle's *actual* mode of being by measuring the given particle.

After measuring the particle, we know with 100% probability where it is. Therefore, we must update our probabilistic study of the particle under consideration accordingly and with it the wave function. This update is known as the "wave function collapse," and it is an observational requirement that stems from the fact that, by measuring the particle, we have achieved a transition from potentiality to actuality. At the level of potentiality, or when we study the potential mode of being of a particle, that particle may be 50% at point A and 50% at point B; at the level of actuality, or when we study the actual mode of being of a particle by managing to measure it, that particle is 100% in a particular position, and we never observe a particle that is 50% at point A and 50% at point B. If we observe a particle at all, then we find that it is either in a particular position or not.

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