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PREDICTION OF MOISTURE FLOW AND RELATED SWELLING OR SHRINKING IN UNSATURATED SOILS

by

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and

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SYNOPSIS

The engineering behavior of unsaturated soils is often most easily understood in terms of changes in natural moisture content. Therefore, the prediction of moisture flow and related swelling or shrinking is of importance to engineers concerned with the design of shallow foundations and other related structures. A theoretical model is presented to predict the moisture flow in unsaturated soil continua as the result of hydraulic, vapor and thermal gradients. Experimentally verified constitutive relationships are used to define the volume changes of the unsaturated soil. A partial differential heat flow equation (i.e., for above freezing conditions) and two partial differential transient flow equations (i.e., one for the water phase and the other for the air phase) are derived and solved simultaneously.

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The solutions give the temperature and the pore pressure distributions with space and time in an unsaturated soil. The computed pore pressures allow the computation of the change in soil suction with time. The computed pressure changes are used to determine the quantity of moisture flow and to predict the rate of swelling and/or shrinking during the transient process as well as the ultimate swell and/or shrinkage in an unsaturated soil. The main part of the proposed analytical model is tested using a case study in the city of Regina, Saskatchewan, Canada. Reasonable agreement is noted with the observed rate and amount of swelling.

INTRODUCTION

A large number of engineering structures founded near the surface of deposits of swelling and shrinking clay soils undergo significant ground movements and pose a serious problem to engineers around the world. In the Prairie regions of Western Canada the glacio-lacustrine soil deposits cover several thousand square miles. In South-Central Saskatchewan there are thick deposits of clay of high plasticity, which are often identified as "problem soils". These soils are generally unsaturated for varying depths for part or all of the year because the moisture received in the form of total precipitation is less than the amount of moisture lost by evaporation and evapotranspiration. As a result, the pore-water pressure is negative and the soil is said to have "suction". The engineering behavior of these soils is commonly described in terms of changes in natural water content.

There are a number of engineering problems in which the movement of moisture through unsaturated soils play a primary role. However, these problems are difficult to analyze since a satisfactory moisture flow formulation, which is coupled with realistic microclimatic boundary condtions, has not been available. The microclimatic condition can be characterized in terms of factors such as temperature, relative humidity, wind velocity, precipitation and evaporation. These factors must be quantified as boundary conditions and used in conjunction with the analytical formulation which considers moisture movement in both the liquid and vapor phases.

An attempt is made in this paper to present briefly the theoretical formulations developed for the moisture flow in the liquid and vapor phases of an unsaturated soil. The use of the proposed analytical model is demonstrated using a study from the city of Regina, Saskatchewan. The case study measurements were made by the Division of Building Research, National Research Council, Saskatoon. The study consisted of observing the vertical slab and subsoil movement of an instrumented slab-on-grade building in Regina, Saskatchewan. This building suffered serious floor movement as the result of a plumbing leak below the floor slab. Periodic records of ground movements were made over a period of several years (1961-1972).

THEORY (REQUIRED PHYSICS)

One-dimensional transient flow equations for saturated soils have basically involved equating the time differential of the constitutive equation for the soil structure to the divergence of the velocity of flow of water from an element (Terzaghi, 1943). Two- and three-dimensional seepage analyses for saturated soils have been either coupled or uncoupled from the equilibrium requirements. Unfortunately, a similar type of formulation for seepage analysis in unsaturated soils has been lacking. This has been the result of an incomplete understanding of the stress state, constitutive relations and other physical relations for an unsaturated soil.

This paper will first outline the necessary physical relations for an unsaturated soil and provide a transient flow model for an unsaturated soil which is consistent with that commonly used for saturated soils. The formulations are for the one-dimensional case.

Stress State Variables

An element of an unsaturated soil is considered to be a mixture with two phases (i.e., soil particles and contractile skin) that come to equilibrium under an applied stress gradient and two phases (i.e., the air and water) that flow under applied pressure gradients. The air phase is assumed to be continuous. $(\sigma_y - u_a)$ and $(u_a - u_w)$ are used as independent stress state variables for an unsaturated soil where

σ_y = total stress in the y-direction, u_a = pore-air pressure, and u_w = pore-water pressure (Fredlund and Morgenstern, 1977). These stress state variables have been tested by means of "null type" tests and have become widely used for describing the behavior of unsaturated soils.

Continuity Equation

The deformation state variables required to describe changes in volumes associated with each of an element must be consistent with the continuity requirement for an unsaturated soil. If the soil particles are assumed to be incompressible and the volume change of the contractile skin assumed internal to the element, the continuity equation is then written,

$$\frac{\Delta V}{V} = \frac{\Delta V_w}{V} + \frac{\Delta V_a}{V} \qquad [1]$$

where: V = total volume of the element,

ΔV = change in volume of soil structure or overall element,

ΔV_w = change in volume of water in the element, and

ΔV_a = change in volume of air in the element.

Constitutive Relations

The volumetric continuity requirement for an unsaturated soil shows that at least two constitutive relations are required to define volume weight relations.

The constitutive relation for an unsaturated soil can be summarized as follows (Fredlund and Morgenstern, 1976).

Soil Structure

$$\Delta V = [m_1^s\, d(\sigma_y - u_a) + m_2^s\, d(u_a - u_w)]V \qquad [2]$$

where: m_1^s = compressibility modulus of soil structure when $d(u_a - u_w)$ is zero.

m_2^s = compressibility modulus of soil structure when $d(\sigma_y - u_a)$ is zero.

Water Phase

$$\Delta V_w = [m_1^w\, d(\sigma_y - u_a) + m_2^w\, d(u_a - u_w)]V \qquad [3]$$

where: m_1^w = slope of the $d(\sigma_y - u_a)$ versus volume of water plot when $d(u_a - u_w)$ is zero.

m_2^w = slope of the $d(u_a - u_w)$ versus volume of water plot when $d(\sigma_y - u_a)$ is zero.

Air Phase

$$\Delta V_a = [m_1^a \, d(\sigma_y - u_a) + m_2^a \, d(u_a - u_w)]V \qquad [4]$$

where: m_1^a = slope of the $d(\sigma_y - u_a)$ versus volume of air plot when $d(u_a - u_w)$ is zero, and

m_2^a = slope of the $d(u_a - u_w)$ versus volume of air plot when $d(\sigma_y - u_a)$ is zero.

Any two of the three constitutive relations can be used in an independent manner. Quantitatively, the air phase constitutive relationship is equal to the difference between the soil structure constitutive relationship and the water phase constitutive relationship.

Flow Laws

Flow equations are required for the water and air phases.

Darcy's Law

Darcy's law can be used to describe the flow under a hydraulic gradient in an unsaturated soil (Childs and Collis-George, 1950).

$$q_1 = -[k_w \frac{\partial h_w}{\partial y}] \qquad [5]$$

where: q_1 = (liquid) water flux,

k_w = coefficient of permeability with respect to the water phase,

h_w = hydraulic head in the water phase (i.e., $\frac{u_w}{g\rho_w} + Y$),

ρ_w = density of liquid water,

g = acceleration due to gravity,

Y = elevation head (is assumed zero in this analysis), and

y = direction of flow.

The coefficient of permeability is shown as a constant; however, it can be made a function of degree of saturation, void ratio, water content or any combination of these properties (Corey, 1957). Its magnitude must simply be revised as the numerical analysis proceeds.

Fick's Law

Blight (1971) used a simple form of Fick's law to describe the mass of air flowing through an unsaturated soil continua.

$$\frac{\partial m}{\partial t} = - D \frac{\partial p}{\partial y} \qquad [6]$$

where: m = mass of air in the element,

D = transmission constant having the same units as coefficient of permeability,

p = absolute air pressure (i.e., $u_a + u_{atm}$),

u_{atm} = atmospheric air pressure, and

t = time.

Modified Fick's Law

de Vries (1975) used a modified Fick's law to describe the vapor flux under conditions of uniform and constant total pressure,

$$q_v = \frac{-1}{g\rho_{wv}} [D_{vap} (\frac{\omega^*}{R\theta}) \frac{\partial p_v}{\partial y}] \qquad [7]$$

where: q_v = water vapor flux,

D_{vap} = molecular diffusivity of water vapor in air,

ω^* = molecular weight of water vapor,

R = universal gas constant,

θ = absolute temperature,

p_v = vapor pressure, and

ρ_{wv} = density of water vapor.

Pore Pressure Parameters

Fredlund (1976) and Hasan and Fredlund (1980) defined a pore pressure parameter, B_{aw}, to predict the changes in pore-water pressure resulting form a change in pore-air pressure. The change in pore-air pressure of interest in this paper is the result of the thermal gradient imposed on the soil.

$$B_{aw} = \frac{\Delta u_a}{\Delta u_w} \qquad [8]$$

where: Δu_a = change in pore-air pressure, and

Δu_w = change in pore-water pressure.

The B_{aw} pore pressure parameter is a function of the compressibility of the air-water mixture and the compressibility of the soil (Fredlund, 1976).

THEORY (TRANSIENT PARTIAL DIFFERENTIAL EQUATIONS)

Three partial differential equations are required for the rigorous non-isothermal analysis for an unsaturated soil.

Heat Flow Equation

The Fourier diffusion equation is used to describe conductive heat transfer for above freezing conditions in the soil mass. The equation expresses the heat flow rate in terms of the thermal conductivity and the temperature gradient. The thermal conductivity and heat capacity values used in this analysis are given in Tables 1 and 2.

$$\frac{\partial\theta}{\partial t} = \alpha \frac{\partial^2\theta}{\partial y^2} \qquad [9]$$

where: θ = temperature,

α = thermal diffusivity factor = $\lambda/c\rho_s$,

λ = thermal conductivity,

c = heat capacity, and

ρ_s = soil bulk density,

The heat flow equation is solved using an explicit finite difference method. The solution gives the temperature dissipation with space and time and must be solved at least one time step in advance of the water and air phase equations.

Water Phase Partial Differential Equation

The water phase partial differential equation considers the total flux of (liquid) water in an element of an unsaturated soil continua considered. The net flux of (liquid) water is the result of imposed hydraulic and vapor pressure (or in other words the relative humidity) gradients at the surface boundary of the element. Equating the net flux of (liquid) water to the time differential of the water phase constitutive relationship, rearranging and simplifying gives the water phase partial differential equation (Dakshanmurthy and Fredlund, 1981b).

$$\frac{\partial u_w}{\partial t} = C_w \frac{\partial u_a}{\partial t} + c_{v1}^w \frac{\partial^2 u_w}{\partial y^2} + c_{vv}^w \frac{\partial^2 p_v}{\partial y^2} \qquad [10]$$

where: $C_w = -(1 - m_2^w/m_1^w) \,/\, (m_2^w/m_1^w)$ and is called the interactive constant associated with the water phase equation. This equation is further simplified by letting $R_w = m_2^w/m_1^w$. When the soil is saturated, R_w approach unity,

$$c_{v1}^w = \frac{1}{R_w} \frac{k_w}{\rho_w g} \frac{1}{m_1^w} \qquad [11]$$

; the coefficient of consolidation for the water (liquid) phase, and

$$c_{vv}^w = \frac{1}{R_w} D_{vap} \left(\frac{\omega^*}{R\theta}\right) \frac{1}{\rho_{wv} g} \frac{1}{m_1^w} \qquad [12]$$

; the coefficient of consolidation for the water (vapor) phase.

The vapor pressure in equation [10] can be expressed as the product of the saturation vapor pressure, p_v^s, and the relative humidity, h (de Vries, 1975).

$$p_v = p_v^s\ h \qquad [13]$$

The relative humity, h, can also be expressed in terms of total potential, ϕ, molecular weight of water vapor, ω^*, specific volume of water vapor, V_w^o, universal gas constant, R, and the absolute temperature, θ (van Haveren and Brown, 1972).

$$h = e^{\left(\frac{\phi \omega^* V_w^o}{R\theta}\right)} \qquad [14]$$

where: $\phi = [(u_a - u_w) + \pi]$ (i.e., total potential)

π = osmotic suction; assumed to be constant equal to 102 kPa for this study.

Air Phase Partial Differential Equation

The air phase is compressible and flow occurs in response to a pressure gradient (i.e., the graviety term is negligible). The constitutive relationship for the air phase defines the volume of air in the element for any combination of the total, water and air pressures. Equating the net mass flux of dry or moist air through the element to the time differential of the air phase constitutive relationship, rearranging and simplifying, gives the modified air phase partial differential equation (Dakshanamurthy and Fredlund, 1981a).

$$\frac{\partial u_a}{\partial t} = C_a \frac{\partial u_w}{\partial t} + C_\theta \frac{\partial \theta}{\partial t} + c_v^a \frac{\partial^2 u_a}{\partial y^2} \qquad [15]$$

where: $C_a = \dfrac{-m_2^a/m_1^a}{(1 - m_2^a/m_1^a) + \dfrac{(1 - S)n}{m_1^a(\Delta u_a + u_{atm})}}$; the interactive [16]

pressure constant associated with the air phase equation.

This equation is further simplified by letting $R_a = m_2^a/m_1^a$.

Δu_a = the difference between the pore-air pressure and standard atmospheric conditions.

$$C_\theta = \frac{1}{\theta}\left[\frac{(1 - S)n\,(\Delta u_a + u_{atm})}{(1 - R_a)(\Delta u_a + u_{atm})m_1^a + (1 - S)n}\right]; \quad [17]$$

the interactive thermal constant associated with the air phase equation,

where: S = degree of saturation, and

n = porosity.

$$c_v^a = \frac{DR\theta}{\omega}\left[\frac{1}{(1 - R_a)(\Delta u_a + u_{atm})m_1^a + (1 - S)n}\right]; \quad [18]$$

the coefficient of consolidation for the air phase.

The dissipation of the excess pressures of the pore-air and pore-water phases are obtained by solving the equations [10] and [15] simultaneously. The magnitudes of coefficients of consolidation for the water and air phases viz., c_{v1}^w, c_{vv}^w and c_v^a are primarily dependent upon the coefficients of permeability values for the water and air phases.

TRANSLATION OF PORE PRESSURES TO VOLUME-WEIGHT SOIL PROPERTIES

The solution to the equation [9], [10] and [15] enables the prediction of changes in the temperature, the pore-water pressure, the pore-air pressure, the vapor pressure, the partial pressure during the transient process under complex environmental changes at the boundary. The pressure changes are, in turn, used to determine the change in void ratio, degree of saturation, and the moisture content. The volume change of an unsaturated soil can be converted into a prediction of swell or shrinkage by substituting the change in stress state variables, for a given period of time, into the respective constitutive relations.

Void Ratio (Swell or Shrinkage)

The change in volume of the element computed as per equation [2] is equal to the change in void ratio, Δe. The total volume of the element, V, is $(1 + e)$. The void ratio at various times during the transient process can be written.

$$e = e_i + \Delta e \qquad [19]$$

where: e = void ratio at any time,

e_i = void ratio at the beginning of the transient process, and

Δe = change in void ratio during the transient process.

The change in void ratio enables the computation of the amount of swell or shrinkage, ΔH, during the transient process.

$$\Delta H = H_i \left[\frac{\Delta e}{1 + e_i}\right] \qquad [20]$$

where: H_i = initial thickness of soil layer.

Water Content

The change in volume of water in a unit volume of soil can also be computed by substituting the change in stress state variables at any time, into the water phase constitutive relation (i.e., equation [3]).

The gravimetric water content at any time during the transient process can then be written:

$$w = w_i + \Delta V_w \frac{\rho_w}{\rho_d} \qquad [21]$$

where: w = water content at any time,

w_i = water content (gravimetric) at the beginning of the transient process, and

ρ_d = dry density of the soil.

Degree of Saturation

The degree of saturation is calculated using the following volume-weight relationship.

$$Se = wG_s \qquad [22]$$

where: G_s = specific gravity of the soil solids.

A CASE STUDY

The main part of the theoretical model is checked using a case study from the city of Regina, Saskatchewan. The study was conducted by the Division of Building Research, National Research Council, Saskatoon, starting in 1961 (Hamilton, 1965). The case study analyzed involved the movement of the slab-on-grade in a light industrial structure, Lorne Street, North Central Regina, Saskatchewan. The site was instrumented with neutron moisture meter access tubes to estimate the moisture content profile in-situ. Three vertical ground movement, spiral-foot movement gauges were also installed and referenced to a deep bench mark. Several tensiometers were installed and surveys were performed on the floor levels. The movement gauges were embedded at 2' (0.6 m), 3' (0.9 m) and 8 1/2' (2.6 m) depths, respectively, below the concrete floor slab. The vertical ground movements were recorded periodically over the period of years (1961-1972). In 1962 a rather sudden floor heave of 3 1/2" (8.9 cm) had developed in the central portion of the slab directly above the center line of a subfloor plumbing trench. This heave caused a maximum angular distortion of 1/20 and seriously damaged interior partitions. At the point of maximum heave a leak was discovered in the hot water line from which it was estimated that some 7500 gallons of water had escaped. The ground under the floor slab became saturated and the water migrated for a considerable distance along the length of the back-filled plumbing trench.

EXAMPLE PROBLEMS

Measurements from the case study do not allow the verification of all aspects of the proposed analytical model. Therefore, several example problems are also considered to demonstrate the solution of the three partial differential transient flow equations.

The simultaneous solutions of all three differential equations enables the prediction of the transient and ultimate values of ground movements (i.e., swelling or shrinking) and the resulting volume-weight soil properties during the transient process. Four examples are solved in this paper. These are: i) isothermal swelling), ii) non-isothermal swelling, iii) isothermal consolidation, and iv) non-isothermal consolidation. Example No. 1 (i.e., isothermal swelling) is used to test the theoretical analysis by attempting to fit the analysis with the field data observed in a case study carried out at Regina, Saskatchewan. The other three example problems are used to demonstrate the effect of enviornmental changes such as imposed thermal gradients and vapor pressure gradients. No attempt has been made in this paper to compare the solution of these three example problems with independently observed values, for lack of data.

The first two example problems (i.e., isothermal swelling and non-isothermal swelling) are solved assuming a two-meters thick layer of Regina clay. As well, the last two example problems (i.e., isothermal consolidation and non-isothermal consolidation) are solved assuming a ten centimeter thick Regina clay. The soil properties of the first series of example problems are shown in Table 1. The soil properties of the second series of example problems are shown in Table 2. The index properties and other classification tests were carried out by Fredlund (1964). Both soils are similar. As a result of solving many example problems, it was observed that only if a relatively small thickness of soil layer is assumed in the analysis, it is possible to clearly show the effects of thermal and vapor pressure gradients. Therefore, a

Table 1 Summary of Classification Tests, Compressibility and Permeabilities of Regina Clay

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Specific Gravity	2.73
Atterberg Limits	
Liquid Limit	82%
Plastic Limit	33.7%
Shrinkage Limit	11.9%
Plasticity Index	48.3%
Grain Size Distribution	
% Sand Sizes	41%
% Silt Sizes	10%
% Clay Sizes	49%

Compressibility*

$m_1^w = 0.000170/\text{kPa}$

$m_1^a = 0.00004/\text{kPa}$

$R_w = 0.95, \ R_a = -2.75$

Permeability*

$k_w = 0.8 \times 10^{-9}$ m/sec

$D_{vap} = D_{air} = 3.0 \times 10^{-9}$ m^2/sec

Volume-Weight Properties

Soil Density, $\rho_s = 1.8$ gm/cc

Porosity, n = 52.56%

Degree of Saturation, S = 52%

Thermal Properties

λ = Thermal Conductivity = 0.4574 cal/m. sec. °K

c = Heat Capacity = 30 cal/gm °K

Physico Chemical Properties

ω^* = Molecular weight of water vapor - 18.015 gm/mole

ω = Molecular weight of moist air (80% of water vapor and 20% of air) = 20.2048 gm/mole

R = Universal gas constant = 847.825 gm.m/mole/°K

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* Estimated values from constant volume odeometer tests performed on samples from Lorne Street, North Central Regina.

Table 2 Summary of Classification Tests, Compressibility and Permeabilities of Regina Clay

Specific Gravity	2.83
Atterberg Limits	
Liquid Limit	75.5%
Plastic Limit	24.9%
Shrinkage Limit	13.1%
Plasticity Index	50.6%
Grain Size Distribution	
% Sand Sizes	8
% Silt Sizes	41
% Clay Sizes	51

Compressibility*

$m_1^w = 0.0007614/\text{kPa}$

$m_1^a = 0.0003263/\text{kPa}$

$R_w = 0.7$, $R_a = -0.01$

Permeability*

$k_w = 0.6 \times 10^{-10}$ m/sec

$D_{vap} = D_{air} = 1.0 \times 10^{-9}\ m^2/\text{sec}$

Volume-Weight Properties

Soil Density, $\rho_s = 1.8$ gm/cc

Porosity, n = 50%
Degree of Saturation, S = 70%

Thermal Properties

λ = Thermal Conductivity = 0.4574 cal/m. sec. °K
c = Heat Capacity = 30 cal/gm °K

Physico Chemical Properties

ω^* = Molecular weight of water vapor = 18.015 gm/mole
ω = Molecular weight of moist air (80% of water vapor and 20% of air) = 20.2048 gm/mole
R = Universal gas constant = 847.825 gm.m/mole/°K

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* Estimated values

ten-centimeter thick soil layer is assumed in the second series of the example problems.

In all the above example problems it is assumed that the subsoil is initially in a known state of stress. It is then assumed that a sudden environmental change such as flooding (i.e., infiltration) or drying (i.e., evaporation) is imposed at the boundary. The boundary conditions used in the example problems are given in Table 3 through Table 8. Sudden environmental changes such as infiltration or evaporation create a change in the initial boundary conditions. For non-isothermal conditions it is assumed that the pore-air pressure can change as a result of a temperature change and the corresponding change in the pore-water pressure is computed using the B_{aw} parameter. B_{aw} values used in the example problems are 0.1 and 0.3. The unsaturated soil mass will eventually equilibriate to the new boundary conditions. The solution showing the temperature, the pore pressure changes with time is obtained by solving equations [9], [10] and [15]. A computer program has been developed to solve these partial differential equations (Dakshanamurthy and Fredlund, 1980a).

RESULTS AND DISCUSSIONS

Based on the solution to equations [9], [10] and [15], the distribution of pore-water pressure, pore-air pressure, vapor pressure and partial pressure can be computed. The void ratio can be computed corresponding to any time from equation [2] and [19]. Subsequently, the amount of heave can be computed from equation [20]. The water content

Table 3

Boundary Conditions for Example No. 1 (Isothermal Swelling)

Variable	Depth (m)	Boundary Conditions	
		Initial	Final
Pore-Water (kPa) (absolute)	0 2	-816 -200	102 102
Pore-Air (kPa) (absolute)	0 2	102 102	102 102
Temperature (°C)	0 2	20 20	20 20
Relative Humidity (%)	0 2	99.2480 99.7015	99.9246 99.9246
Vapor Pressure (kPa)	0 2	2.338 2.338	2.338 2.338

Table 4

Boundary Conditions For Example No. 2
(Non-Isothermal Swelling)

Variable	Depth (m)	Boundary Conditions	
		Initial	Final
Pore-Water (kPa) (absolute)	0 2	-816 -200	102 102
Pore-Air (kPa) (absolute)	0 2	102 102	102 102
Temperature (°C)	0 2	10 10	25 25
Relative Humidity (%)	0 2	99.2278 99.6894	99.9256 99.9256
Vapor Pressure (kPa)	0 2	1.210 1.220	3.156 3.156

Table 5

Boundary Conditions for Example 3(a) (Isothermal Consolidation)

Variable	Boundary Conditions	
	Initial	Final
Pore-Water (kPa) (absolute)	-280	-420
Pore-Air (kPa) (absolute)	102	102
Temperature (°C)	20	20
Relative Humidity (%)	99.6425	99.5393
Vapor Pressure (kPa)	2.294	2.294

Table 6

Boundary Conditions for Example 3(b) (Isothermal Swelling)

Variable	Boundary Conditions	
	Initial	Final
Pore-Water (kPa) (absolute)	-420	-280
Pore-Air (kPa) (absolute)	102	102
Temperature (°C)	20	20
Relative Humidity (%)	99.5393	99.6425
Vapor Pressure (kPa)	2.294	2.294

Table 7

Boundary Condition for Example 4(a)
(Non-Isothermal Consolidation)

Variable	Boundary Conditions	
	Initial	Final
Pore-Water (kPa) (absolute)	-280	-420
Pore-Air (kPa) (absolute)	102	102
Temperature (°C)	10	25
Relative Humidity (%)	99.6328	99.5455
Vapor Pressure (kPa)	1.188	3.156

Table 8

Boundary Condition for Example 4(b)
(Non-Isothermal Swelling)

Variable	Boundary Conditions	
	Initial	Final
Pore-Water (kPa) (absolute)	-420	-280
Pore-Air (kPa) (absolute)	102	102
Temperature (°C)	10	25
Relative Humidity (%)	99.5268	99.6473
Vapor Pressure (kPa)	1.188	3.156

is computed from equations [3] and [21]. The degree of saturation is computed from equation [22].

Example No. 1 (Table 3)

Figure 1 shows the pore-water pressure distribution throughout the clay layer as the result of sudden flooding at the surface. The initial (i.e., equilibrium) pore-water pressure was assumed to be -816 kPa at ground surface and the upper boundary was then instantaneously changed to a value of 102 kPa (i.e., atmospheric pressure) at the surface. It was assumed that the initial pore-water pressure varied linearly with depth and that the initial pore-water pressure at the two-meters depth was -200 kPa. Although there is no direct verification of these initial boundary values, they are consistent with laboratory test results from one-dimensional, oedometer tests at the light industrial structure, Lorne Street, North Central Regina (Yoshida et al, 1981). The relative humidity at the surface was assumed to be essentially the same as the relative humidity at the soil boundary. In other words, it was assumed that there was no water vapor movement when the site was flooded. As well, under isothermal condition the pore-air pressure was assumed to remain essentially unchanged.

Figure 2 shows the distribution of (gravimetric) water content throughout the soil layer, during the swelling process. The final water content throughout the soil layer equilibriates to a final value in response to the change in pore-water pressure and the soil modulus. Figure 2 also shows the observed water content profile after an elasped time period of one year, during which time the plumbing leak was noticed.

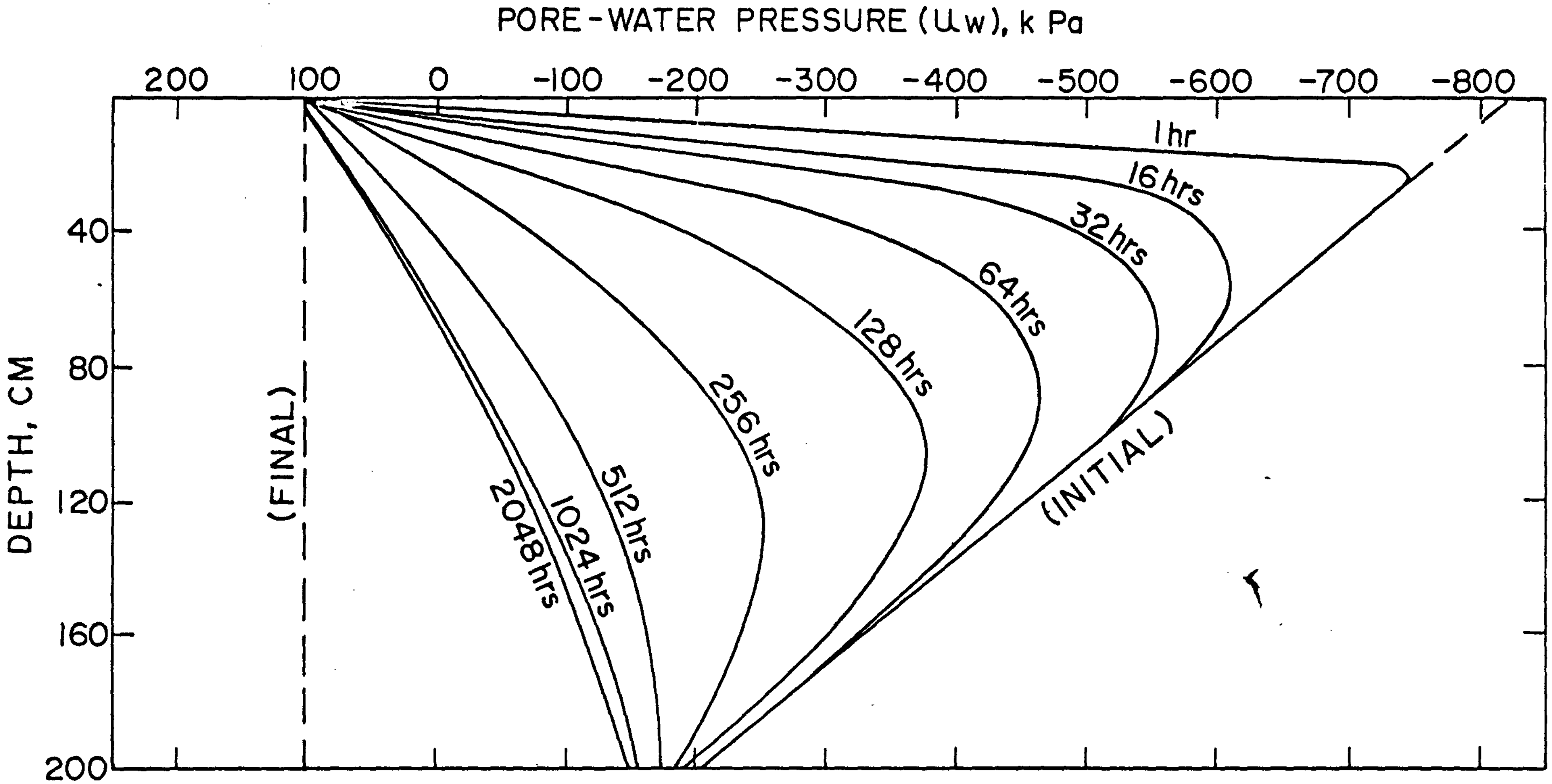

FIGURE 1 PORE-WATER PRESSURE DISTRIBUTION FOR THE EXAMPLE NO. 1

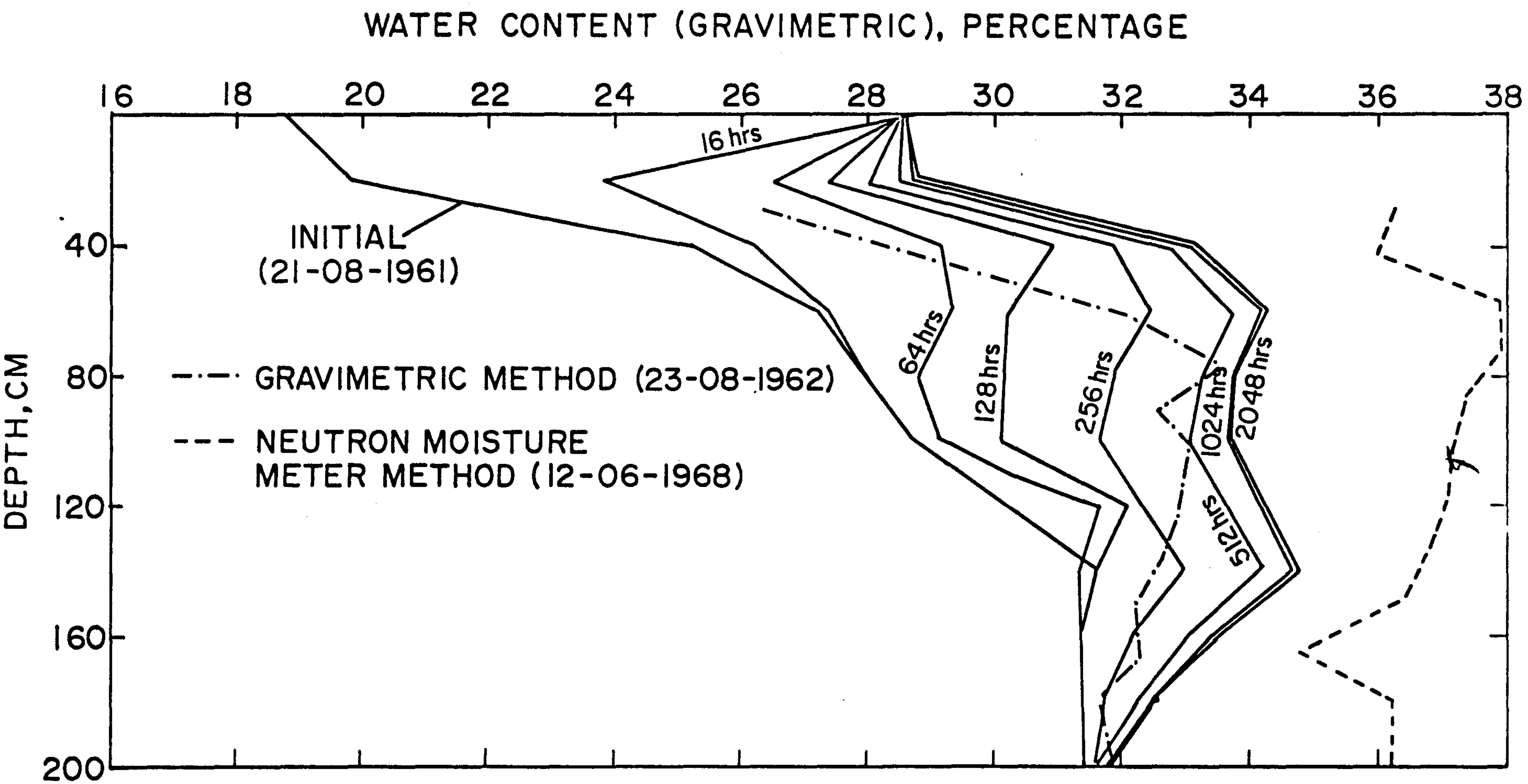

FIGURE 2 WATER CONTENT (GRAVIMETRIC) CHANGE THROUGHOUT THE SOIL LAYER FOR EXAMPLE PROBLEM NO. 1

The observed water content shows a reasonable agreement between the predicted and the observed values. The scatter seen in Figure 2 can be attributed to local test plot conditions, such as the presence of extensive, shrinkage cracks and fissures in the subsoil at the site. These conditions are responsible for a greater and faster absorption of water in the soil.

Figures 3 and 4 show the distribution of degree of saturation, S, and the void ratio, e, during the transient process under isothermal conditions. The changes in the degree of saturation depend upon the moduli used for soil structure (i.e., m_1^s and m_2^s).

Figure 5 shows the prediction of the total heave for the entire two-meters thick soil layer during the transient process. The predicted values of net heave compare well with the test site data (Hamilton, 1969).

Example No. 2 (Table 4)

Figure 6 shows the temperature isotherms within the clay layer as a result of an increase in temperature from 10° C (283.2° K) to 25° C (298.2° K). The imposed thermal gradient at the surface slowly dissipiates to the bottom of the soil layer and should eventually equilibriate to the new boundary condition. In other words, the lower boundary of the soil layer is assumed to be insulated. It should be pointed out that the dissipation of the temperature within the two-meters thick soil layer takes considerable time. Figure 6 shows even at the end of 2048 hours (i.e., 86 days) the temperature remains unchanged approximately 90 cm below. Because of the relatively slow change in tempera-

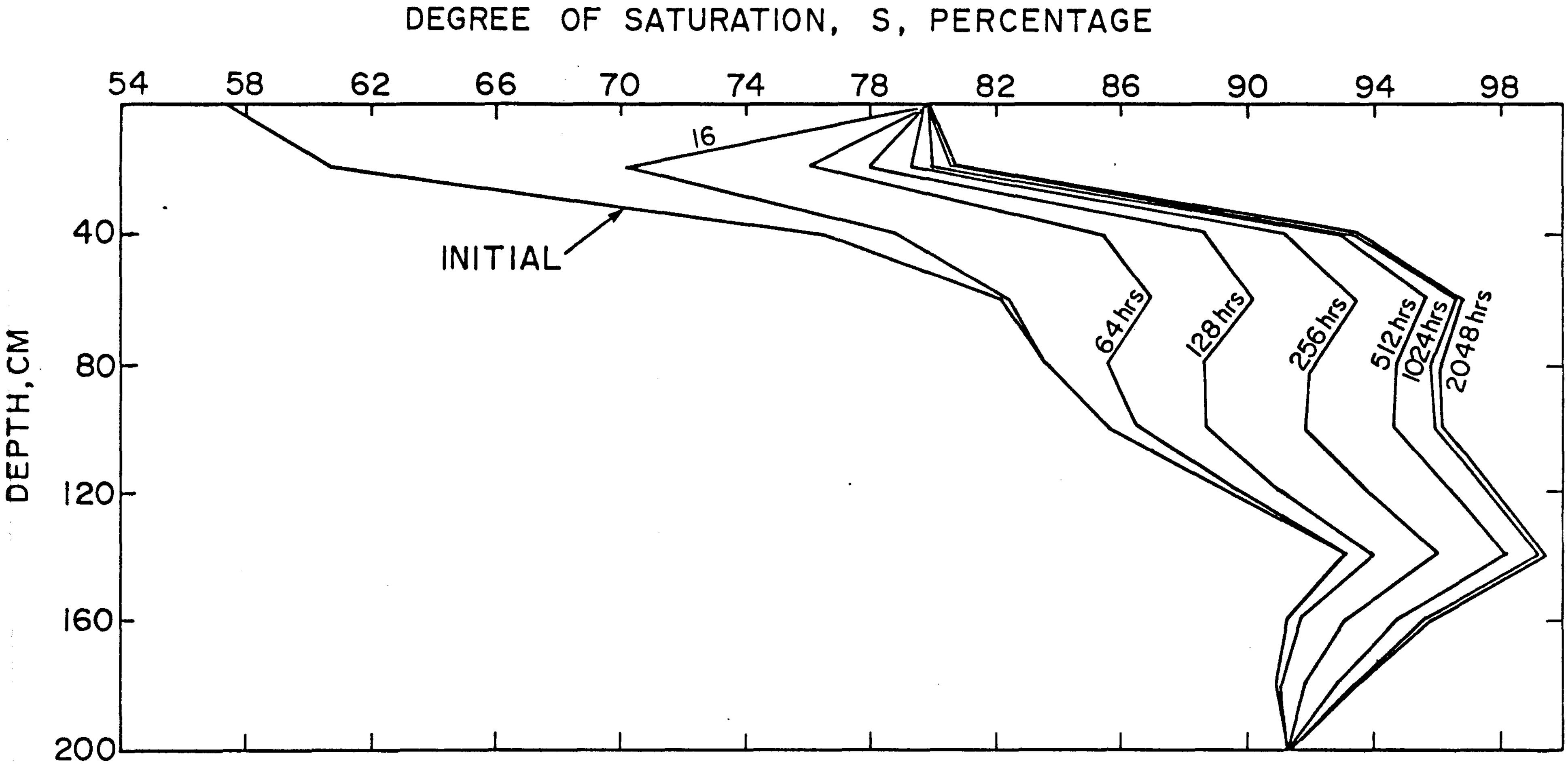

FIGURE 3 DEGREE OF SATURATION CHANGE THROUGHOUT THE SOIL LAYER FOR EXAMPLE PROBLEM NO. 1

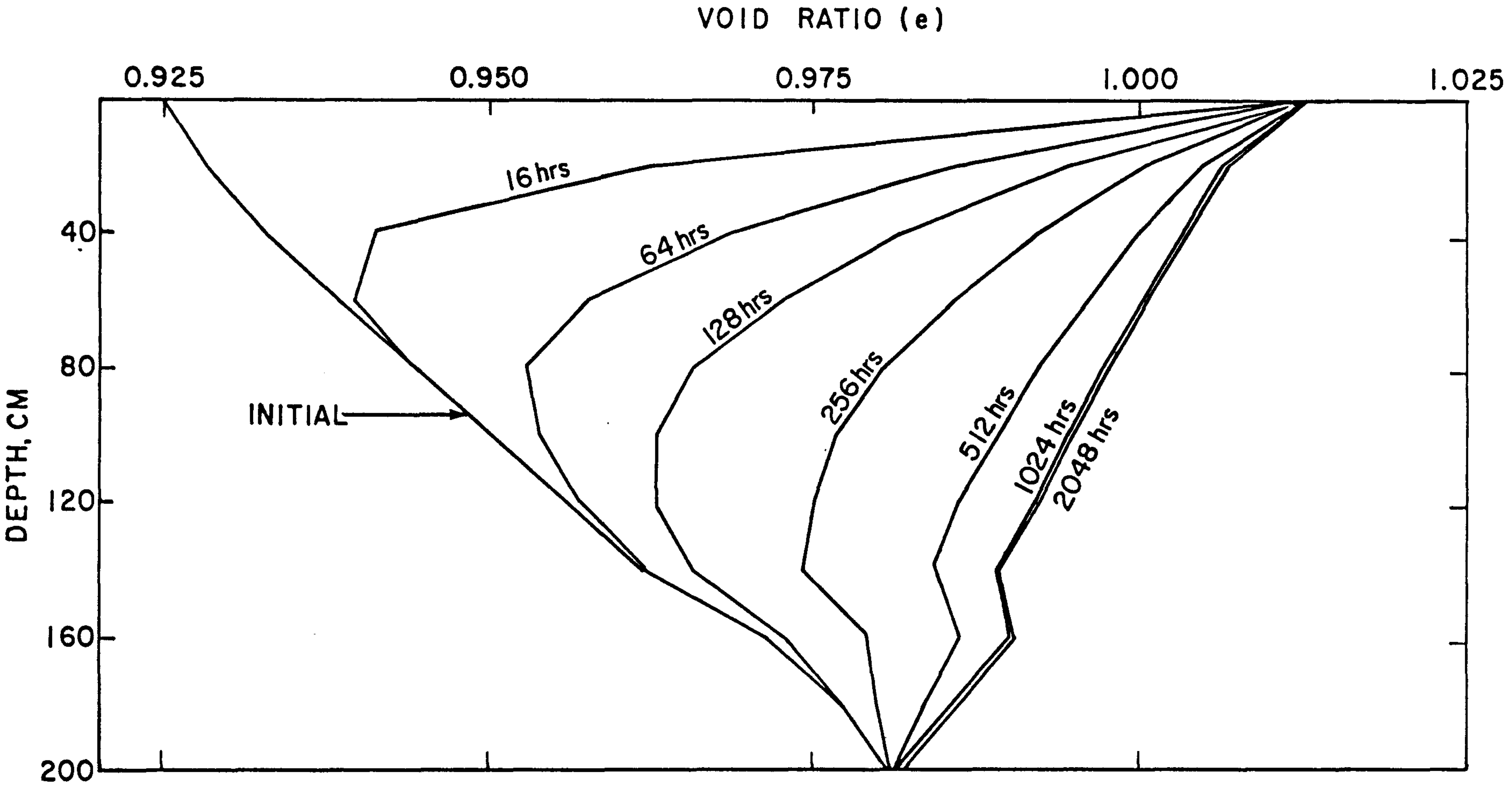

FIGURE 4 VOID RATIO CHANGE THROUGHOUT THE SOIL LAYER FOR EXAMPLE PROBLEM NO. 1

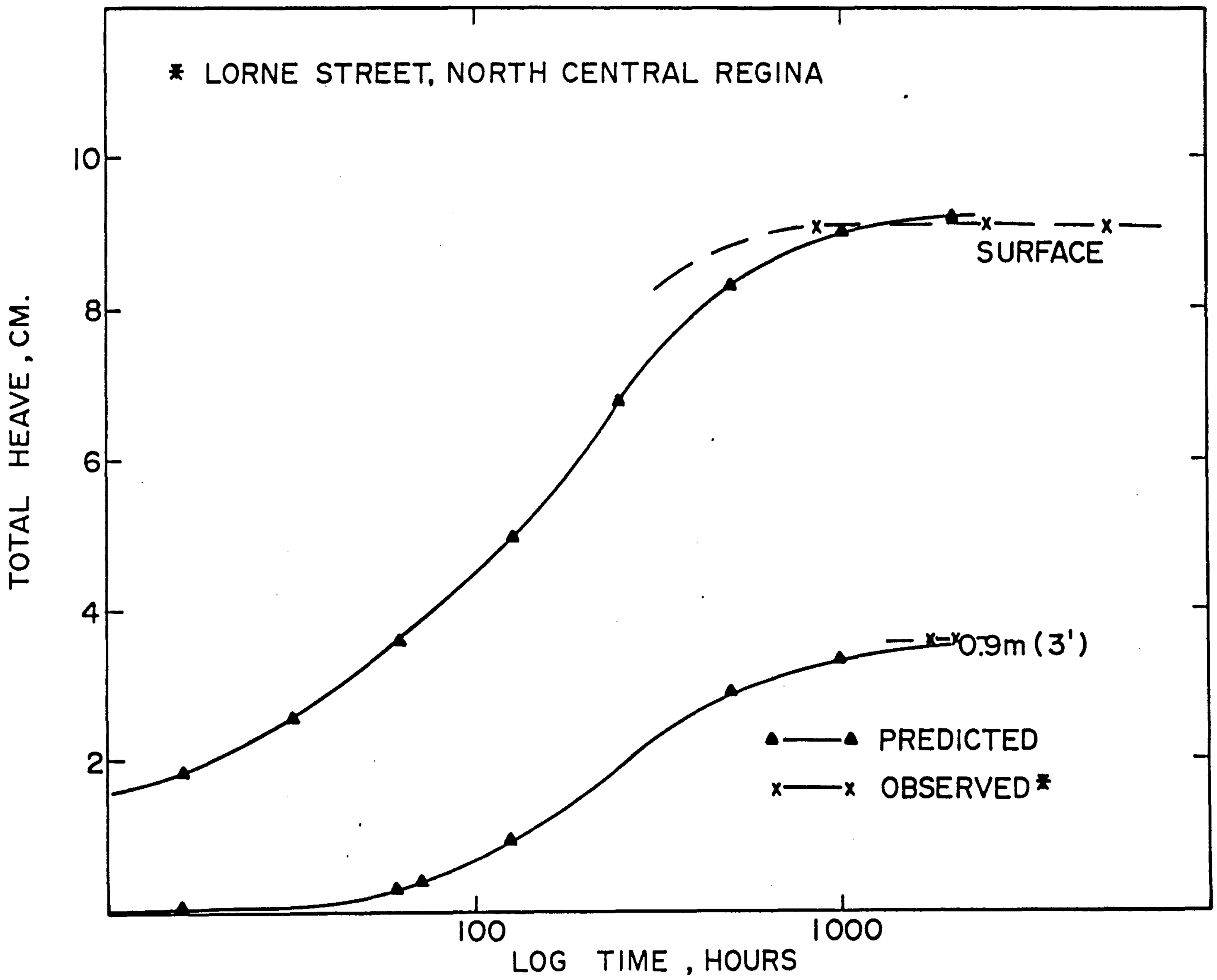

FIGURE 5 TOTAL HEAVE VERSUS LOG TIME

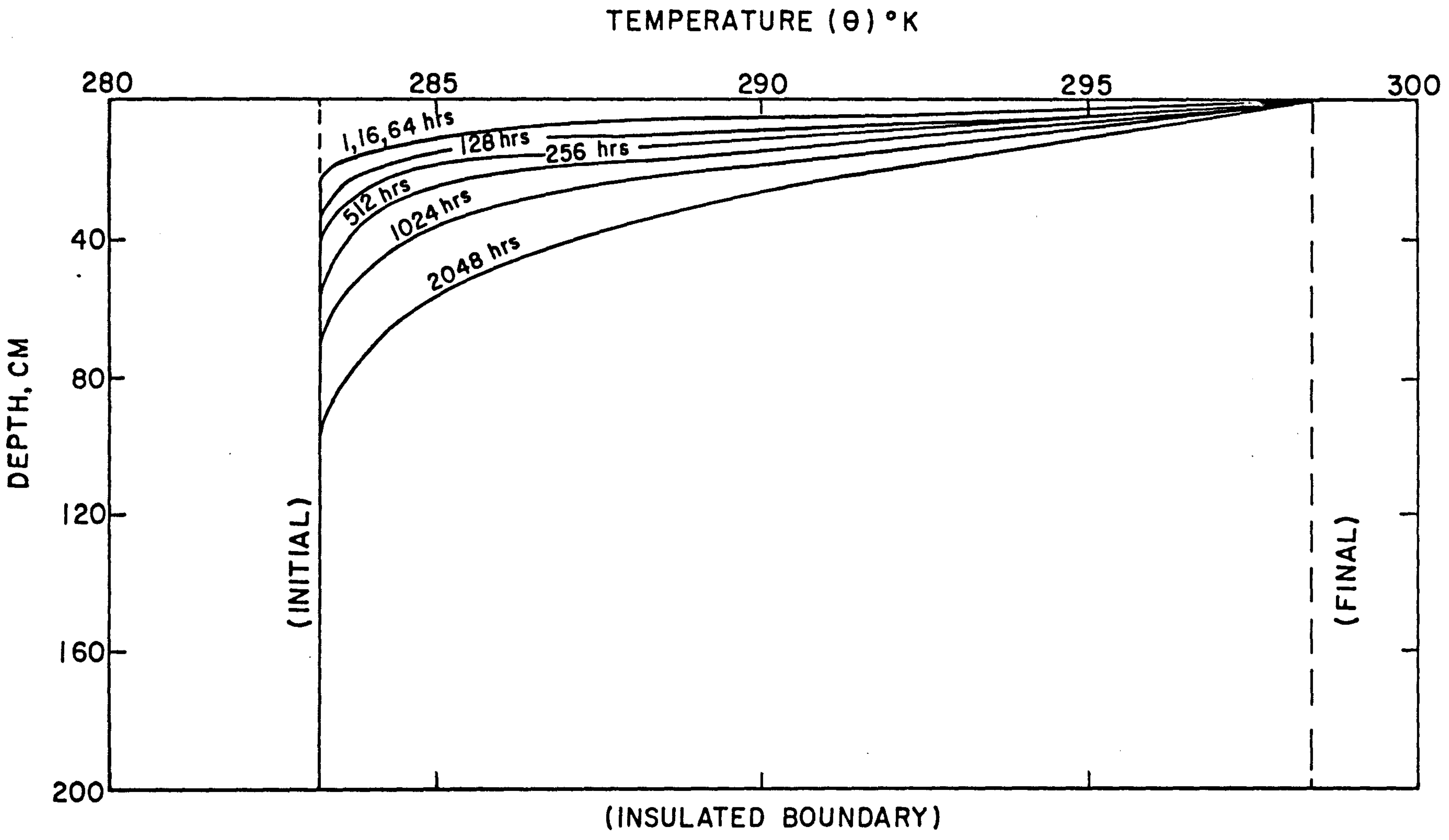

FIGURE 6 TEMPERATURE ISOTHERMS FOR THE EXAMPLE NO. 2

ture within the soil layer, the pore-air pressure changes under non-isothermal condition are insignificant. Figure 1 shows the pore-water pressure under non-isothermal swelling conditions are the same as those for isothermal conditions. In order to explicitly bring out the effect of thermal and vapor pressure gradients, another series of example problems are solved and discussed later.

Example No. 3 and 4 (Tables 5 to 8)

The second series of the example problems was selected to demonstrate the effects of thermal and vapor pressure gradients. A ten-centimeter thick compacted Regina clay is assumed. The family of curves shown in Figures 7 through 15 represent the solution of the transient flow equations.

Figure 7 shows the temperature isotherms within the clay layer as a result of an increase in the temperature from 10° to 25°. The figure illustrates how the imposed thermal gradient at the surface dissipates to the bottom of soil and eventually equilibriates to the new boundary condition. As in example No. 2, the lower boundary of the soil is assumed to be completely insulated. It should be pointed out here that a comparison of Figures 6 and 7 shows a significant difference in temperature distribution under the same thermal gradient (i.e., 15° C) and thermal properties of the soil but with different soil thickness assumed (i.e., 200 cms as opposed to 10 cms).

Figure 8 shows the pore-water pressure distribution for both isothermal and non-isothermal conditions under the consolidation process. The initial (equilibrium) pore-water pressure was -280 kPa, and the

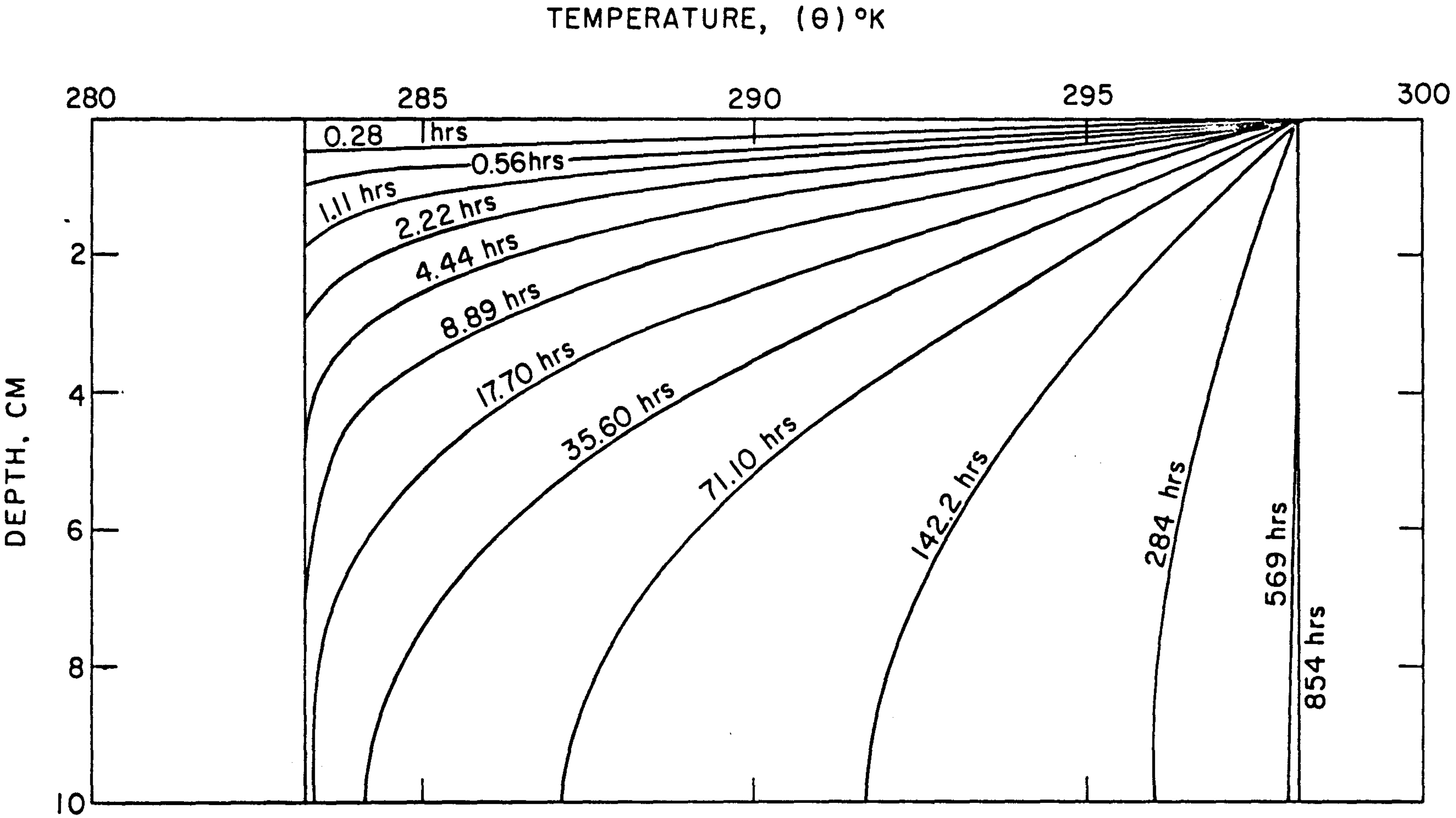

FIGURE 7 TEMPERATURE ISOTHERMS FOR THE EXAMPLE PROBLEM NO. 4

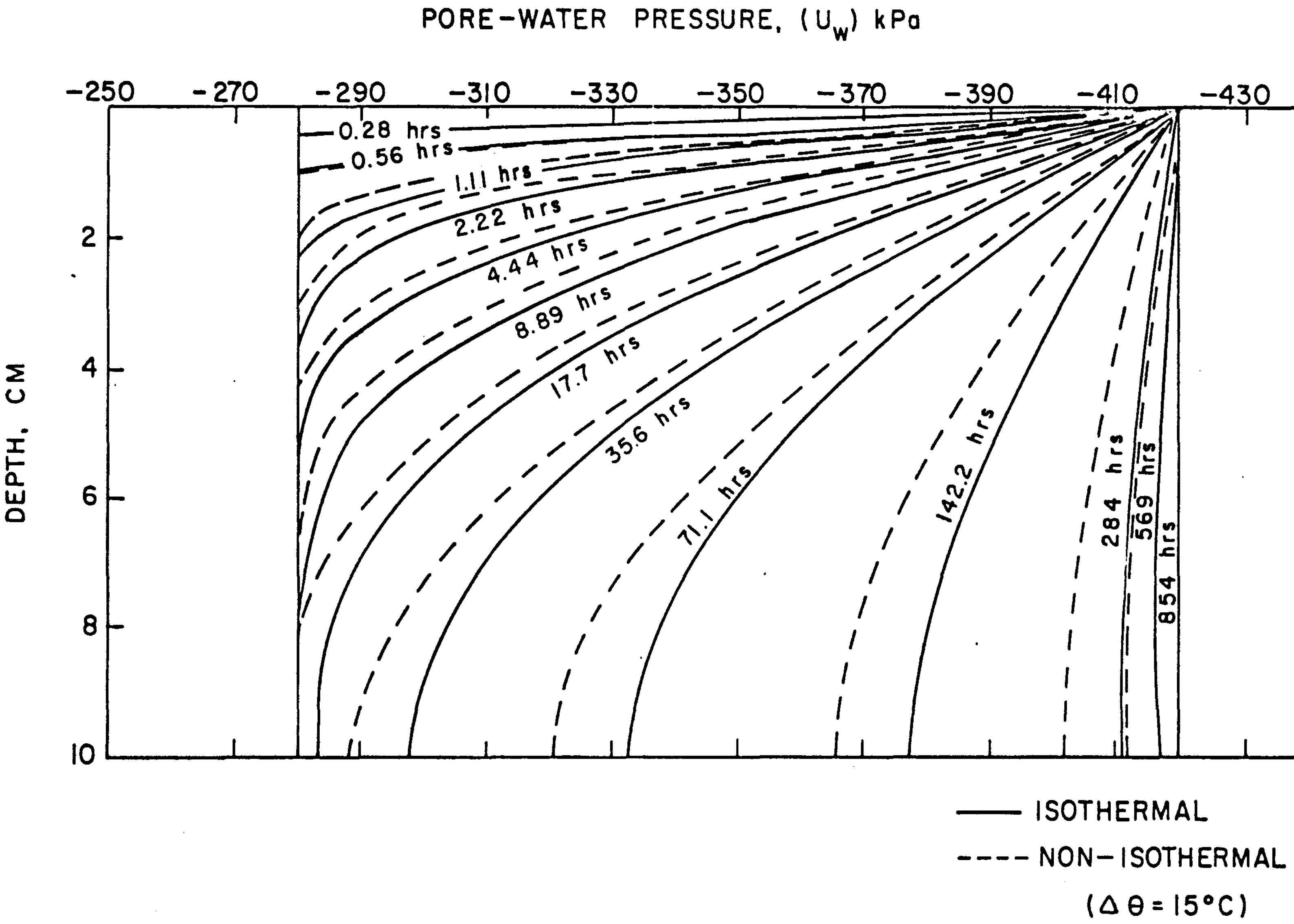

FIGURE 8 PORE-WATER (LIQUID) PRESSURE DISTRIBUTION (CONSOLIDATION PROCES
(EXAMPLE NO. 3 (a) AND 4 (a))

upper boundary was instantaneously changed to a value of -420 kPa. The temperature gradient was 15° C. The dashed and solid lines show the difference between isothermal and non-isothermal conditions. Similarly, Figure 9 shows the numerically identical but reverse process to that presented in Figure 8.

Figure 10 shows the distribution of (gravimetric) water content throughout the soil layer, under the consolidation and swelling processes for both isothermal and non-isothermal conditions. The final water content throughout the soil layer equilibriates to a value consistent with the change in pore-water and vapor pressures at the boundary.

Figures 11 and 12 show the related distribution of the degree of saturation (S) and the void ratio (e), during the isothermal and non-isothermal transient process.

Figure 13 shows the pore-air pressure distribution throughout the soil layer under non-isothermal conditions. Under isothermal conditions the pore-air pressure remains unchanged. However, for non-isothermal conditions the excess pore-air pressure generated due to the imposed thermal gradient varies with space and time as seen in Figure 13.

Figure 14 shows the vapor pressure distribution throughout the soil layer for the example problems No. 3 and 4. For the Example 3 (Isothermal Consolidation/Swelling) the pore-water pressure, which was initially at -280 kPa was instantaneously changed to a value of -420 kPa at the boundary (Figure 8). This change in the pore-water (liquid) pressure results in consequent change in the equilibrium relative humidity from 99.6425 percent to 99.5393 percent (Table 5). This small change in equilibrium relative humidity results in essentially a

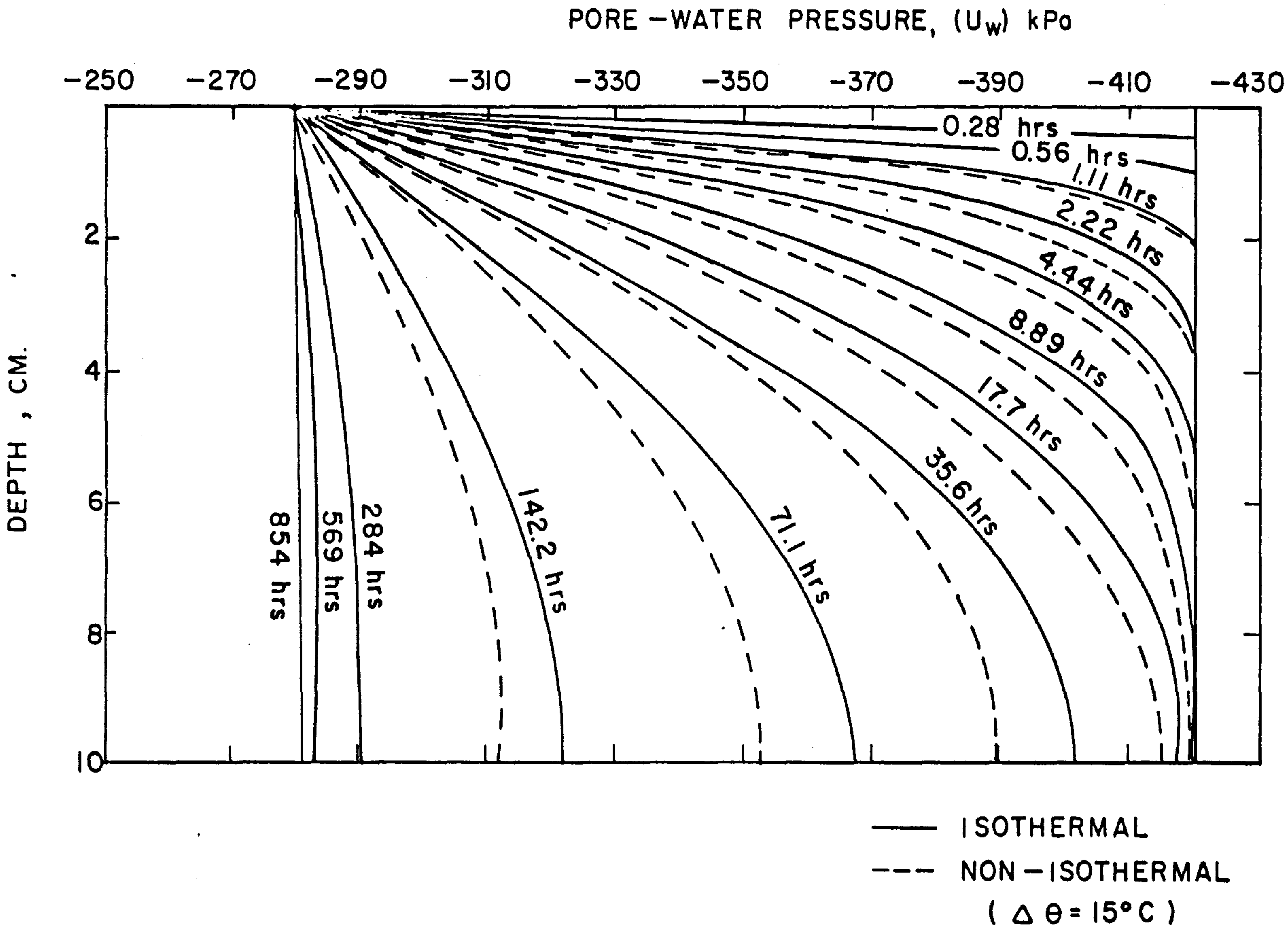

FIGURE 9 PORE – WATER (LIQUID) PRESSURE DISTRIBUTION (SWELLING PROCESS) (EXAMPLE NO. 3 (b) AND 4 (b))

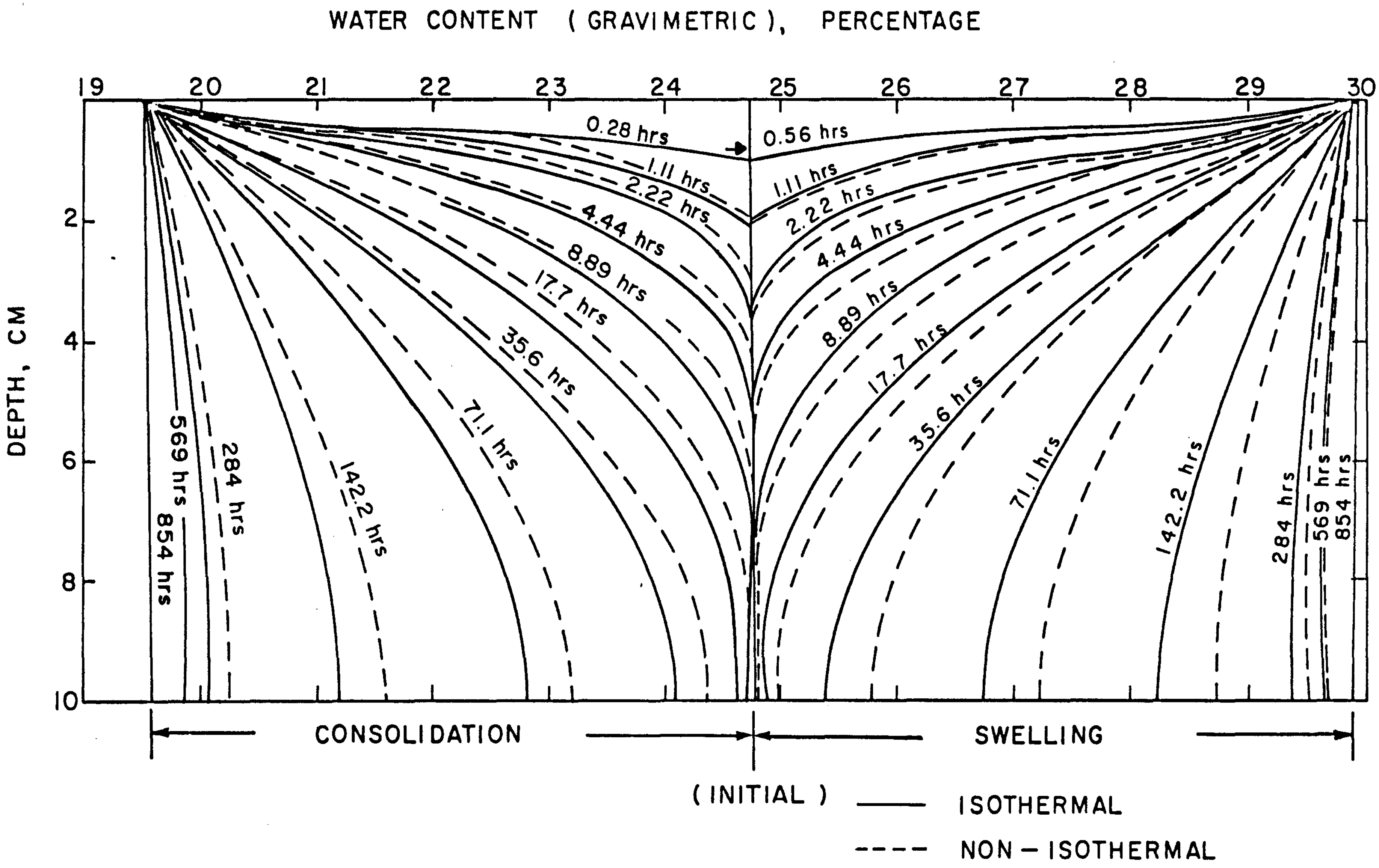

FIGURE 10 WATER CONTENT (GRAVIMETRIC) CHANGE THROUGHOUT THE SOIL LAYER (EXAMPLE NO. 3 AND 4)

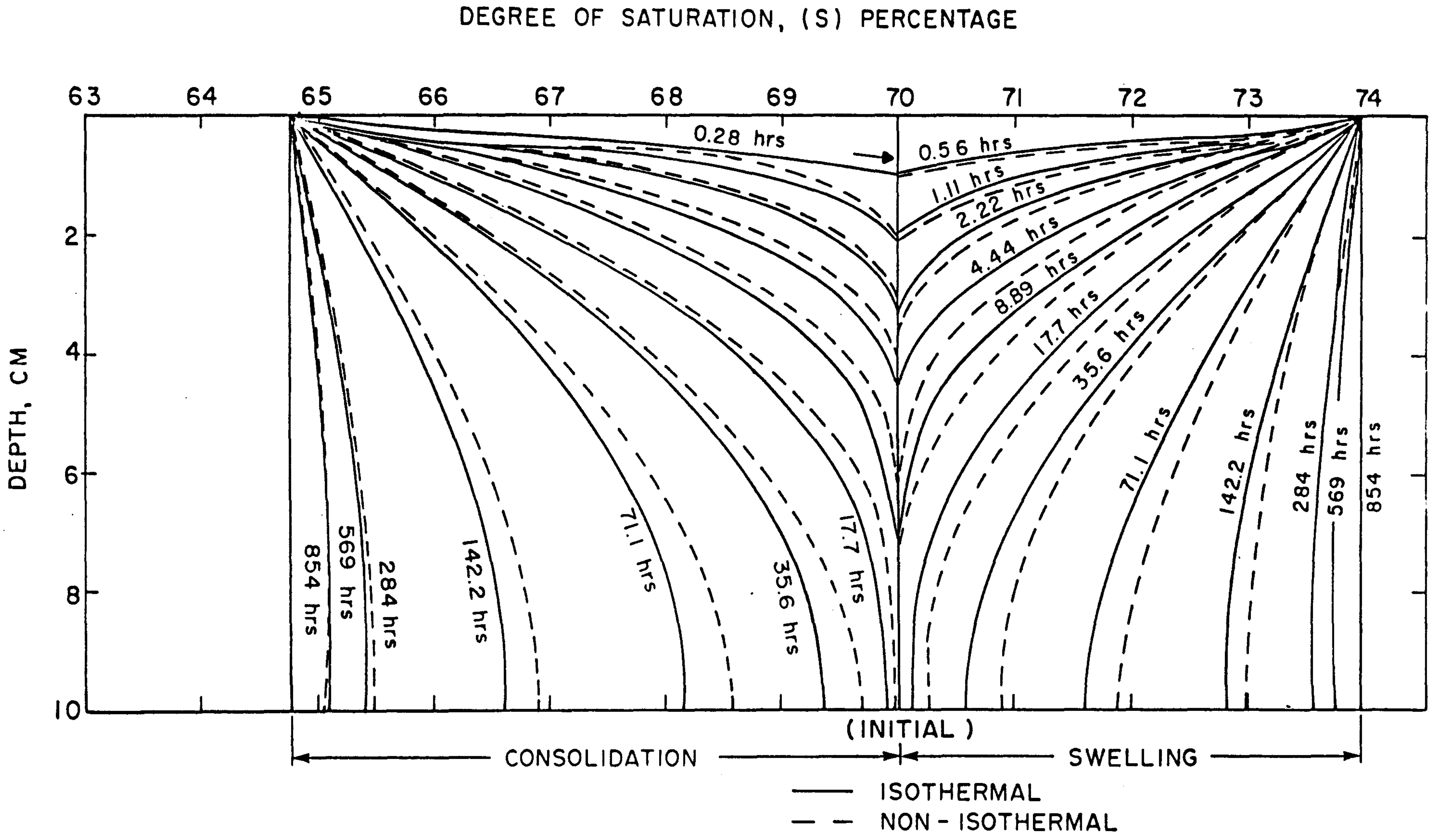

FIGURE 11 DEGREE OF SATURATION CHANGE THROUGHOUT THE SOIL LAYER (EXAMPLE NO. 3 AND 4)

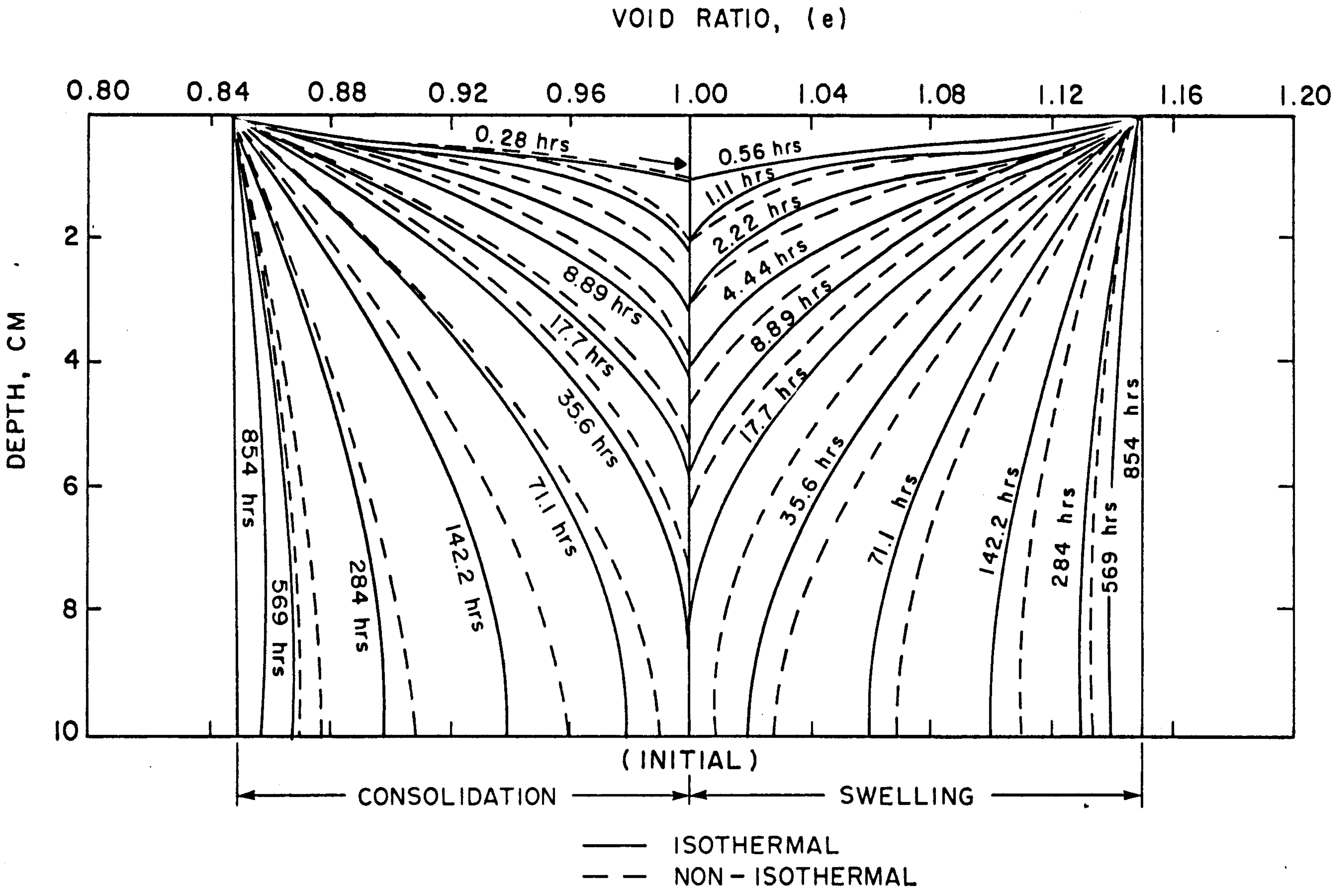

FIGURE 12 VOID RATIO CHANGE THROUGHOUT THE SOIL LAYER (EXAMPLE NO. 3 AND 4)

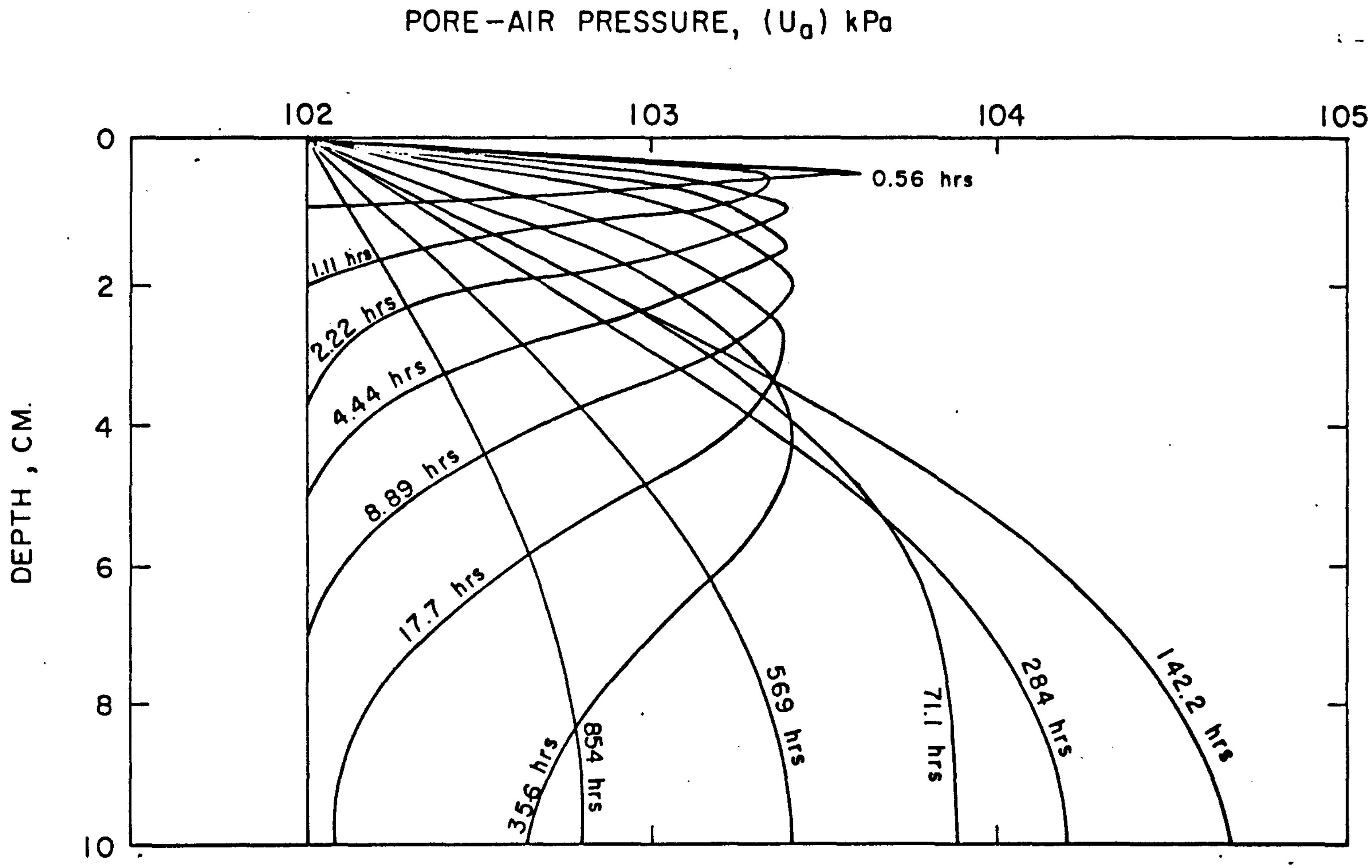

FIGURE 13 PORE-AIR PRESSURE DISTRIBUTION UNDER NON-ISOTHERMAL CONDITION

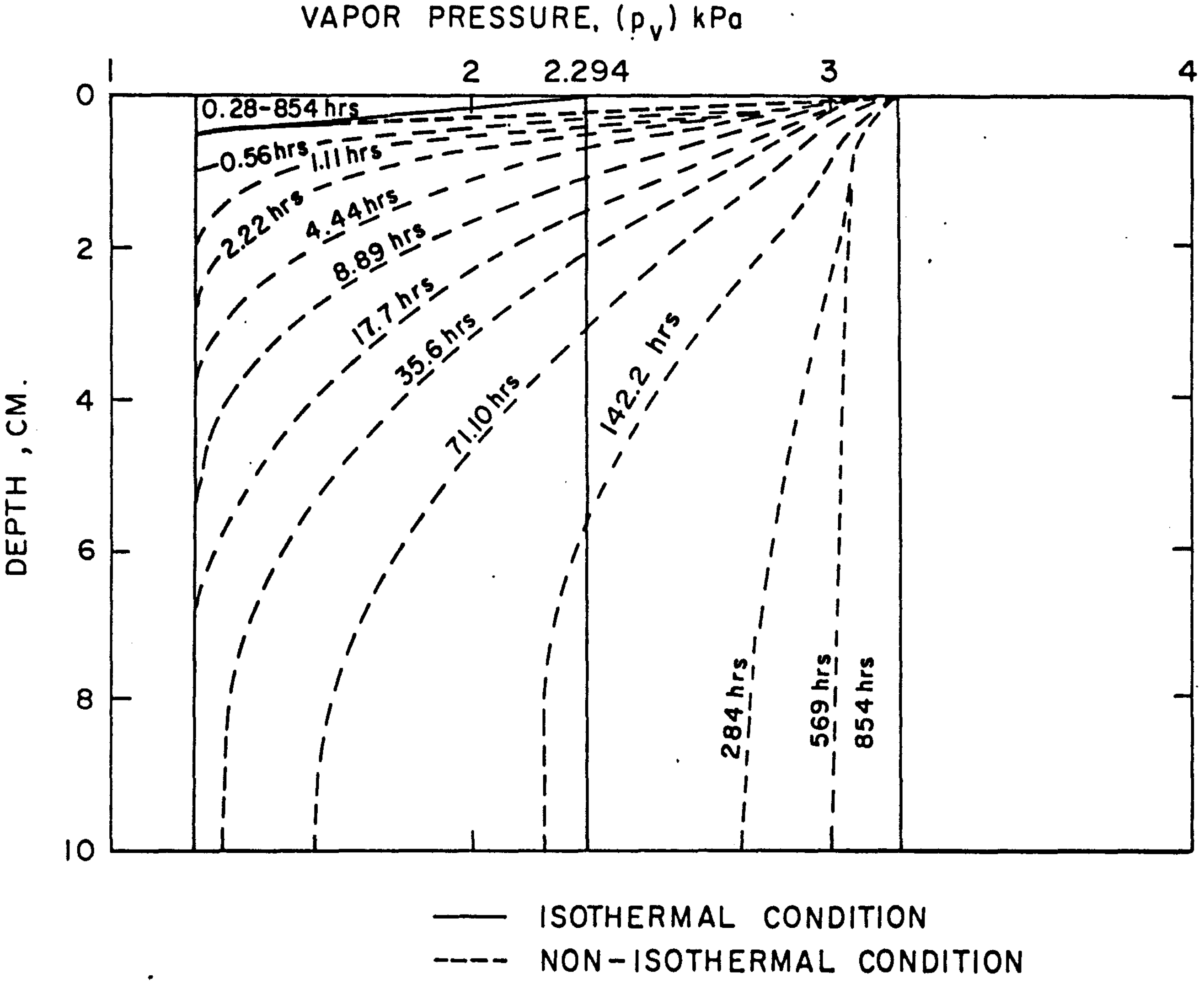

FIGURE 14 VAPOR-PRESSURE DISTRIBUTION FOR THE EXAMPLE NO. 3 AND 4

constant vapor pressure (i.e., 2.294 kPa) throughout the soil layer during the transient process under isothermal condition. The vapor pressure essentially remained constant when the pore-water pressure was reversed (Figure 9) as the result of infiltration.

Under non-isothermal conditions (even though the initial and final pore-water (liquid) pressure remains the same) the equilibrium relative humidity changes as the result of temperature changes. Consequently, the vapor pressure which was initially at 1.188 kPa (corresponding to 10° C) was changed to 3.156 kPa (corresponding to 25° C) at the surface. This vapor pressure difference is eventually dissipated throughout the soil layer during the transient process (Figure 14).

Figure 15 shows the partial pressure (i.e., total pressure minus vapor pressure) distribution throughout the soil layer under isothermal and non-isothermal conditions. A constant partial pressure (i.e., 99.706 kPa) value is obtained throughout the soil layer under isothermal conditions. Under non-isothermal conditions the partial pressure which was initially at 100.812 kPa, was instantaneously changed to 98.844 kPa at the boundary. This partial pressure difference is eventually dissipated throughout the soil layer during the transient process (Figure 15).

The analysis of the above example problems allows the distinguishing of the primary and secondary effects (i.e., significant and insignificant effects) as the result of complex environmental changes at the boundary. For example, the pore-water pressure change as the result of infiltration or evaporation cause a primary or significant change in volume-weight properties of an unsaturated soil over a short period of time. On the other hand, the thermal gradients and relative humidity gradients

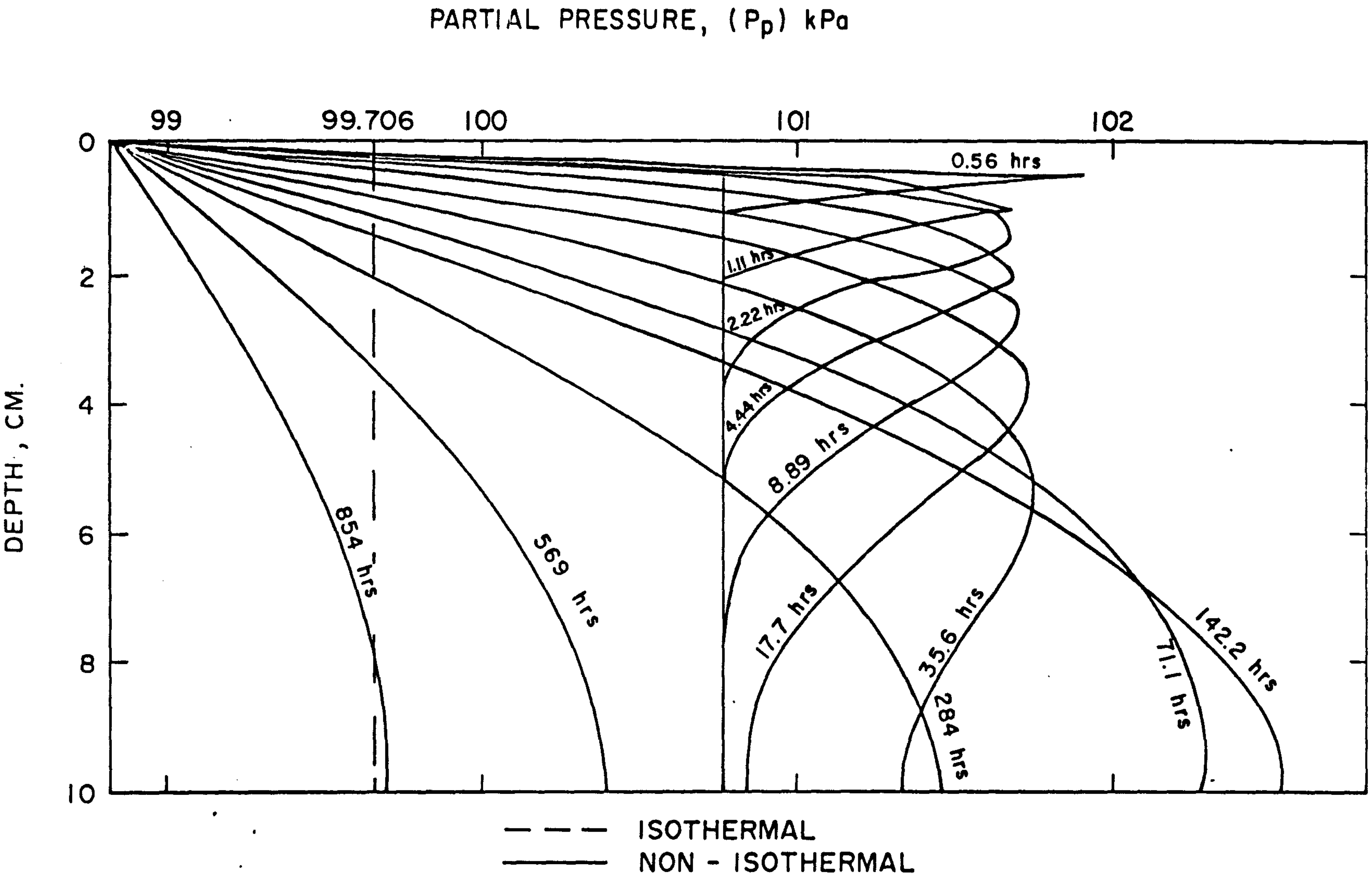

FIGURE 15 PARTIAL PRESSURE DISTRIBUTION FOR THE EXAMPLE NO. 3 AND 4

imposed at the surface cause a secondary effect on volume-weight properties of the soil over a long period of time. This paper presents a rigorous theoretical analysis which considers both primary and secondary environmental effects. The practicing engineer must be able to identify the primary effects of the complex environmental changes at the boundary and simplify the design procedure so as to predict reliable and realistic ground movements and related volume-weight properties for an unsaturated soil.

SUMMARY

All the necessary physical relations are available to formulate a rigorous theoretical model to describe the transient flow process under combined hydraulic, relative humidity, and thermal gradients in an unsaturated soil. The heat flow equation is first solved using a forward finite difference technique. Then two partial differential equations (i.e., one for the water phase and the other for the air phase) are solved simultaneously. Families of curves show temperature, pore-water pressure, pore-air pressure, vapor pressure, partial pressure, moisture content, degree of saturation, and void ratio distribution throughout the soil layer as a result of combined boundary condition changes.

The distribution of moisture content and overall volume change enables the prediction of all volume-weight properties of the soil throughout the transient process. The model shows good promise for describing the behavior of an unsatusrated soil systems under highly complex environmental changes. As well, this model allows the assessment of the primary and secondary effects of complex environmental changes. The distinction between primary and secondary effects should assist practicing engineers in producing realistic design procedures.

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LIST OF FIGURES